Public Listing Choice with Persistent Hidden Information

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PRELIMINARY AND INCOMPLETE

Abstract

The propensity of U.S. firms to be listed on public stock exchanges has been in steady decline since the late 1990s, while firm intangibility has increased. We build a model of firm contracting with competition between private and public financiers. We assume managers have private information over intangible cash-flows. Persistence in these cash-flows implies the optimal contract of public financiers induces excessive volatility but higher growth in compensation. Private, specialist financiers can avoid the information friction yielding savings in compensation. Limited funding of private financiers induces a selection of higher volatility, more intangible firms being privately funded. We test these predictions on a large sample of public and private firms. Our results suggest that the changing composition of firms may be driving both the decline in publicly listed firms and the growth in public CEO compensation.

Keywords: Stock market listing, intangible capital, CEO compensation, optimal contracts, persistent private information

JEL: E02, G24, D21, D22

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1 Introduction

The number of U.S. firms listed on a public stock exchange has been in decline since the mid/late '90s and is now below the level of the '70s (see Kahle and Stulz (2017)). At the same time, the cumulative value of private U.S. firms with more than \$1 billion in valuation (so-called 'unicorns') has ballooned from below \$20 billion across fewer than 10 firms in 2006 to over \$500 billion across 138 firms in 2018 (source: Pitchbook). This suggests the relative benefits of having access to U.S. public equity markets have declined over time. Finally, U.S. firms have become increasingly R&D intensive and have valuations less tied to tangible assets, while CEO compensation has grown but also become increasingly volatile (Corrado and Hulten (2010), Edmans et al. (2017)).

We construct a model of optimal firm contracting with external financiers in public and private markets that rationalizes these three dynamics. Furthermore, we empirically test the model's additional predictions on data of both private and public firms.

In the model, firms can elect to contract with either a representative shareholder in public equity markets or a private equity specialist in order to obtain additional capital. Firm managers have private information over the realizations of their productivity and hence have the opportunity to divert cash-flow from the project. In light of this persistent hidden information, the optimal contract will induce excessive volatility and higher consumption growth for the manager to incentivize truth-telling. Private equity specialists are assumed, due to their subject-matter expertise, to have access to a monitoring technology. This allows them to observe the realized productivity path of the manager. Consequently, they can obtain higher returns from financing a given firm than the non-specialized public shareholders. Unlike the deep-pocketed shareholders, the private equity specialists are subject to a budget constraint and hence must restrict the sample of firms to which they provide financing.

In equilibrium, firms with (i) a larger share of revenue coming from the private component of cash-flows (i.e. more intangible), (ii) higher cash-flow volatility, and/or (iii) greater persistence in their hidden productivity evolution process will be funded by private equity specialists. Further, for low levels of intangibility, volatility and/or idiosyncratic productivity shocks, all funding will occur via the public equity market. Finally, higher drift and volatility in CEO compensation will be associated with higher persistence in the private component of the productivity process as well as a larger weight of intangible capital on total revenues.

We use data collected on both private and public firm characteristics obtained from Capital IQ between 1993 and 2015 to empirically validate our model predictions. Our results suggest that SOX is not the fundamental driver of this decline, but rather fundamentals in firm characteristics consistent with arguments by Corrado and Hulten (2010) about the rise in intangibles.

Our model adapts the optimal contracting, continuous time framework in Williams (2011) into an equilibrium setting with heterogeneous, competing principals and a continuum of firms. This contracting framework allows for a tractable analysis of moral hazard in the presence of persistent hidden information. DeMarzo and Sannikov (2006) formulate the model with hidden action, but the source of private information in their setting are increments of a Brownian motion and hence i.i.d., while in our case, the hidden information process itself is an Ornstein-Uhlenbeck process (a continuous time AR(1) process) and hence persistent.

Our paper is thematically close to Holmstrom and Tirole (1997) who study the endogenous sorting of firms with heterogeneously informed financiers and the implications for firms' sensitivity to aggregate credit shocks. In contrast to their model in which firms only differ based on their own net worth, we focus on firm heterogeneity based on the volatility and persistence of their productivity (and hence cash-flow) process. This difference is motivated by Doidge et al. (2017), who find that the propensity for U.S. firms to be listed has declined across all firm sizes.

The puzzle of the declining number of U.S. publicly listed firms has been widely documented. Gao et al. (2013b) show that U.S. IPOs have collapsed, with annual IPOs dropping by roughly two-thirds in the 2000s compared to the previous two decades. From a different angle, Doidge et al. (2017) compare the listings of firms across countries and show that the number of U.S. listings is much lower than predicted relative to a comparison group of countries. Kartashova (2014) documents that the private equity premium has become positive and increased substantially since Moskowitz and Vissing-Jorgensen (2002). Altogether, the evidence seems to suggest that the relative benefits of being publicly listed has deteriorated.

Central to our argument is that asymmetry between information of management and shareholders has grown due to fundamental differences in firms' production technology, resulting in public contracts which induce excessively high volatility to managerial compensation. Kahle and Stulz (2017) document that the decline is associated with the increasing importance of R&D expenditures amongst public firms. Corrado and Hulten (2010) provide some evidence that intangible assets have become increasingly prevalent and necessary for a firm's success. We argue these assets are more difficult for outside investors to assess and hence balance sheet reports offer less information on future cash-flows for shareholders. As a consequence, our model suggests the contract will change to account for the increased agency concerns. Indeed, Frydman and Saks (2010) report the increase of the stock and stock option portions of the

CEO compensation, which, together with the increase in idiosyncratic volatility of firms documented by Campbell et al. (2001), implies an increase of the volatility of public CEO compensation.

We also contribute to the CEO compensation literature, where concern has grown over the vast growth in public CEO compensation. In a recent survey of CEO compensation, Edmans et al. (2017) state, "much can be learned from papers that do not attempt to identify causal effects and instead carefully study how firms endogenously choose compensation contracts in different settings." Our paper falls precisely in this category. Within our model, compensation structures are endogenously determined based on fundamental firm characteristics and the selection of financiers. Consistent with the work of Gao and Li (2015), our model gives rise to lower performance sensitivity of private firms' CEO compensation. It addresses both the documented rise in the level and volatility of CEO compensation derived from increasing performance pay, as well as CEO turnover. Finally, it also helps rationalize the association found in Glover and Levine (2017) that the size of agency issues implied by compensation contracts observed correlate strongly with firm intangibility.

Finally, we also contribute to the empirical characterization of the cross-sectional differences of private and public firms and how these differences have evolved over time. To our knowledge, relatively little work has examined these differences across public/private firms due to the historic dearth of data on private firms. Notable exceptions include Gao et al. (2013a), Gao and Li (2015) and Acharya and Xu (2017), who all use Capital IQ data to contrast the behaviour of a large sample of public and private firms with respect to cash, CEO compensation and innovation respectively. We use the same data source, but contrast firm intangibility, R&D intensity and cash-flow volatility, along with the associated CEO compensation structure.

2 Financing with persistent hidden information

We start by outlining the core model wherein a representative public market financier and private market financier compete to provide funds to a unit measure of heterogeneous entrepreneurs. From this partial equilibrium model we obtain static predictions on firm sorting into public and private based on project characteristics. In a later section, we will extend the model into a full dynamic, general equilibrium economy to endogenize the supply of funding and obtain additional predictions on wealth inequality, dynamics of the private equity premium.

2.1 Environment

Time is continuous and runs forever. There is a unit measure of entrepreneurs and two representative financiers: a public market financier and private market financier. Each entrepreneur is endowed with a project and assumed to have decreasing curvature over wealth, and so has preference over consumption $u(c)$ which is strictly increasing, concave and $u''' > 0$ (consistent for instance with either CARA or DARA utility). At time-0 each entrepreneur needs a capital injection k at price p_0^k in order to start-up the project. The two representative financiers (principals) on the other hand are risk-neutral $v(c) = c$ and are each endowed with a supply of capital. Assuming that the entrepreneurs' have internal funds $M_0 < p_0^k k$, they need to exchange some of the future cash-flows of the project for initial financing. Private information held by the entrepreneur about the realized cash flows gives rise to a contracting problem between the financier and entrepreneur (principal-agent problem). The two principals compete with each other to fund the pool of projects subject to their heterogeneous capital endowments and monitoring technologies.

The project of a given entrepreneur is indexed by $\theta = (\tau, \mu(\cdot), \mu^x(\cdot), \sigma, \sigma^x)$ where $\theta \sim G(\cdot)$ which for a given level of capital k yields cash flows at time t of

$$y_t = \underbrace{[(1-\tau)z_t + \tau x_t]}_{\text{TFP}} f(k).$$

Here $f(k)$ is a standard neoclassical production function and the TFP process $[(1-\tau)z_t + \tau x_t]$ is decomposed between a tangible component, x_t, which follows some publicly observable diffusion process

$$dx_t = \mu^x(x)dt + \sigma^x dW_t^x$$

and an intangible component, z_t, which is directly observable only by the entrepreneur and follows the diffusion

$$dz_t = \mu(z_t)dt + \sigma dW_t$$

where $\mu^x(x) = \mu_0^x - \frac{x_t}{\lambda^x}$ and $\mu(z) = \mu_0 - \frac{x_t}{\lambda}$ with initial TFP levels $x_0 = \mu_0^x \lambda^x$ and $z_0 = \mu_0 \lambda$ both publicly known.

With λ finite, z_t is an Ornstein-Uhlenbeck process - the continuous time equivalent of a stationary AR(1). Notice λ here governs the persistence of the process of z_t, so $\lambda \to 0$ implies that innovations to z_t dissipate immediately, while $\lambda \to \infty$ implies innovations to z_t are permanent.[1] Finally, the parameter

[1] With the appropriate scaling of $\sigma = \sigma\sqrt{\lambda}$, the limit of $\lambda \to 0$ approximates an idiosyncratic productivity shock (i.e. an iid process). In this case entrepreneur's private information is perfectly transient.

$\tau \in [0,1]$ captures the degree of tangibility of the firm, with $\tau = 1$ implying the firm is entirely tangible and $\tau = 0$ the firm has no tangible TFP. Thus, the amount of private information held by the entrepreneur decreases in τ and increases in λ and σ.

Both principals have access to funds to finance projects and compete with each other by posting a contract for each entrepreneur. The public financier, denoted P for public investor, has deep-pockets and so can fund all available projects but has no monitoring technology, hence must take into account the agency frictions when designing the offered contract. The private financier, denoted S for specialist investor, on the other hand has a monitoring technology incurring fixed cost ν and fixed pool of funds B^S.

At time 0, both financiers make bids of promised utility contingent on the amount of funding required f and project characteristics ($\theta = (\tau, \mu(\cdot), \mu^x(\cdot), \sigma, \sigma^x)$). Denote the bids $b_0^P(f,\theta), b_0^S(f,\theta)$ for the public financier and specialist financier respectively. Observing the offered contracts, a given entrepreneur can choose amongst the two contracts or to go it alone (i.e. have their scale limited to M_0) yielding payoff $V^A(M_0, \theta)$. Once in a contract with the financier, the entrepreneur reports some fraction of the realized cash-flows at each instant time and receives compensation from the financier based on the reported cash flows. Projects stochastically die with the same Poisson arrival rate η.

In the next subsection, subsection 3.2, we outline the reporting problem for the entrepreneur and define a contract in this setting. In subsection 3.3 we setup the contracting problem for the public financier. In subsection 3.4 we do the same for the private, specialist financier. Finally, we describe the financier bidding and entrepreneur contract selection problems in subsection 3.5.

2.2 Entrepreneur's Reporting Problem

After the point of financing at time 0, for the time horizon that the project is active, for now suppose fixed from $[0,T]$ (but which we will extend later to $T \to \infty$), the cash-flows of the project $y_t = [\tau x_t + (1-\tau)z_t]f(k)$ are realized by the entrepreneur. As was stated above, while x_t is observed by both the financier and the entrepreneur, the realizations of z_t are not directly observed by the financier and hence total realized cash-flows is private information of the entrepreneur. Assuming the entrepreneur must hand-over any reported cash-flows before receiving compensation by the financier in the contract, the entrepreneur has the ability to under-report and consume any un-reported cash-flows.

In general, given the two components of the cash-flows, the entrepreneur's reporting strategy is $y : \mathbb{C}[0,T] \times \mathbb{C}[0,T] \to \mathbb{C}[0,T]$. The financier with whom the given entrepreneur has contracted with, assuming no monitoring technology is used, observes the realized tangible component of cash-flows $\{x_s f(k)\}$ and the total cash-flow report $\{y_v\}$, $\{x_s f(k), y_s : s \leq t\}$ and provides compensation c subject to the realized/reported cash-flows, $c : \mathbb{C}[0,T] \times \mathbb{C}[0,T] \to \mathbb{C}[0,T]$, $c_t(\{x_s, y_s : s \leq t\})$. Since the project characteristics θ and initial conditions (set at the long-run means) are commonly known, given $\{x_s f(k), y_s : s \leq t\}$ the principal can back out the intangible cash-flow report, $\tilde{y}_t = y_t - \tau x_t f(k) = (1-\tau) z_t f(k)$. Thus, this residual component of the cash-flows is the component of cash-flows subject to potential mis-reports.

The commonly known parameters of the cash-flow processes combined with the continuous reports by the entrepreneur imply that the principal can back out the reported increments of the intangible cash-flow process:

$$dW_t^{\tilde{y}} = \frac{d\tilde{y}_t - (1-\tau) f(k) \mu\big(\frac{\tilde{y}_t}{(1-\tau)f(k)}\big) dt}{(1-\tau) f(k) \sigma}.$$

Further, with this information, any discontinuous jumps or other deviations in the instantaneous volatility of the reported cash-flows from those given by the true processes x^0, z^0 can be immediately detected and punished by the financier.[2] Consequently, feasible mis-reports by the entrepreneur can affect only the drift not the volatility of the reported cash-flow process relative to the true cash-flows.

Let $\Delta_t \leq 0$ denote the size of under-report to the drift of the intangible cash-flow process z_t at time t. Since the financier has perfect recall, the entrepreneur must track the history of lies $m_t = \int_0^t \Delta_s ds$. Thus, feasible residual reporting processes of the entrepreneur follow the evolution,

$$d\tilde{y}_t = (1-\tau) f(k) \left[\mu\big(\frac{\tilde{y}_t}{(1-\tau) f(k)} - m_t \big) + \Delta \right] dt + (1-\tau) f(k) \sigma dW_t.$$

Fix a prior reporting strategy for the entrepreneur by the financier such that $\tilde{y}_t$ follows a martingale $d\tilde{y}_t = (1-\tau) f(k) \sigma dW_t^0$. Then taking a given path of the diversion drift Δ by the entrepreneur relative to this prior, we can define the likelihood process of the cash-flows following the prior report strategy of the entrepreneur $\frac{dP_\Delta}{dP_0} = \Gamma_T(\Delta)$.[3] Since there is a one-to-one mapping of $\tilde{y}$ to z, we will from hereon track z_t, rather than $\tilde{y}_t$.

Given a prior and contracted payment process $c(\bar{x}, \bar{z})$, diversion process Δ and evolution of tangible and intangible cash-flows $\bar{x}, \bar{z}$, the discounted life-time

[2] For simplicity, we assume that the financier has unbounded punishments available in the case of provable mis-reports. This can be thought of firm boards/shareholders can litigate CEOs who breach contracts.

[3] For the exact form and details on this change of variables see the appendix.

value of the entrepreneur within the contract is

$$V(\bar{x},\bar{z};c,\Delta)=E_0^{\Delta}\left[\int_0^T e^{-\rho t}u(c_t-m_t^y)dt+e^{-\rho T}U(c_T-m_T^y)\right].^4$$

The entrepreneur therefore solves the following reporting problem

Problem 1: <u>Entrepreneur Reporting Problem:</u>

$$\max_{\Delta_s\le 0} V(\bar{x},\bar{z};c,\Delta) \tag{1}$$

s/t

$$dz_t=[\mu(z_t-m_t)+\Delta_t]dt+\sigma dW_t \tag{2}$$

$$dm_t=\Delta_t dt \tag{3}$$

$$m_t^y=(1-\tau)f(k)m_t dt \tag{4}$$

$$y_t=\tau x_t+(1-\tau)z_t$$

$$dx_t=\mu^x(x_t)+\sigma^x dW_t^x.$$

To make this problem tractable and avoid having history dependence in the decision rules of the entrepreneur, we follow DeMarzo and Sannikov (2006) and Williams (2011) and use a change of variables so that rather than taking $(\{x_s,z_s,m_s:s\le t\}$ as the state variables the entrepreneur controls the belief process of the financier that the entrepreneur is reporting according to his prior, that is the likelihood process $\frac{dP_\Delta}{dP_0}=\Gamma_T(\Delta)$ over the possible intangible cash-flow sample paths $(\bar{z})$, so that the relevant state variables are (Γ_t,b_t,x_t) where $b_t=\Gamma_t m_t$. With this change of variables, the entrepreneur's reporting problem then boils down to:

Problem 1' - <u>Entrepreneur's Transformed Reporting Problem</u>

$$\max_{\Delta_s\le 0} V(\bar{x},\bar{z};c,\Delta)=E_0^0\left[\int_0^T \Gamma_t e^{-\rho t}u(c_t(\bar{x},\bar{z},m_t^y)-m_t^y)dt+\Gamma_T e^{-\rho T}U(c_T(\bar{x},\bar{z},m_T^y)-m_T^y)\right]$$

subject to

$$d\Gamma_t=\frac{\Gamma_t}{\sigma}[\mu(z_t-m_t)+\Delta_t]dW_t^0 \tag{5}$$

$$db_t=\Gamma_t\Delta_t dt+\frac{b_t}{\sigma}[\mu(z_t-m_t)+\Delta_t]dW_t^0 \tag{6}$$

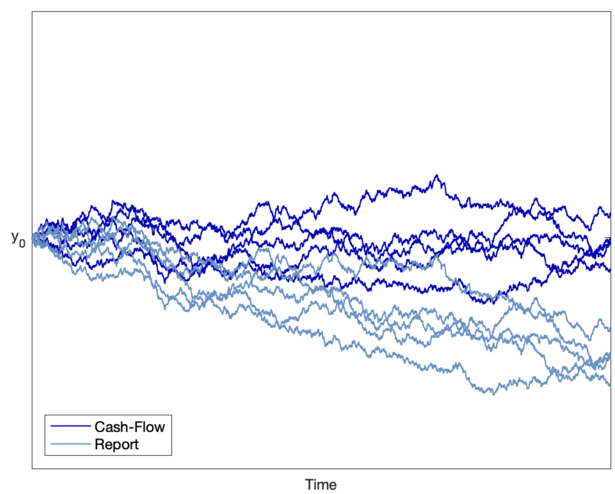
$$b_t=\Gamma_t m_t$$

[4] The limit of this finite horizon problem will be taken in the following section as in Williams (2011).

$$m_t^y = (1-\tau) f(k) m_t dt.$$

The advantage of this transformation is that we no longer need to condition on the whole history of the sample path of z_t or x_t in the agent's choice of report each instant but instead chooses the global diffusion of information to the principal as a function of the evolution of the principal's posterior. As opposed to Williams (2011) we have a second cash-flow history of x_t that the compensation depends on, however, since these cash-flows are directly observed/transferred to the financier, the reporting problem is unaffected beyond the effect on compensation of its history. Further, it will be shown that, given the risk-aversion of the entrepreneur and risk-neutrality/deep pockets of the financier, the optimal compensation contract is independent of the realizations of x_t (ie financier provides full insurance on the compensation of x_t) and so the realizations of x_t do not affect the reporting incentives of the entrepreneur in any way. We depict in Figure 1 some sample paths of the entrepreneur and the observed paths of reports by the financier based on a constant diversion Δ.

Figure 1: Simulated distribution of cash-flows and reports under constant diffusion



The above figure illustrates how the distribution of realized reports is biased downwards due to lies of the entrepreneur. The simulated paths of realized cash-flows follow an Ohrnstein-Uhlenbeck process are depicted in dark blue, while the taupe paths are the corresponding cash-flow reports given to the financier by the entrepreneur assuming a constant diversion process Δ. As can be seen, the diversion process, restricted to under-reports, biases downwards the drift of the real cash-flows. Reports which change the volatility of the cash-flow process are not feasible, since they are instantly detectable by the financier.

2.3 Contracting Problem for Public Financier (P)

The public financier, lacking a monitoring technology, must take the optimal decision-rules (from Problem 1') of the entrepreneurs reporting of the private component of the cash-flows given the compensation contract they provide to the entrepreneurs contingent on their reports. By the revelation principle, we can restrict to contracts which induce truth-telling so that $\Delta_t = 0$ is optimal for all t. Appealing to a stochastic maximum principle (see Bismut (1978)), the current-value hamiltonian for the entrepreneur's reporting problem with co-states (q, γ) for state Γ and (p, Q) for state b is

$$\mathcal{H}(\Gamma, b) = \Gamma u(c - (1-\tau)f(k)[\frac{b}{\Gamma}]) + (\Gamma\gamma + Qb)[\mu(z - \frac{b}{\Gamma}) + \Delta] + p\Gamma\Delta \tag{7}$$

where q and p adjust the drift of the states while γ, Q adjust the noise. The local optimality condition from this maximum principle for the diversion under truth-telling is

$$\Gamma\left[p + \gamma + Qm\right] \geq 0.$$

In other words, due to the negativity constraint on the mis-reports, the financier will distort the local incentives of the entrepreneur so that reporting the maximum possible at each instant is optimal. With truth-telling optimal at each instant prior to the current period, then $m = 0$ hence the incentive constraint for truth-telling reduces to

$$\Gamma\left[p + \gamma\right] \geq 0. \tag{8}$$

Finally, the stochastic maximum principle also pins down the evolution of the co-states as:

$$dq_t = [\rho q_t - \frac{\partial H}{\partial \Gamma}]dt + \gamma_t \sigma dW_t^0$$
$$dp_t = [\rho p_t - \frac{\partial H}{\partial b}]dt + Q_t \sigma dW_t^0$$

with terminal conditions $q_T = U(c_T)$, $p_T = -(1-\tau)f(k)U'(c_T)$.

With the imposition of truthful revelation we have $\frac{\partial H}{\partial \Gamma} = u + \gamma\mu$ and $\frac{\partial H}{\partial b} = (1-\tau)f(k)u' - \gamma\mu' + Q\mu$, and so (using the change of variables) these process simplify to:

$$dq_t = [\rho q_t - u(c_t)]dt + \gamma_t \sigma dW_t \tag{9}$$
$$dp_t = [\rho p_t + (1-\tau)f(k)u'(c_t) - \lambda\gamma_t]dt + Q_t \sigma dW_t \tag{10}$$

Thus, we have that individually rational truthful revelation by the entrepreneur of the TFP process for given diffusion processes γ_t, Q_t and transfers c_t is characterized by processes q_t, p_t as given in (9) - (10). From (9) we have that the co-state q_t is in fact a promised utility process:

$$q_t = E_t^* \left[\int_0^T e^{-\rho(\tau - t)} u(c_\tau) d\tau + e^{-\rho(T-t)} U(s_T) \right] \tag{11}$$

and, if the IC constraint binds almost everywhere (which will be later verified), $\gamma_t = -p_t$ and so p_t is the negative of the agent's marginal utility process with discounting adjusted by the persistence of the private information λ:

$$p_t = -(1-\tau) f(k) E_t^* \left[\int_0^T e^{-(\rho+\lambda)(\tilde{t}-t)} u'(c_\tau) d\tilde{t} + e^{-(\rho+\lambda)(T-t)} U'(c_T) \right]. \tag{12}$$

Equipped with the local and dynamic incentive conditions for truth-telling given in (8), (9) and (10), the financier can tune the contract using the volatility coefficients on the promised utility and marginal utility (Q_t, γ_t, c_t) as well as the initial conditions of marginal and promised utility p_0, q_0 resp. to maximize their share of the compensation within the contract, $y_t - c_t$. In other words, for a given p_0, q_0, with the necessary contract characteristics to induce truth-telling, the principal solves:

Problem 2 - Public financier contracting problem:

$$J^P(p_0, q_0, z_0, x_0) = \max_{\gamma \geq -p, s, Q} E_0 [\int_0^\infty e^{-\rho t} (y_t - c_t) dt] \tag{13}$$

s/t

$$y_t = (1-\tau) f(k) z_t + \tau f(k) x_t$$

$$dz_t = \mu(z_t) dt + \sigma dW_t$$

$$dx_t = \mu^x(x_t) dt + \sigma^x dW_t^x$$

$$dq_t = [\rho q_t - u(c_t)] dt + \gamma_t \sigma dW_t^* \tag{14}$$

$$dp_t = [\rho p_t + (1-\tau) f(k) u'(c_t) - \lambda^z \gamma_t] dt + Q_t \sigma dW_t, \tag{15}$$

where we have taken $T \to \infty$ and replaced the finite terminal conditions with the transversality conditions $\lim_{T\to\infty} e^{-\tilde{\rho}T} q_T = \lim_{T\to\infty} e^{-\tilde{\rho}T} p_T = 0$.

Given a solution to the above problem, the principal then solves for the optimal initial promised marginal utility $-p_0$:

$$\max_{p_0 \leq 0} J(p_0, q_0, y_0, x_0).$$

2.4 Contracting Problem for Specialist Financier (S)

The above problem applies to a principal facing the full set of information frictions. Now, the specialist financier, endowed with the costly monitoring technology (per project cost ν), doesn't need to worry about satisfying the incentive compatibility (IC) constraint and thus evidently sets $Q = 0$, $p_0 = 0$ which reduces to:

$$J^S(q_0, z_0, x_0) = \max_{\gamma \geq 0, s} E_0[\int_0^\infty e^{-\rho t}(y_t - c_t)dt] - \nu \tag{16}$$

s/t

$$y_t = (1 - \tau)f(k)z_t + \tau f(k)x_t$$
$$dz_t = \mu(z_t)dt + \sigma dW_t$$
$$dx_t = \mu^x(x_t)dt + \sigma^x dW_t^x$$

$$dq_t = [\rho q_t - u(c_t)]dt + \gamma_t \sigma dW_t.$$

Comparing the public financiers contracting problem from the previous subsection and the private specialist's problem above, you can see that the specialist financier faces a relaxed version of the public financiers problem, since the private specialist doesn't have to ensure incentive compatibility, and must only satisfy promise-keeping dq_t. Since their problem is a relaxed version of the public financiers, it is obvious that the specialist financier (ignoring the sunk costs of monitoring) will be able to extract weakly higher surplus from a given entrepreneur under the optimal contract relative to the public financier.

Although potentially obfuscated by the math, this comparative advantage of the specialist financier in the contracting problem depends crucially on the loading on z_t in the cash-flow process. If the project is fully tangible ($\tau = 1$) so that cash-flows perfectly track the publicly observable component of firm TFP x_t then as can be seen from the solved marginal utility process (12) becomes identically zero everywhere even for the public financier. In this case the two contracting problems are of course equivalent.

2.5 Financing Competition

At time 0 before a contract has begun, financiers compete with each other for entrepreneurs projects by offering optimal contracts (conditional on project type θ) with different levels of initially promised utility of the entrepreneur, q_0, from the duration of the contract. Entrepreneurs observing a pair of contracts, one from each financier, chooses whichever contract yields highest payoff.

Both financiers thus solve the following project bidding problem, given the bidding strategy of their opponent financier q_0^{-f}, and entrepreneur listing choice selection rule, $i^f(q_0^f, q_0^{-f}, M_0; \theta)$ and the financier's own budget constraint B^f

$$W_0^f = \max_{q_0(\theta)} \int_\theta [J^f(p_0(\theta), q_0(\theta), z_0(\theta), x_0(\theta); \theta) - \left(p_0^k k(\theta) - M_0(\theta)\right)] i^f(q_0^f, q_0^{-f}; \theta) dG(\theta) \tag{17}$$

s/t

$$q_0(\theta) \geq V^A(\theta, M_0)$$

$$\int [p_0^k k(\theta) - M_0(\theta)] i^f(q_0^f, q_0^{-f}; \theta) dG(\theta) \leq B^f$$

where in the case of the public financier $B_P \to \infty$ (reflecting deep pockets) while the specialist financier has $B^S < \infty$.

2.6 Financing Equilibrium Definition

Given individual firm cash holdings M_0, price of initial capital p_0^k, financial resources of specialist financiers, B and monitoring cost ν, a public listing equilibrium consists of (i) contracts $(s, \gamma, Q, q_0, p_0)^f$, $f \in \{S, P\}$ yielding ex-ante promised utility q_0 to the entrepreneur with diversion process $\Delta = 0$, (ii) financiers bidding rules $q_0^P(\theta), q_0^S(\theta)$ and (iv) entrepreneur financier selection rules, $i^S(q_0^S, q_0^P; \theta, M_0)$, $i^P(q_0^P, q_0^S; \theta, M_0)$ indicating which (if any) of the principal's offered contracts to choose given project θ and internal financing M_0 such that:

1. Contracts $(c, \gamma, Q, p_0)^j$ offered induce truth-telling and are optimal principal-agent contracts given principal's information
   (a) $(c, \gamma, Q, p_0)^P$ solves (13) yielding $J^P(q_0; \theta)$
   (b) $(c, \gamma, Q, p_0)^S$ solves (16) yielding $J^S(q_0; \theta)$
2. Financing allocation is a sub-game perfect Nash equilibrium where given the type-contingent contracts above
   (a) Given beliefs about the specialist financier's bidding strategy $b^S(\theta) = q_0^S(\theta)$, and entrepreneur's financier choice rules $i^P(q_0^P, q_0^P; \theta, M_0)$, Public financier value from contract $J^P(\cdot)$ and funding B_P, P chooses bidding strategy $b^P(\theta) = q_0^P(\theta)$ for each project that is a best-response, i.e. solving (17)
   (b) Given beliefs of $q_0^P(\theta)$, entrepreneurs' financier choice rules $i^S(q_0^G, q_0^S; \theta, M_0)$, value from contract $J^S(\cdot)$, financing demands $f(\theta, M_0) = p_0^k k(\theta) - M_0$, monitoring cost ν and budget constraint B^S, financier S chooses the bid $q_0^S(\theta)$ that is a best-response, i.e. solving (17)

(c) Entrepreneur's make financing choice $i^P(q_0^P, q_0^S; \theta, M_0), i^P(q_0^G, q_0^S; \theta, M_0) \in \{0,1\}$, $i^P(q_0^P, q_0^S; \theta, M_0) + i^S(q_0^S, q_0^P; \theta, M_0) \leq 1$ maximizing promised utility $q_0(\theta)$ subject to their outside option $V^A(M_0; \theta)$

(d) Financier's P, S and each entrepreneur θ's beliefs are consistent.

3 Model predictions

3.1 Solution overview

In this section, we solve the competitive financing model outlined above. We first solve for the optimal contracts offered by the public and private, specialist financier for a given entrepreneur of type θ and an initial level of promised utility under the contract q_0. This gives us predictions for compensation differences between public and private firms, as well as differences in the mean level and performance sensitivity amongst heterogeneous publicly listed firms. We then solve the financier's optimal bidding strategies on the pool of entrepreneurs $G(\theta)$ subject to their funding constraints, B, as well as the optimal financier selection rules for the entrepreneurs. This provides us with cross-sectional predictions over the types of firms which will be publicly or privately financed as well

To emphasize our core channel, we will make the following assumptions: (i) internal funds are homogenous across entrepreneurs, $M_0(\theta) = M_0$, (ii) the scale of all projects is the same $f(k) = 1$ for $k \geq \bar{k}$ and $f(k) = 0$ for $k < \bar{k}$) and by appropriate adjustment of the units of M_0 and capital, net financing is $p_k^0 k - M_0 = 1$ and (iii) the expected return $\mu_0^z = \mu_0^x = \mu_0$ are homogenous across projects, and $\lambda^x = \lambda \forall \theta$. With these assumptions firms do not differ in size or productivity, but simply differ in the degree of information frictions the project is subject to based on the observability of their cash flows (τ). Finally, while this model can be solved numerically for the general preferences of entrepreneurs stated in the environment, we will restrict attention to CARA utility $u(c) = -\exp(-\psi c)$ for the entrepreneurs' to obtain closed form solutions to facilitate intuition.[5]

3.2 Contracting results

With the simplifying assumptions listed above, the solution to the public financier's contracting follows a very similar approach to Williams (2011). We assume (and verify ex-post as in Williams (2011)) the IC constraint is binding at each point in time so that the volatility tuning parameter of promised utility

[5]Though other functional forms have not been explicitly solved, the broad strokes (testable implications) should be fairly robust to the preference specifications assuming the relative risk-aversion differences of the entrepreneur and principals is maintained.

γ is set equal to the promised level of marginal utility, $\gamma_t = -p_t$. leaving two free variables of the compensation to the entrepreneur c and the tuning parameter on the volatility of promised marginal utility Q_t , as well as the initial condition for promised marginal utility $-p_0$.

The optimal contract between a given entrepreneur of type θ and the public financier is summarized in the following theorem:

Theorem 1 (Optimal contract under public financier) *Let $\tilde{\rho} = \rho + \eta$ and define $\phi = \frac{\psi\tilde{\rho}}{\tilde{\rho}+\frac{1}{\lambda}}$. Under Assumptions (i) - (iii), and Assumption (iv) $u(c) = -\exp(-\psi c)$, the optimal contract for the public financier and project of type θ has the following characteristics:*

1. *Entrepreneur's compensation has a positive drift and positive sensitivity, ϕ, to realized (intangible component of) cash-flows*

$$c_t^P = \frac{-log(-\tilde{\rho}q_0^P)}{\psi} + \frac{((1-\tau)\sigma\phi)^2}{2\psi}t + \frac{(1-\tau)\sigma\phi}{\psi}W_t. \tag{18}$$

2. *Public financiers time-0 expected payoff from the contract is*

$$J^P(y_0, q_0^P; \theta) = (1-\tau)\left(\frac{z_0 + \frac{\mu_0}{\tilde{\rho}}}{\frac{1}{\lambda} + \tilde{\rho}}\right) + \tau\left(\frac{x_0 + \frac{\mu_0^x}{\tilde{\rho}}}{\frac{1}{\lambda^x} + \tilde{\rho}}\right) - \left[-\frac{log(-\tilde{\rho}q_0^P)}{\psi} + \frac{(1-\tau)^2\phi^2\sigma^2}{2\psi}\right]. \tag{19}$$

In other words, compensation of the entrepreneur follows a Brownian motion with positive drift to compensate the agent for the greater volatility in promised utility in the future. From here it is easy to see that the more risk-averse the agent (the higher is ψ) the higher is the level and drift component of consumption. Further, the larger the 'size' of private information (namely higher σ or λ or lower τ) the higher is the drift and variance the financier must pay the entrepreneur.

Observe that in this setting unlike in discrete time contracting models, there is no intertemporal wedge of the entrepreneur trading off lower consumption today for higher consumption tomorrow. Instead, the evolution of future utility and marginal utility are distorted based on the reports. As time goes by, to incentivize truth-telling, the public financier must promise increasingly larger shares of the pie.[6]

We now turn to the optimal contract between an entrepreneur with project θ and the fully-informed, specialist financier (S).

[6] Note here that over a sufficiently long time horizon the financier will have to actually subsidize the compensation of the manager from his own funds. However, with sufficiently high job-destruction arrival rate η, the probability of this occurring will be effectively zero.

Theorem 2 (Optimal contract with specialist financier) *Under Assumptions (i) - (iv), the optimal contract between an entrepreneur with project θ and the specialist financier (S) paying the monitoring cost ν has the properties that:*

1. *Entrepreneur's compensation is invariant to performance*

$$c_t^S = \frac{-log(-\tilde{\rho} q_0^S)}{\psi}.^{7} \tag{20}$$

2. *Private specialists payoff from the contract is*

$$J^S(x_0, z_0, q_0^S; \theta) = J^P(x_0, z_0, q_0^S; \theta) + \frac{(1-\tau)^2\sigma^2\psi}{2(\tilde{\rho} + \frac{1}{\lambda})^2} - \nu. \tag{21}$$

Comparing the compensation from the two contracts, for a given entrepreneur and level of initially promised utility q_0 (i.e. $q_0^P = q_0^S$) it is immediate that the expected compensation paid under the public contract is higher than the specialist for the same project:

$$\mathbb{E}_0[c_t^P - c_t^S] = \frac{((1-\tau)\sigma\phi)^2}{2\psi} t > 0;$$

however, the volatility of the public contract is much larger than the private contract. These results imply the compensation evolution under private specialist and public financiers depicted in Figure 2.

Since the entrepreneur's compensation comes out of the pocket of the financier, the value of a given project with the same level of initially promised utility to the public financier is lower than the specialist. Using the closed form expressions for the optimal contract and the derived form of the value function, the private specialist information premium

$$\pi(\theta, z_0, q_0) \equiv J^S(z_0, x_0, q_0; \theta) - J^P(z_0, x_0, q_0; \theta)$$

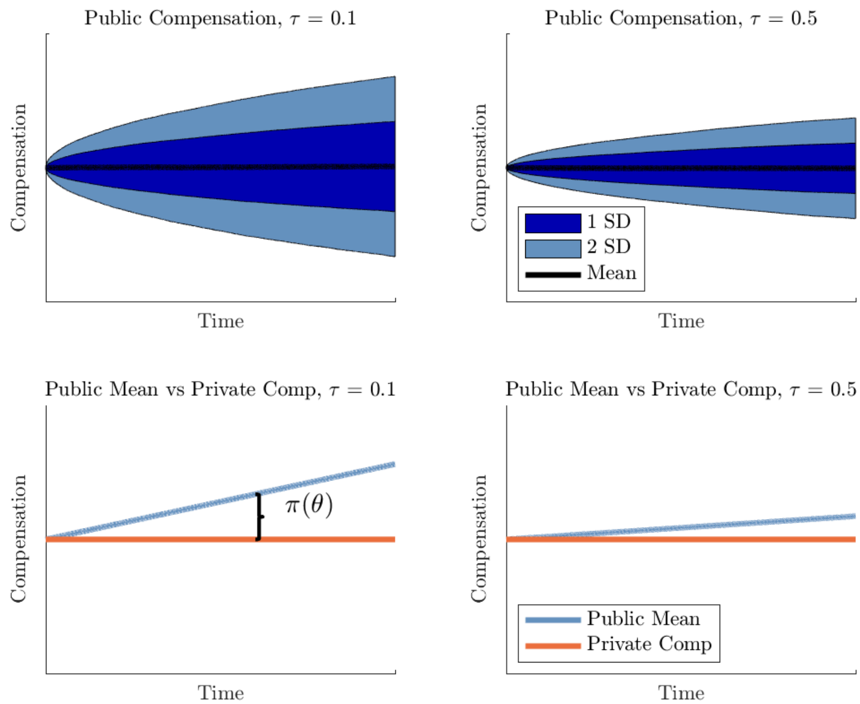
is given in the next corollary.

Corollary 2.1 (Private specialist information premium) *Under Assumptions (i) - (iv), the premium of the specialist financier of a contract with entrepreneur type θ over the public financiers value from the contract is:*

$$\pi(\theta, z_0, x_0, q_0, \nu) = \pi(\theta) - \nu \equiv \frac{(1-\tau)^2\sigma^2\psi}{2(\rho + \frac{1}{\lambda})^2} - \nu. \tag{22}$$

[7] Note that with $u(c)$ of negative exponential and q is promised utility then $q \leq 0$.

Figure 2: Optimal Compensation for the Manager with Public or Private Financier



The top two panels depict the distribution of consumption paths of the entrepreneur under the optimal contract with the public financier for low and high levels of tangibility respectively. The bottom two panels are the entrepreneur's consumption under the optimal contract of the private financier in red, contrasted with the mean compensation over time if contracted with the public financier. $\pi(\theta)$ is the wedge in the drift between the mean compensation of the public and private financier contracts (see equation 22 for the equilibrium expression.

Observe that with these assumptions on cash-flow and preferences, this information premium of specialist financiers, $\pi(\theta)$, is independent of the level of promised utility q_0. While this is certainly an artifact of the CARA preferences of the entrepreneur, the existence of an information premium, coming from cheaper compensation to the entrepreneur with full-information is not. The latter comes from the relaxation of a binding IC constraint (only preferences such that the IC constraint does not bind almost everywhere would imply no premium, but in this case, the agency friction is irrelevant). Also, notice that sufficient statistics for the optimal contract for both the specialist and public financier is given by θ and q_0.

Notice that the information premium that the specialist financier receives from avoiding the agency issues in the public contract is increasing in the volatility of the un-observable portion of the firms cash-flows σ, increasing in the persistence of private information $\frac{1}{\lambda}$ and in the degree of intangibility $(1-\tau)$. As will be made explicit in the next subsection, these comparative statics will drive the core listing predictions of the model.

3.3 Firm listing results

Equipped with the optimal contracts for the specialist and public financier for a given project type θ and initial promised utility q_0, we can now solve for the equilibrium sorting of projects financed by the specialist and the public. We first solve for the optimal decision rules for a given entrepreneur of type θ given the contracts solved for above. We then solve the optimal bidding rules for the two financiers.

First, given the optimal contracts described in (1) and (2), we know that promised utility q_0 is a sufficient statistic for the entrepreneurs selection rule. That is, the entrepreneur is indifferent between either contract despite the public financier contracting inducing more volatility, thus the entrepreneur's best-response is to accept the offer of the financier which gives the highest initial promised utility. It then follows directly that provided the promised utility from either contract weakly exceeds their value of autarky, $V^A(\theta, M_0)$. This result is summarized in the next lemma.

Lemma 1 (Entrepreneurs equilibrium financier selection rule) *Under Assumptions (i) - (iv) and assuming (v) homogenous outside options for* $V^A(\theta, M_0) = \bar{V}^A$, *the entrepreneur best response selection rule is*

$$i^S(q_0^S, q_0^P; \theta) = \begin{cases} 1 & q_0^S > q_0^P, \bar{V}^A \\ [0,1] & q_0^S = q_0^P \geq \bar{V}^A \\ 0 & q_0^S < \max\{q_0^P, \bar{V}^A\} \end{cases}$$

with $i^P = 1 - i^S$ *provided* $\max\{q_0^P, q_0^S\} \geq \bar{V}^A$ *and* $i^P = 0$ *otherwise.*

Both principals observe the distribution of entrepreneur's projects $dG(\theta)$ and make competing offers of initial promised utility q_0 to the entrepreneur for the duration of the contract. Recall that for a given offer of initial promised utility q_0 the entrepreneur is indifferent between either contract despite the public financier contracting inducing more volatility, thus the entrepreneur's best-response is to accept the offer of the financier which gives the highest initial promised utility.

Now turning to the financiers optimal bidding strategies, recall that both principals observe the distribution of entrepreneur's projects $dG(\theta)$ and make competing offers of initial promised utility q_0 to the entrepreneur for the duration of the contract. From the previous subsection, the information advantage of the specialist confers a positive information premium $\pi(\theta)$ in the contract for a given level of promised utility relative to the public financier, however, this information advantage comes at the per project fixed cost ν. Consequently, if the net specialist premium $\pi(\theta) - \nu$ is strictly positive, then the specialist can always outbid the public principal by offering promised utility q_0 that makes the public financier indifferent to financing the given entrepreneur.

Given the equilibrium contracts solved for in the previous subsection, it is always individually rational for public financier to bid on any project provided the value of the contract with promised utility of autarky to the entrepreneur net the cost of injecting capital is positive;

$$J^P(y_0, V^A; \theta) - 1 \geq 0. \tag{23}$$

Provided this condition is satisfied for all projects θ, the time-0 equilibrium financing selection decisions for the specialist and public financier can be boiled down to:

Theorem 3 (Equilibrium financing decisions) *Suppose Assumptions (i) - (v) hold, and also Assumption (vi), that all projects are individually rational for the public financier to finance (i.e. (23) holds for all θ). Then the equilibrium financing rules (bidding and a entrepreneur sorting) is characterized by*

1. *The private specialist financier solves*

$$W^S(B^S) = \max_{q_0^S(\cdot)} \int_\theta \underbrace{[\quad J^f(q_0^S(\theta); \theta) - 1 \quad]}_{\text{Principal payoff under contract net cost of financing}} i^S(q_0^S, q_0^P; \theta) dG(\theta) \tag{24}$$

s/t

$$\int_\theta i^S(q_0^S, q_0^P; \theta) dG(\theta) \leq B^S \qquad \textit{(Budget constraint)}$$

2. *the public financier makes bids according to:*

$$q_0^P = \begin{cases} \bar{V}^A & q_0^S \le \bar{V}^A \\ \inf\{q_0 : J^P(q_0;\theta) = 0\} & q_0^S > \bar{V}^A \end{cases} \tag{25}$$

3. *entrepreneur has equilibrium selection rule*

$$(i^S, i^P) = \begin{cases} (1,0) & q_0^S \ge q_0^P > \bar{V}^A \\ (0,1) & q_0^P \ge \bar{V}^A \ge q_0^S \\ (0,0) & \bar{V}^A > q_0^P, q_0^S. \end{cases} \tag{26}$$

First, by inspection of the objective function of (24) it is clear that a cutoff rule is optimal where for any $\pi(\theta) \ge \underline{\pi}$ the specialist will bid to fund the project and otherwise not. There are two cases. In the first case, the measure of projects where $\pi(\theta) - \nu \ge 0$ is too small for the budget constraint constraint to bind and so the optimal cutoff is $\underline{\pi} = \nu$ (since any lower cutoff would result in funding a negative NPV project for the specialist). In the second case, $\underline{\pi} > \nu$ and is pinned down implicitly by the binding budget constraint. Taking the cutoff level in this case $\underline{\pi}(B^S)$ as given, and using the closed form expression for $\pi(\theta)$ obtained in the contracting section, we get the following volatility cutoff for projects funded by the specialist:

$$\sigma \ge \underline{\sigma}(\lambda, \tau; \underline{\pi}) \equiv \frac{\sqrt{\frac{\underline{\pi}}{\tilde{\psi}}}(\rho + \frac{1}{\lambda})}{(1-\tau)} \tag{27}$$

where to reduce clutter $\tilde{\psi} = \frac{\psi}{2}$.

Notice that the volatility cutoff is downward sloping in the degree of persistence λ of the private information process, and increasing in the tangible share of cash-flows τ and profit cutoff $\underline{\pi}$, where (τ, λ, σ) above this threshold satisfy an indifference condition for the specialist, while those projects θ below this line the specialist will prefer not to fund.

The solution for $\underline{\pi}$ in this case is given implicitly by:

$$\int_0^1 \int_0^\infty \int_{\sigma \ge \underline{\sigma}(\tau,\lambda;\underline{\pi})} dG^\sigma(\sigma) dG^\lambda(\lambda) dG^\tau(\tau) \le B^S.$$

The above arguments combined yield our key result on firm sorting between financing by specialists and generalists (public financiers).[8]

[8] In a GE extension, we can endogenize the funds supplied to the private specialist by having households equipped with heterogeneous monitoring abilities which they provide as input to the private specialist firm.

Theorem 4 (Equilibrium firm sorting) *Under Assumptions (i) - (v), in equilibrium, firm financing by the specialist is characterized by the set* (σ, λ) : $\sigma \geq \underline{\sigma}(\tau, \lambda, \underline{\pi}(B^S, \nu)))$

where

$$\underline{\sigma}(\tau, \lambda; \underline{\pi}) \equiv \sqrt{\frac{\underline{\pi}}{\tilde{\tilde{\psi}}} \frac{(\rho + \frac{1}{\lambda})}{1 - \tau}} \tag{28}$$

$\underline{\pi} = \max\{\pi(\underline{\sigma}), \nu\}$ *and* $\pi(\underline{\sigma})$ *solves*

$$\int_0^1 \int_0^\infty \int_{\sigma \geq \underline{\sigma}(\tau, \lambda; \underline{\pi})} dG^\sigma(\sigma) dG^\lambda(\lambda) dG^\tau(\tau) = B^S$$

when $\underline{\pi} > \nu$.

The theoretical sorting predictions of the model are given in Figure 3a. All else equal, more volatile cash-flows and more persistent deviations in cash-flows provide more cover for an executive to hide mis-behaviour and hence is more costly for optimally designed compensation contracts to preclude for an uninformed financier.

We have thus-far pinned down the equilibrium sorting and compensation by heterogeneously informed principals and heterogeneous projects in terms of the degree and persistence of private information. It remains of interest to examine how compensation and sorting patterns change as a function of the fundamentals in the environment. Notice that the information premium cutoff $\underline{\pi}(B^S, \nu) - \nu$ directly determines the mass of firms which sort to be private:

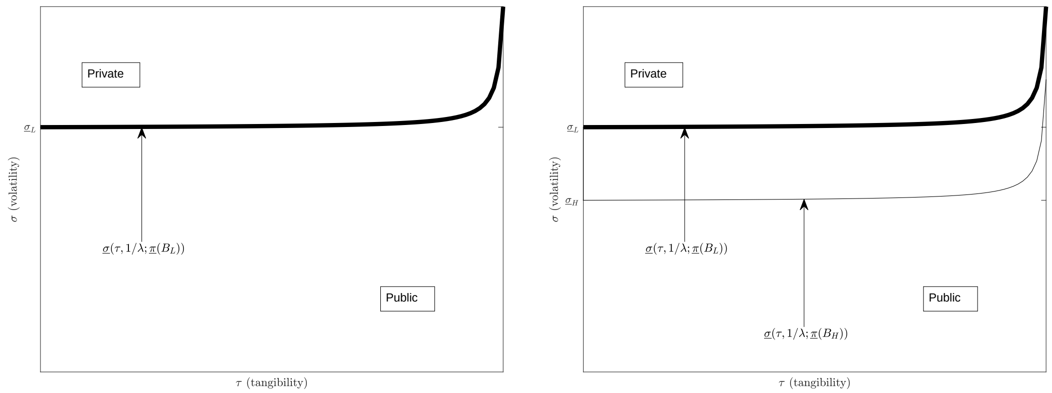
$$\underbrace{\mathbb{M}(B^S, \nu)}_{\text{Mass of specialist (private) financed firms}} = \int_\theta \{\pi(\theta) \geq \underline{\pi}(B^S, \nu)\} dG(\theta).$$

Now by inspection of the information premium cutoff $\underline{\pi}(B, \nu) - \nu$, when the budget constraint is binding then increasing B will reduce the information premium cutoff and hence increase the mass of firms privately financed.

Corollary 4.1 (Comparative statics I) *Assume* $\pi(\theta) - \nu > 0$ *for some mass* $G(\theta) > 0$*, but* $\pi(\theta) - \nu < 0$ *for some positive measure of* θ*. Then as specialist financing* B^S *moves from* $(0, \bar{B})$, $\bar{B} < \infty$*, the threshold information premium and hence volatility/persistence cutoffs will reduce and so the measure of private firms will increase but with lower private specialist premia.*

After some threshold however, $\underline{\pi}(B^S, \nu) = \nu$ *and the measure and types of firms funded by private equity will be locally invariant to changes in funding. That is,*

Figure 3: Firm sorting predictions



(a) Baseline sorting predictions (b) Sorting with increased funds, $B_H > B_L$.

The left-hand graph depicts the model sorting predictions implied by Theorem 4 over tangibility τ and volatility σ of the intangible cash-flow process for a fixed level of private financier funding B and given level of persistence λ. All firms with (τ, σ) above the solid line are predicted by the model to be funded by the private financier, while all those below will be funded by the public financier. The right-hand graph shows the same sorting line, as well as the new lower threshold given by a higher amount of funds available to the private financier B_H.

$$\frac{\partial \underline{\pi}(B, \nu)}{\partial B} = \begin{cases} < 0 & B < \bar{B} \\ 0 & B \geq \bar{B} \end{cases}$$

and the mass of privately financed firms is non-decreasing in B:

$$\frac{\partial \mathbb{M}(B, \nu)}{\partial B} = \begin{cases} > 0 & B < \bar{B} \\ 0 & B \geq \bar{B} \end{cases}.$$

By similar logic, we obtain the following comparative statics of the mass of firms privately financed based on other parameters.

Corollary 4.2 (Comparative statics II) *The mass of specialist (privately) financed firms, $\mathbb{M}(B^S, \nu)$, is non-increasing in ν, and in the discount rate ρ and non-decreasing in the risk-aversion parameter ψ.*

Finally, the average private equity premium is the return earned by private specialist financiers net of the monitoring cost

$$\bar{\Pi} = \int \pi(\theta)\{\pi(\theta) \geq \underline{\pi}\} dG(\theta) - \nu.$$

Straightforward computations yield the following results.

Corollary 4.3 (Comparative statics III) *The average private equity premium $\bar{\Pi}$ is decreasing in B, the monitoring cost ν, and the discount factor (interest rate) ρ and is increasing in the rightward skew of the marginal distribution of lambda dG^{λ} and leftward skew of the marginal distribution of sigma dG^{σ} (ie more mass on greater volatility firms and more persistent cash flow shock firms).*

3.4 Model discussion

We solved in the previous subsections a model of firm financing and CEO compensation. The firm financing in flavour resembles Holmstrom and Tirole (1997) with sorting of firm financing driven by heterogeneously informed principals. While in their model firm heterogeneity is driven by amount of internal funds, in our model it is driven primarily by differences in the degree of tangibility of firm projects and how persistent is the unobservable component of firms productivity evolution.

Our core prediction is that holding the environment fixed except for a change in the fundamental characteristics of firms which increases the wedge between insiders and shareholders, namely increasing intangibility, will lead to a higher private equity premia for a fixed amount of funding. A straightforward extension of the model can endogenize the amount of funds supplied to private specialist financiers based on the level of the private equity premium and hence an increase firm value derived from internal know-how can jointly explain the private equity premium observed in the past two decades by Kartashova (2014). At the same time, our model shows that this shift to firm-value derived from private information will lead to higher average compensation and compensation growth for public CEOs but also higher volatility/performance sensitivity to their compensation. Such dynamics in the level of public CEO compensation has been widely documented (see survey by Edmans et al. (2017)). Furthermore, from the same survey we have that CEO turnover and performance based compensation has increased, implying our model speaks to a number of consequential and puzzling dynamics simultaneously.

In light of the comparative advantage in the contracting problem, ignoring the monitoring cost and constrained budget of the specialist financier, all projects are weakly better funded by the specialist. However, in the presence of a positive, fixed monitoring cost ν per funded project exceeds this information premium of the specialist (defined as π) then the efficient financier for this project θ is in fact the public financier.

The limited budget of the specialist financier rations the measure of projects the specialist can finance. This rationing combined with competition from the generalist financier induces potentially another selection effect whereby the specialist will restrict their funding to the projects where the information premium $\pi(\theta)$ is largest. While the exact form of this information premium and sorting rule of specialist vis-a-vis generalist funded projects will depend on the exact specification of the entrepreneur's preferences, intuitively, the information premium will increase in the amount of leeway, or size of the private information, that the entrepreneur has when he reports. The higher the share of cash-flows derived from non-tangible or independently unobservable sources $(1 - \tau)$ the greater the advantage of the specialist. In particular in the limit of fully tangible cash-flows ($\tau = 1$), the specialist has no advantage in the contracting problem. Further, with $\tau < 1$ the size of private information increases in the volatility of the unobservable component of productivity (σ) and the persistence of these deviations from the mean (λ).

Here we have a partial equilibrium setting where the funds allocated to the private specialist financier are taken exogenously. This can be easily endogenized by embedding this financing model into general equilibrium. In particular, having a pool of households endowed with funds and heterogeneous monitoring ability (i.e. differing costs ν), where investing in the private intermediary also requires providing their monitoring labour. With a continuum of different ν, an endogenous cutoff of households will pin down the supply of funds, where an indifference condition for the marginal private equity investor will hold (but still an average positive private equity premium). Thus, a compositional change in firms will induce more funding to private equity due to a higher equilibrium monitoring cost cutoff.[9]

Finally, while we use some stark assumptions to obtain closed form solutions including that the private financier has a perfect monitoring technology whereas the public financier has none, if we relax the assumption of perfect monitoring we will get contracts and compensation which lie somewhere between the two extremes.

Summary of testable hypotheses

In this subsection we summarize the key testable hypotheses of our theory of CEO compensation and firm listing choice, associating specialist financiers with

[9] Details on this extension available upon request.

private equity and venture capital and publicfinanciers as those which supply funds to public stock markets.

First, generalizing from the results of Theorem 1 and Theorem 2 we have our first two hypotheses on CEO compensation and firm listing status:

Hypothesis 1 *CEO compensation will be more volatile and have higher average growth in a public firm than privately funded.*

Hypothesis 2 *CEO compensation growth and performance sensitivity will increase in firm intangibility and the persistence of private information.*

Second, from Theorems 4 and Corollary 4.3 we have the following predictions on the private equity premium:

Hypothesis 3 *The private equity premium is increasing for higher volatility, and more persistent deviations in cash-flows associated with internally observable factors. All else equal, firms will elect to be listed with lower fundamental volatility, less persistent hidden information, and/or are less tangible.*

Third based on the comparative statics corollaries, Corollary 4.1 and Corollary 4.2 we obtain the following time-series prediction of the private equity premium:

Hypothesis 4 *The higher the funding to private financiers the lower the private equity premium observed.*

Finally, appealing to complete markets and no systemic risk so that the discount rate ρ equals the real interest rate, we obtain the following hypothesis:

Hypothesis 5 *The private equity premium is negatively correlated with the real interest rate.*

4 Empirically testing the model predictions

4.1 Our data of public and 'private' firms

Equipped with the firm sorting and compensation predictions of the model outlined in the previous section, we move to examining the empirical content of these predictions. We use data on publicly listed firms on the historical "top three" U.S. stock exchanges (i.e. NYSE, AMEX and Nasdaq) and a large sample of firms that are not listed on any stock exchange. Following much of the literature on the listings puzzle, we exclude utilities and financials.

Our primary data source is the S&P Capital IQ database, which shares the same provider as the Compustat database. This dataset has recently become popular for studying the differences between public and private firms; papers utilizing this dataset include Gao et al. (2013a), Gao and Li (2015), and Acharya and Xu (2017). The sample of non-listed firms here consist of firms, which, though not listed on a U.S. stock exchange in a given year, have public reporting requirement by the SEC. Prior to the enactment of the 2012 JOBS Act, Section 12(g), any U.S. firms with $10 million or more of total assets and more than 500 shareholders are required to provide balance sheet and compensation data through the annual 10-K and quarterly 10-Q reports.[10] In addition, firms with public debt issues must file Form S-1. Consequently, our sample of non-listed firms are relatively large firms, making them more closely comparable to the listed firms than the private firms examined by Brav (2009) or Asker et al. (2014).[11]

While the SEC reporting requirements are the same for both the listed and non-listed firms, there are still two key distinctions between top U.S. exchanges and the non-listed firms. First, the listed firms have more comprehensive reporting requirements in the SEC and receive much more analyst attention than those with stocks tradable in an OTC market or without a trading platform. Second, by the nature of these markets having less market depth (and implicitly fewer shareholders), the ease of communicating private information to long-term consolidated shareholders while avoiding divulging to the broader public should be higher than that of the firms listed on the top-3. Finally, the lower frequency of trade implies price adjustments of firm value should be lower than that of the top exchanges, so the information sensitivity of stock prices and CEO compensation should lie on a continuum between the totally private firm and the top 3 exchanges.

4.2 Proxying firm private information characteristics (τ, σ)

Core to our predictions on compensation and firm sorting are firm characteristics of: (i) the share of cash-flows with or without information frictions τ, (ii) the persistence of private information shocks $\frac{1}{\lambda}$, and (iii) the volatility of the private information component of cash-flows σ. Since λ is a latent variable, in this section, we will abstract from any heterogeneity of $\frac{1}{\lambda}$ between firms, leav-

[10] Since the JOBS Act, this threshold has been increased to 2000 shareholders or 500 "non-accredited investors."

[11] As emphasized in Acharya and Xu (2017), S&P Capital IQ has only the most recent information about the listing status of firms. For this reason, to determine listing status each year, we use the CRSP and Compustat databases to understand whether a firm is publicly listed or not. While CRSP has historical information about the listing status of firms, the standard version of Compustat does not. For this reason, we use its Snapshot version, which enables users to view the historical listing status of firms (including OTC and minor exchanges). After merging this information with the S&P Capital IQ data, we exclude from our analysis firm-year observations which are traded on minor stock exchanges.

ing firm sorting and compensation heterogeneity being driven by τ and σ. As the volatility of the private information component of cash-flows is also latent, using the linear relationship between total volatility and the private information component, we will simply use the total volatility of the firm's cash-flows over the prior three years, $\widehat{\sigma}_{3y}$ as a proxy for the former. Finally, we proxy the share of cash-flows associated with private information by the share of intangible capital $1-\widehat{\tau}$, so that the more tangible firm (higher $\widehat{\tau}$), the lower the share of private information within the firm.

To construct the measure of intangible capital, we largely follow the procedure of Peters and Taylor (2017). That is, we use a perpetual inventory method to compute the stock of intangible capital for each firm from the year of foundation (as provided in the Capital IQ data and supplemented by the Field-Ritter dataset of founding dates).[12] We use depreciation rates from Li and Hall (2020) for the intangible capital that accumulates from R&D expenses. Following the Bureau of Economic Analysis (BEA) directives, we set the depreciation rate to 15% when it is not reported in the paper.[13]

To our knowledge, the mapping of the stock of firm intangibility to private information has been relatively unexamined in the literature. One notable exception is Glover and Levine (2017) who find a positive correlation between their implied estimate of the importance of CEO effort based on observed compensation contracts and intangibility. In contrast, private information in human capital (e.g. ability, or health), which is partially captured in the intangible capital measure via organizational capital, has been widely acknowledge and explored (see for instance Weiss (1995)). A large component of intangible capital is intellectual property, some of which is comprised of trade secrets. The growth of non-disclosure agreements, trade secret litigation and legislation (e.g. the passage of the "Defend Trade Secrets Act" of 2016) suggests that this aspect of intangible capital is important and tightly tied to private information that is difficult to convey to the broader public (see Almeling et al. (2009)). In contrast, the link of intangible capital to firm productivity and market power as an omitted variable in the production function has garnered substantial attention/support recently (e.g. Corrado et al. (2009), McGrattan (2017), Crouzet and Eberly (2018b), Crouzet and Eberly (2018a)).

[12]For more information about this dataset see Field and Karpoff (2002) and Loughran and Ritter (2004).

[13]For the component of intangible capital that accumulates from from SG&A expenses, often called organizational capital we use the rate from Falato et al. (2017) of 20%. For more discussion of organizational capital, see Eisfeldt and Papanikolaou (2013) and Eisfeldt and Papanikolaou (2014) and Lev and Radhakrishnan (2005).

4.3 Descriptives

Our sample starts in 1993 and ends in 2015. We consider only firm-year observations with a positive and non-missing book value of total assets and we exclude from our analysis financial firms (SIC codes from 6000 to 6999), utilities (SIC codes from 4900 to 4999) and quasi governmental firms (SIC codes from 9000). All the variables are normalized in 2015 U.S. dollars.

Table 1 provides summary statistics of our sample. As expected, the public firms are bigger in terms of both size and sales. On average, public firms are also twice as old as private firms. Public firms have a lower level of net leverage since unlike private firms, they have access to the public equity market and can substitute away from debt. While on average, private firms have a lower ROA (calculated as EBIT over assets) and tend to be closer to default according to the Altman's z score,[14] they have higher sales growth and higher investment rate. This suggests that private firms in our sample are not necessarily firms in distress. Finally, we compare the numerator of the second term of our proxy of τ, the sum of intangible stocks, and our proxy of σ. We see that public firms tend to have more intangible assets than private firms and lower volatility of cash-flows, measured as the the three year volatility of cash-flows.

Our results about age, physical capital expenditures, leverage, and cash-flow volatility are qualitatively similar to those of Gao et al. (2013a) even though we use slightly different proxies for some of these measures. In their sample, the sales growth of public firm is marginally higher than that of private firms, but this difference may be due to the differences between the sample periods that we consider. Indeed, our results about sales growth are qualitatively equivalent to those of Acharya and Xu (2017). Our results about the stock of intangible seem to be consistent with their findings using patent data. While there are some differences between our ROA and that reported in Acharya and Xu (2017) (driven by slight differences in proxies and sample period), our median ROA for the public is 7% is slightly higher than the 5.3% found in Gao and Li (2015).

The last column of Table 1 compares the differences between the listed and non-listed sample. We find that across the board, the differences between firms listed and non-listed are significantly different both statistically and economically.[15]

Finally, in Figure 4 we depict the average firm intangibility conditional on listing status. Here we see that in the '90s intangibility rose for all listing

[14]The Altman's z-score measure is provided by S&P Capital IQ, which is adjusted for private firms. Curiously the average ROA of the full sample is negative, however, when computed in terms of EBITDA over assets we get numbers consistent with Acharya and Xu (2017).

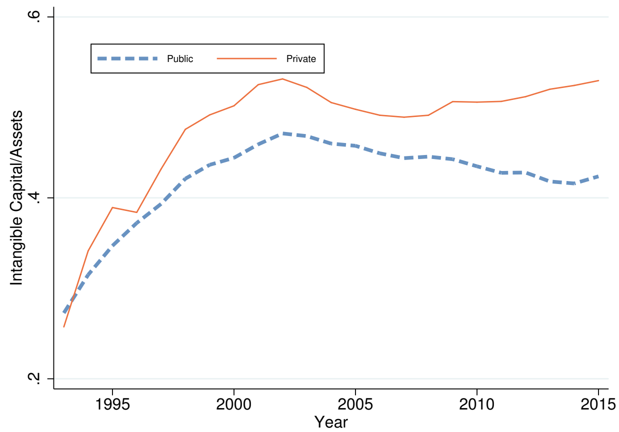
[15]While not reported here we obtain similar results if we compare the means of the sample of publicly listed and non-listed firms that are matched according to size measured by sales.

Table 1: Summary statistics by listing type

Variable	Total	Listed	Unlisted	t-stat
Ln(assets)	4.265	5.525	2.500	-3.025^{***}
	(2.873)	(2.067)	(2.915)	
Ln(sales)	4.639	5.565	3.049	-2.516^{***}
	(2.796)	(2.273)	(2.892)	
Intangibles	595.0	862.5	220.1	-642.4^{***}
	(2087.0)	(2521.2)	(1145.2)	
$\widehat{\sigma}_{3y}$	0.111	0.0828	0.173	0.0900^{***}
	(0.188)	(0.120)	(0.274)	
Age	27.23	34.22	17.43	-16.80^{***}
	(28.61)	(30.41)	(22.49)	
Altmann Z score	-1.878	5.599	-12.38	-17.98^{***}
	(212.5)	(14.54)	(328.8)	
Physical Investment	0.247	0.234	0.273	-0.0394^{***}
	(0.539)	(0.476)	(0.650)	
ROA	-0.475	-0.0312	-1.098	-1.067^{***}
	(1.829)	(0.499)	(2.651)	
Net Leverage	0.347	0.109	0.681	0.572^{***}
	(1.166)	(0.469)	(1.663)	
Ln(Δ sales)	0.117	0.106	0.140	0.0336^{***}
	(0.465)	(0.396)	(0.585)	
Observations	137451	80208	57243	

The sample consists of 80,208 observations of listed firm-year observations and 57,243 of private firm-year observations obtained from Capital IQ from 1993 to 2015. Net leverage is total debt minus cash and equivalents divided by total book assets, profitability is gross profits divided by assets, investment is capital expenditures divided by lagged gross total property, plant and equipment, return on investment (ROA) is earnings before interests (EBIT) divided by assets, the sales growth variable equals the log change in sales. We build the stocks of r&d and sg&a capital using a similar procedure to the one described in Peters and Taylor (2017). Intangible stock equals the sum of r&d and sg&a stocks, goodwill and other intangibles reported on the balance sheet. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Test statistics of the t-test of the differences in firm characteristics between listed and private firms are given in superscript ***,**,* denoting statistical significance at the 1%, 5% and 10% levels, respectively.

Figure 4: Average intangible capital of U.S. listed vs non-listed firms



Average intangible capital for a given listing status, computed on an annual basis using Capital IQ data. The top red line is private (non-listed) firms, which include equity traded on the OTC and public debt. The bottom, blue dotted line is the annual average amongst US firms listed in the top 3 US stock exchanges. Details of the intangible capital computation and listing status given in the appendix. The denominator of the ratio includes total assets plus intangible assets.

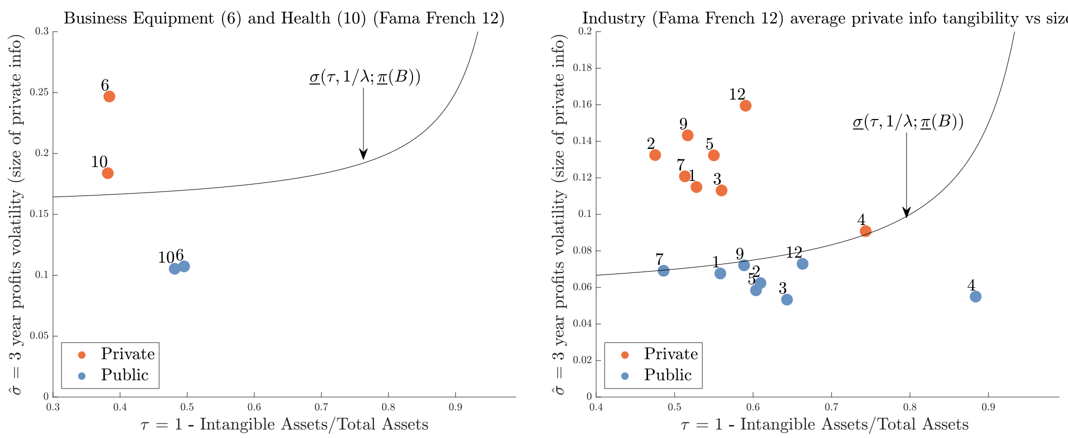
types, but at an accelerated rate for the private firms. However, since the mid 2000s the two trends diverged, with public listed firms becoming steadily more tangible while private firms becoming more intangible. Interestingly the divergence occurs around the time of SOX suggesting that the additional regulatory burden and disclosure may have exacerbated the selection effect.

4.4 Testing cross-sectional firm sorting predictions

In this subsection, we examine the sorting predictions of our model. As summarized in Hypothesis 3, our model predicts that the more intangible or volatile the cash-flows of the firm, the more likely the firm is to be financed by concentrated, private shareholders who can avoid the information asymmetry problem. In Figures 5a and 5b, we present our first validation of our model predictions. Within industry average tangibility $\widehat{\tau}$ and average three-year firm cash-flow volatility $\widehat{\sigma}$ are depicted as points. The within-industry averages are computed over the entire sample period from 1993-2015, conditional on the firm listing status. Our model predicted sorting line is superimposed on the figures, with parameters selected to minimize misclassification.

In the left-hand figure, Figure 5a, we depict the results for business and medical equipment industries (Fama-French 6 and 10 respectively, two outlying industries with much higher volatility and share of cash-flows associated with private information. The non-listed ('private') firms are associated with higher intangibility and firm volatility relative to their listed counterparts. Moving to the remaining industries in the right-hand figure, Figure 5b, we see that we are still able to sort firms perfectly using our predicted sorting pattern based on intangibility and volatility. This is by no means mechanical as can be seen by industry 4 (energy), in which, the non-listed firms barely fall above the sorting line. With a small drop in their volatility (say, to the level of public firms in industry 12) it would be impossible to obtain a sorting line of the form solved for in the model. With the inclusion of all the industries together, we can not find a single best-fit line; however, if we drop the energy sector or simply look at the sorting predictions within industry, we see that non-listed firms cluster in the north-west relative to their listed counterparts across all industries besides industry 7 (telecom).

Figure 5: Firm sorting based on firm intangibility and cash-flow volatility



(a) High private info industries

(b) All other Fama-French 12 industries

Within-industry average tangibility and 3-year cash-flow volatility conditional on listing status as Public or 'Private' (non-listed). Data from 1993-2015 (first three years excluded to allow for the computation of $\widehat{\sigma}$). Super-imposed is the theoretical sorting prediction line given in Figure 3a with cutoff levels selected to minimize mis-classification based on our model.

Having demonstrated the model is consistent on the industry level during our sample period, we now estimate the firm-level listing propensity conditional on firm characteristics. We fit variants of the following model

$$\text{Listed}_{i,t} = g(\alpha + \beta_1 \widehat{\tau}_{i,t-1} + \beta_2 \widehat{\sigma}_{i,t-1} + \gamma X_{i,t-1} + \zeta_t + \eta_k + \varepsilon_{i,t}) \tag{29}$$

where $\text{Listed}_{i,t}$ is an indicator variable, i is a firm level index, k is the industry associated with firm i, t is year and $g(\cdot)$ is some link function.

In Table 2 we present the results from estimating a linear probability model (i.e. $g(x) = x$) while in Table 5 we use a logistic link. We include lagged sales as a control for size across all but the first specification. In the latter specifications we also include industry and year fixed effects. Finally, in the last column we include additional firm level controls of net leverage, investment, ROA and sales growth.

Our results are consistent with the theoretical predictions across all specifications. The higher the tangibility of the firm, the more likely a firm is to be listed, while the higher the volatility of cash-flows, the less likely a firm is to be public. These results are all statistically significant at the 5% level, nearly all at the 1% level, and are economically meaningful. In particular, from the linear probability model with all the controls specified in the last column of Table 2, we see that a 10 percentage point increase in firm tangibility will induce a 1.6% increase in the firm likelihood to be listed. Similarly, a 10 percentage point increase in firm cash-flow volatility will reduce the likelihood of the firm being listed on a top 3 exchange by 68 basis points. In the appendix, Table 5, we show that similar results go through using a logistic link function.

4.5 Testing cross-sectional executive compensation

In this subsection, we test the model predictions on CEO compensation, summarized in Hypotheses 1 and 2. Namely, that (1) CEO compensation will be more volatile and have higher average growth in a listed firm than a non-listed firm, and (2) conditional on listing status, CEO compensation growth and performance sensitivity will increase in firm intangibility and firm volatility. We combine the firm characteristics and listing data utilized above with data obtained from Capital IQ about executives' compensation. Unfortunately, S&P Capital IQ has only the latest information about the position covered by each executive. Consequently, we proxy the CEO with the highest paid executive within each company in each year. Our sample for executives starts from 2000.

In Table 3 we present estimation results from regressing our proxies of private information exposure, $\widehat{\tau}$ and $\widehat{\sigma}$, and listing status on top executive compensation (in millions). We include the same firm level controls (and industry and year controls) as in the previous subsection, as well interaction effects of ROA and our proxies to examine the marginal contribution of performance sensitivity with higher exposure to private information (i.e. lower τ or higher σ).

Table 2: Listing choice - linear probability model

	(1)	(2)	(3)	(4)
$\widehat{\tau}_{t-1}$	0.2131***	0.0635***	0.2184***	0.1618***
	(0.0171)	(0.0162)	(0.0160)	(0.0167)
$\widehat{\sigma}_{t-1}$	−0.4578***	−0.2281***	−0.0737***	−0.0681***
	(0.0142)	(0.0146)	(0.0113)	(0.0121)
$Sales_{t-1}$		0.0606***	0.0425***	0.0490***
		(0.0015)	(0.0027)	(0.0031)
Other firm controls	No	No	No	Yes
Year FE	No	No	Yes	Yes
Industry FE	No	No	Yes	Yes
N	89060	83894	83894	79612
R^2	0.0564	0.1762	0.0508	0.0614

The above table presents the estimation results using a linear probability model testing the cross-sectional predictions of firm sorting into publicly listed / non-listed. The dependent variable is a dummy variable that equals 1 if a firm-year observation is listed on a top 3 US stock exchange. Our primary variables of interest are our proxies pertaining to the private information cash-flows of a firm, namely firm tangibility $\widehat{\tau}$ and cash-flow volatility $\widehat{\sigma}$. The first column specification includes no controls. The second column adds lagged log sales as a size control. The third column adds industry and year fixed effects. Finally, the last column adds additional firm level controls of ROA, investments, sales growth and net leverage in addition to the fixed effects. The last row presents the adjusted R^2 for the first two columns and the within R^2 for the last two. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Superscript ***,**,* denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 3: Top executive compensation in levels

	(1)	(2)	(3)	(4)	(5)	(6)
Listed_{t-1}	0.2589**	0.2350**	0.1561	0.2969***	0.2637**	0.2596**
	(0.1095)	(0.1138)	(0.1185)	(0.1081)	(0.1134)	(0.1182)
$\widehat{\tau}_{t-1}$	-2.3164***	-1.8017***	-1.7149***	-1.7526***	-1.6164***	-1.6137***
	(0.2857)	(0.3033)	(0.3054)	(0.2802)	(0.3005)	(0.3030)
$\widehat{\sigma}_{t-1}$	1.6180***	0.7296***	0.7765***	0.5028**	1.2169***	1.2144***
	(0.2015)	(0.2159)	(0.2160)	(0.2096)	(0.2549)	(0.2540)
sales_{t-1}	1.5551***	1.7708***	1.7998***	1.7230***	1.8457***	1.8465***
	(0.0436)	(0.0490)	(0.0502)	(0.0480)	(0.0510)	(0.0513)
roa_{t-1}		-1.0005***	-0.9291***		-0.7361***	-0.7345***
		(0.0609)	(0.0578)		(0.0744)	(0.0736)
$\text{roa}_{t-1} \times \text{listed}_{t-1}$			-0.8797***			-0.0432
			(0.1578)			(0.1660)
$\text{roa}_{t-1} \times \widehat{\tau}_{t-1}$				-4.9475***	-3.7834***	-3.7518***
				(0.2698)	(0.2973)	(0.3079)
$\text{roa}_{t-1} \times \widehat{\sigma}_{t-1}$				-0.4087***	0.6261***	0.6197***
				(0.0941)	(0.1572)	(0.1541)
Other Firm Controls	Yes	Yes	Yes	Yes	Yes	
Year FE	Yes	Yes	Yes	Yes	Yes	
Industry FE	Yes	Yes	Yes	Yes	Yes	
Observations	63586	61404	61404	63576	61404	61404
Adjusted R^2	0.3216	0.3359	0.3363	0.3304	0.3386	0.3386

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

We regress top executive compensation for each firm in millions on listing choice, our proxy of firm intangibility $(1 - \widehat{\tau})$ and private cash flow volatility $(\widehat{\sigma})$, whether a firm is publicly listed on a top 3 stock exchange. Other firm controls are investment (capex/pp&e), sales growth and net leverage. All regressions have industry and year fixed effects. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Superscript ***,**,* denote statistical significance at the 1%, 5% and 10% levels, respectively.

We find that across all the specifications, our primary three variables of interest (listed status, tangibility and volatility) support our predictions. Indeed, we find in our full specification in column (6) of Table 3 that (i) being publicly listed increases average compensation by about \$260,000, (ii) a 10 percentage point increase in firm intangibility raises the expected CEO compensation by \$160,000, and (iii) a 10 percentage point increase in cash-flow volatility implies a \$120,000 average pay bump.

We find that the size of the firm, as proxied by sales, is strongly associated with increased executive compensation. In contrast to the results of Gao and Li (2015), we find that the level of performance proxied by ROA is not positively associated with pay after controlling for sales. This is likely driven by our inclusion of OTC and other non-listed firms. Somewhat contrary to our prediction that listed firms will require compensation, which is more sensitive to performance, the interaction of ROA with listed status predicts lower compensation. However, this effect loses statistical significance when we include the interactions of ROA with firm tangibility and volatility in column (6) of Table 3. These last two interactions suggest that CEO performance pay sensitivity is increasing in firm intangibility and firm volatility. The interaction effect of performance with intangibility is not insubstantial, with a 10 percentage point increase in intangibility and ROA implying a \$30,000 increase in top executive compensation.

In Table 6, we present the same regressions as above but with top executive pay normalized by firms assets. Here, we show that the same qualitative first-order effects of τ and σ hold, but the interaction effects with performance go in the opposite way. Furthermore, the interaction of listed and performance is large and significant, suggesting a higher share of the firm is assigned to the CEO as a reward when in a publicly listed firm.

As a final test for this subsection, we hone in on the variable component of compensation and test the corresponding compensation predictions. We interpret this as another measure of how performance-sensitive the contract is. Using the same regressors as before but with the level of variable compensation in millions assigned to the top executive, we find that listing status is positively associated with the variable share consistent with our hypothesis. In addition, we find large and significant effects of firm tangibility and volatility consistent with our model predictions. Interestingly, the size of these effects are at a similar magnitude of our proxy for firm size, with, for instance, a 10 percentage point increase in intangibility $(1-\widehat{\tau})$ implying a \$150,000 increase in the level of the variable component of compensation. Finally, we find that the interaction effects of the performance and tangibility/firm volatility are qualitatively the same as those found for the total level of compensation.

Table 4: Top executive variable compensation

	(1)	(2)	(3)	(4)	(5)	(6)
Listed_{t-1}	0.280***	0.256**	0.188	0.315***	0.281**	0.283**
	(2.65)	(2.32)	(1.64)	(3.01)	(2.56)	(2.47)
$\widehat{\tau}_{t-1}$	-2.097***	-1.640***	-1.566***	-1.581***	-1.472***	-1.473***
	(-7.64)	(-5.62)	(-5.33)	(-5.86)	(-5.09)	(-5.05)
$\widehat{\sigma}_{t-1}$	1.578***	0.772***	0.812***	0.573***	1.210***	1.211***
	(8.19)	(3.74)	(3.93)	(2.85)	(4.95)	(4.97)
$Sales_{t-1}$	1.408***	1.608***	1.632***	1.562***	1.676***	1.675***
	(33.65)	(34.12)	(33.76)	(33.80)	(34.07)	(33.87)
roa_{t-1}		-0.927***	-0.866***		-0.686***	-0.687***
		(-16.03)	(-15.75)		(-9.74)	(-9.83)
$\text{roa}_{t-1} \times \text{Listed}_{t-1}$			-0.749***			0.0186
			(-4.93)			(0.12)
$\text{roa}_{t-1} \times \widehat{\tau}_{t-1}$				-4.539***	-3.433***	-3.447***
				(-17.52)	(-12.05)	(-11.74)
$\text{roa}_{t-1} \times \widehat{\sigma}_{t-1}$				-0.358***	0.562***	0.565***
				(-3.99)	(3.72)	(3.82)
Other Firm Controls	Yes	Yes	Yes	Yes	Yes	
Year FE	Yes	Yes	Yes	Yes	Yes	
Industry FE	Yes	Yes	Yes	Yes	Yes	
Observations	63551	61370	61370	63541	61370	61370
Adjusted R^2	0.3023	0.3160	0.3164	0.3103	0.3185	0.3185

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

We regress the variable component of compensation of the highest paid executive for each firm in millions on listing choice, our proxy of firm intangibility $(1 - \widehat{\tau})$ and private cash flow volatility $(\widehat{\sigma})$, whether a firm is publicly listed on a top 3 stock exchange. Other firm controls are investment (capex/pp&e), sales growth and net leverage. All regressions have industry and year fixed effects. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Superscript ***,**,* denote statistical significance at the 1%, 5% and 10% levels, respectively.

4.6 Discussion

In general, our empirical results support the core predictions of our model for both sorting and CEO compensation. The performance of these tests rests on the assumption that our proxies of the private cash-flow share τ and its volatility σ by firm tangibility and three-year cash-flow volatility respectively are valid. This is all but impossible to verify due to the latent nature of these private information parameters. A structural estimation procedure may provide an alternative means of identification.

Our results on firm sorting pool together both firms with shares traded on an OTC market and those with public bonds with the wholly private firms in the capital IQ sample. While these firms are decidedly different from the median private enterprise in other samples of private firms, we believe that the inclusion of OTC and public bond firms is proper for our question since we wish to consider large firms with the option to be publicly listed on a top 3 exchange, but simply not the willingness to be. Nonetheless, there are limitations to the extent that one should generalize the empirical results to private firms not included in the Capital IQ database.

Our findings on firm sorting complement some of the recent studies of the differences between private and public firms. Acharya and Xu (2017) find that firm innovation intensity is higher for publicly listed firms than their private counterparts in strongly externally finance dependent industries, but that this does not hold for internal finance industries. Gao et al. (2013a) also find that cash-holdings are substantially lower in private firms than matched public counterparts, suggesting lower agency frictions in the private firms.

Although we have focused on the listing decline in the U.S. and Doidge et al. (2017) argue that there is a U.S. 'listings gap', to the extent that rising firm intangibility is a global phenomenon, our channel would predict a decline in public listings to also be occurring abroad. Indeed, using the same data source as Doidge et al. (2017), World Development Indicators (WDI), we find that the decline has been similar although slightly lagged for the larger developed economies (see Figure 6 for the case of UK, Germany and France). There is also some evidence that this decline is occurring in Canada despite the numbers suggested in WDI, which seem to include the growth of financial vehicles being listed on the TSX.[16]

As mentioned in the previous subsection, our results on compensation pertain to the top compensated executive in each firm according to the Capital IQ data rather than the CEO. In most cases this should not lead to a discrepancy; however, there will naturally be more measurement error here than if the iden-

[16]See for instance https://cirano.qc.ca/actualite/2016-10-25/pdf/20161025_Are-the-Canadian-Public-Markets-Broken_J-Ari-Pandes.pdf.

tity of the CEO was known and tracked. In addition, this measure prevents us from studying other important components of CEO compensation volatility, namely CEO turnover.

The CEO compensation results are also broadly consistent with much of the previous work. For instance, Gao and Li (2015) find that performance sensitivity is higher for publicly listed CEOs than privately listed CEOs. To our knowledge, there is little work linking CEO compensation to firm intangibility. One notable exception is Glover and Levine (2017) who structurally estimate the value of CEO effort implied by compensation contracts and find that firm intangibility is strongly correlated to the firm value of CEO effort. While our paper abstracts from the intertemporal moral hazard effort trade-off, much of the same agency conflict is present in our model with dynamic and hidden diversion of cash-flows, which can be used to shed additional light on their findings.

5 Conclusion

The listing propensity of U.S. firms on one of the three largest stock markets has declined over the past two decades. At the same time, public CEO compensation, performance-based compensation and turnover have markedly increased. We link these dynamics to the growing intangibility of firms, which we map to an increasing wedge between the information of insiders and outsiders. Adapting a continuous time, optimal contracting framework with persistent private information developed by Williams (2011) to an equilibrium financing setting, our model rationalizes these dynamics. In particular, through an informational advantage in contracting, our model generates an endogenous private equity premium which grows with the average firm intangibility and induces the growth and increased performance sensitivity of compensation paid to public executives.

We validate the model by empirically testing cross-sectional predictions of sorting and compensation with data on publicly listed and non-listed firms. We abstract from size and lifecycle considerations for firm listing choice and compensation, but study of the interaction of these considerations with the firm characteristics analyzed here are no doubt important and left for future work.

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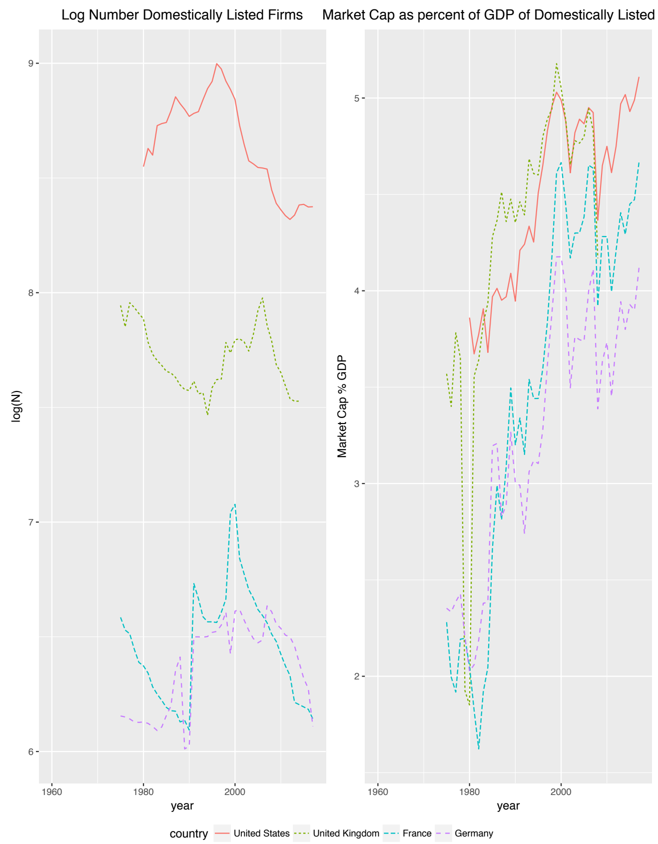
A Appendix

A.1 Supplemental Tables and Figures

Figure 6: Public listing dynamics across highly developed countries



Domestic public stock listings (left) and market cap scaled by GDP (right) for different countries. Data is obtained from World Development Indicators following Doidge et al. (2017). France listings data is interpolated for 1994. Here we see that a decline in listings is occurring in the past decade or two across all four countries depicted, while the market cap to GDP has also increased suggesting a similar pattern of consolidation. Notice that the decline in listings in the U.S. however begins somewhat earlier than the others.

Table 5: Listing choice - logistic model

	$Listed_t$	$Listed_t$	$Listed_t$	$Listed_t$
$\widehat{\tau}_{t-1}$	1.0818***	0.3073***	0.7261***	0.2632**
	(0.0888)	(0.0954)	(0.1079)	(0.1227)
$\widehat{\sigma}_{t-1}$	−2.1077***	−1.0323***	−1.3477***	−1.6564***
	(0.0811)	(0.0778)	(0.0884)	(0.1137)
$Sales_{t-1}$		0.3531***	0.4230***	0.4997***
		(0.0110)	(0.0121)	(0.0167)
Other Firm Controls	No	No	No	Yes
Year FE	No	No	Yes	Yes
Industry FE	No	No	Yes	Yes
N	89060	83894	83894	79612
R^2	0.0443	0.1561	0.2154	0.2847

The above table presents the estimation results using logistic regression testing the cross-sectional predictions of firm sorting into publicly listed / non-listed. The dependent variable is a dummy variable that equals 1 if a firm-year observation is listed on a top 3 US stock exchange. Our primary right-hand side variables of interest are our proxies pertaining to the private information cash-flows of a firm, namely firm tangibility $\widehat{\tau}$ and cash-flow volatility $\widehat{\sigma}$. The first column specification includes no controls, the second column adds lagged log sales as a size control. The third column adds industry and year fixed effects. Finally the last column adds in addition to the fixed effects, additional firm level controls of ROA, investments, sales growth and net leverage. For the logistic regressions, we report the pseudo-R^2. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Superscript ***,**,* denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 6: Top executive compensation scaled by firm total assets

	(1)	(2)	(3)	(4)	(5)	(6)
Listed_{t-1}	-0.0170***	-0.0054***	-0.0007	-0.0148***	-0.0061***	-0.0021*
	(0.0017)	(0.0015)	(0.0013)	(0.0014)	(0.0014)	(0.0012)
$\widehat{\tau}_{t-1}$	-0.0850***	-0.0160***	-0.0213***	-0.0573***	-0.0210***	-0.0236***
	(0.0044)	(0.0027)	(0.0026)	(0.0034)	(0.0026)	(0.0025)
$\widehat{\sigma}_{t-1}$	0.1716***	0.0795***	0.0767***	-0.0144	0.0528***	0.0553***
	(0.0149)	(0.0118)	(0.0116)	(0.0112)	(0.0096)	(0.0096)
sales_{t-1}	-0.0121***	-0.0051***	-0.0068***	-0.0094***	-0.0068***	-0.0075***
	(0.0005)	(0.0004)	(0.0004)	(0.0005)	(0.0004)	(0.0004)
roa_{t-1}		-0.0500***	-0.0542***		-0.0484***	-0.0499***
		(0.0034)	(0.0036)		(0.0056)	(0.0057)
$\text{roa}_{t-1} \times \text{listed}_{t-1}$			0.0528***			0.0426***
			(0.0048)			(0.0057)
$\text{roa}_{t-1} \times \widehat{\tau}_{t-1}$				-0.0072	0.0684***	0.0372***
				(0.0118)	(0.0123)	(0.0140)
$\text{roa}_{t-1} \times \widehat{\sigma}_{t-1}$				-0.1643***	-0.0378***	-0.0316**
				(0.0109)	(0.0142)	(0.0142)
Other Firm Controls	Yes	Yes	Yes	Yes	Yes	
Year FE	Yes	Yes	Yes	Yes	Yes	
Industry FE	Yes	Yes	Yes	Yes	Yes	
Observations	63586	61404	61404	63576	61404	61404
Adjusted R^2	0.2094	0.3808	0.3917	0.3293	0.3888	0.3947

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

We regress top executive compensation for each firm scaled by firm assets on listing choice, our proxy of firm intangibility $(1 - \widehat{\tau})$ and private cash flow volatility $(\widehat{\sigma})$, whether a firm is publicly listed on a top 3 stock exchange. Other firm controls are investment (capex/pp&e), sales growth and net leverage. All regressions have industry and year fixed effects. All the ratio variables are winsorized at 1% and 99%. All dollar values are in 2015 dollars. Superscript ***,**,* denote statistical significance at the 1%, 5% and 10% levels, respectively.

A.2 Variable Construction

We build net leverage as total debt minus cash and equivalents divided by total book assets, profitability as gross profits divided by assets, investments as capital expenditures divided by lagged gross total property, plant and equipment, return on investment (ROA) as earnings before interests (EBIT) divided by assets, the sales growth variable equals the log change in sales.

Our proxy of $1 - \tau$ equals to one minus the sum of all the intangible assets divided by the sum of all assets where the sum of all the intangible assets equals the sum of R&D capital, SG&A capital, goodwill and other intangible assets as reported on the balance sheet while the sum of all assets equals the sum R&D capital, SG&A capital and total book value of assets. Our proxy of σ is the 3 years firm standard deviation of profitability. Table 7 reports all the definitions of the variables. Capital IQ provides the information about the Altman's z scores. All the variables involving ratios are winsorized at the 1% and 99%. We use the Fama and French 12 industry classification when we use industry fixed effects. Using a more granular industry classification like the Fama and French 48 industry classification doesn't change our results.

Finally, as dependent variable for the executives' analysis we consider the ratio of the total compensation of the manager divided by the total book value of assets where total compensation is computed as the sum of salary, bonuses, restricted stock awards, option awards, deferred compensation earnings due to changes in the compensation plan, non equity incentive plan, long term incentives plan, total value of stock awards, other annual compensation and all other forms of compensation.

Table 7: Construction of the variables used for the analysis

Variable	Formula
Net Leverage$_t$	$\frac{\text{Total debt}_t - \text{Cash and equivalents}_t}{\text{Total Assets}_t}$
Profitability$_t$	$\frac{\text{Gross Profits}_t}{\text{Total Assets}_t}$
Investments$_t$	$\frac{\text{CAPEX}_t}{\text{PPEGT}_{t-1}}$
Sales Growth$_t$	$ln\left(\frac{\text{CAPEX}_t}{\text{PPEGT}_{t-1}}\right)$
$\widehat{\tau}_t$	$1 - \frac{\text{R\&D Stock}_t + \text{SG\&A Stock}_t + \text{Goodwill}_t + \text{Other Intangibles}_t}{\text{Total Assets}_t + \text{R\&D Stock}_t + \text{SG\&A Stock}_t}$
$\widehat{\sigma}_t$	$SD_{3Y}(\text{Profitability}_{\text{from } t-2 \text{ to } t})$

A.3 Details on sample construction

We obtain data about balance sheets, age of foundation and executives' compensation of public and private firms from the S&P Capital IQ database which is produced by Standard & Poor (which is also the provider of the commonly used Compustat database). Our sample starts in 1993 and ends in 2015. We consider only firm-year observations with a positive and non-missing book value of total assets and we exclude from our analysis financial firms (SIC codes from 6000 to 6999), utilities (SIC codes from 4900 to 4999) and quasi governmental firms (SIC codes from 9000). All the variables are normalized in 2015 US dollars.

As emphasized in Acharya and Xu (2017), S&P Capital IQ has only the most recent information about the listing status of firms. For this reason we use the CRSP and Compustat databases to understand whether a firm is publicly listed or not. While CRSP has historical information about the listing status of firms, the standard version of Compustat doesn't and for this reason we use its Snapshot version which enables users to view data over time. After merging this information with the S&P Capital IQ data, we exclude from our analysis firm-year observations which are traded on minor stock exchanges.

Since S&P Capital IQ doesn't have information about the foundation year for all the firms, we use the Field-Ritter dataset of company founding dates to complement our data and we impose the restriction that a firm can't be founded before 1900 as they do.[17] Since after the merger between this dataset and the S&P Capital IQ data an observation might still have missing founding year, we assume that the first time it appears in our panel is at the end of the first year of life. This assumption doesn't change any of our core results since the latter ones hold both using all the firms and using only firms whose foundation year is reported in S&P Capital IQ.

We construct the measures about intangible stocks using a similar procedure to the one described in Peters and Taylor (2017). Given that our data offers more information about private firms, we consider the year of foundation as their first year and we assume that during the first year the firm doesn't accumulate any intangible capital. We use the perpetual inventory method to compute the stocks of intangible capital of each firm at the end of the first year. We use the depreciation rates from Li and Hall (2018) for the intangible capital that accumulates from R&D expenses. Following the Bureau of Economic Analysis (BEA) directives, we set the depreciation rate to 15% when it's not reported in the paper. Following Falato et al. (2017) we use a depreciation rate for the intangible capital that accumulates from SG&A expenses, often called

[17] For more information about this dataset see Field and Karpoff (2002) and Loughran and Ritter (2004).

organizational capital [18], equal to 20%.

Ultimately, we merge the data about firms with the one about executives' compensation. Unfortunately, S&P Capital IQ has only the latest information about the position covered by each executive. For this reason, we consider the most paid executive within each company in each year. Our sample for executives starts from 2000.

[18] See Eisfeldt and Papanikolaou (2013) and Eisfeldt and Papanikolaou (2014) and Lev and Radhakrishnan (2005).

B Proofs

B.1 Proofs for Section 2.2 (Entrepreneur Reporting Problem)

Change of measure from P_0 to P_Δ

We follow Williams (2011). For a given path for Δ, define

$$\Gamma_t(\Delta) = \exp\left(\int_0^t \left[\frac{\mu\left(\frac{\tilde{y}_t}{(1-\tau)f(k)} - m_t\right) + \Delta_s}{\sigma} \right] dW_s^0 - \frac{1}{2} \int_0^t \left[\frac{\mu\left(\frac{\tilde{y}_t}{(1-\tau)f(k)} - m_t\right) + \Delta_s}{\sigma} \right]^2 ds \right).$$

Using this definition of Γ, it is clear that $E_0[\Gamma_T(\Delta)] = 1$ and Γ_t is a martingale. Thus, an application of Girsanov's Theorem gives us the above result.

Notice that with this, the financier can construct a Wiener process under the diversion distribution P_Δ where

$$W_t^\Delta = W_t^0 - \int_0^t \left[\frac{\mu\left(\frac{\tilde{y}_t}{(1-\tau)f(k)} - m_t\right) + \Delta_s}{\sigma} \right] ds. \quad (30)$$

Proof for transformation of Entrepreneurs problem

This transformation follows the same approach as Williams (2011), except integrating as well over the sample paths of x_t.

C Proofs for Section 2.3, Deriving static / dynamic IC constraints

Define

$$A_t = \begin{bmatrix} \Gamma_t \\ b_t \end{bmatrix}, \Omega_t = \begin{bmatrix} q_t \\ p_t \end{bmatrix}, \Lambda_t = \begin{bmatrix} \gamma_t \\ Q_t \end{bmatrix}.$$

From the evolutions of Γ_t, b_t given in (5) and (**??**) we have

$$dA_t = \underbrace{\Gamma_t \begin{bmatrix} 0 \\ \Delta_t \end{bmatrix}}_{M_t} dt + \underbrace{\frac{\Gamma_t}{\sigma}[\mu(z_t - \frac{b_t}{\Gamma_t}] \begin{bmatrix} 1 \\ m_t \end{bmatrix}}_{N_t}.$$

Then the current value Hamiltonian of the Entrepreneur's transformed Problem (Problem 1') is

$$\mathcal{H}(\Gamma, b) = \Gamma H = \Gamma u(c - (1-\tau)f(k)[\frac{b}{\Gamma}]) + \Omega_t' M_t + \Lambda_t' N_t.$$

By the stochastic maximum principle of Bismut (1978):

$$(1) \qquad H_\Delta - \lambda^{LM} = 0$$

where λ^{LM} is the Lagrange multiplier on the non-positivity constraint of Δ, and

$$(2) \qquad d\Omega_t = \rho \begin{bmatrix} q_t \\ p_t \end{bmatrix} - \frac{\partial \mathcal{H}}{\partial A} dt + \Lambda_t \sigma dW_t^0,$$ [19]

with terminal condition given by

$$\Omega_T = \frac{\partial \Gamma_T U(c_T - m_T^y)}{\partial A_T}.$$

Direct calculation gives that $\frac{\partial \mathcal{H}}{\partial \Gamma} = H$,

$$\frac{\partial \mathcal{H}}{\partial b} = \frac{\partial \mu(z - \frac{b}{\Gamma})}{\partial b}[\gamma + Qm] + Q[\mu(z - \frac{b}{\Gamma}) + \Delta] - \Gamma \frac{\partial u}{\partial b}$$

where $\frac{\partial u(c - m^y)}{\partial b} = -u'(c - m^y)(1 - \tau)\frac{1}{\Gamma}$.

Finally, using the change of measure from W_t^0 to W_t^*, given in (30) we obtain the final form of (9), (10).

C.1 Proof that stochastic job-destruction arrival simply scales the discount rate of both agents: $\tilde{\rho} = \rho + \eta$.

With stochastic death following a poisson arrival process, assuming the contracted relationship is destroyed at this point for both parties with outside consumption c^A obtained for each instant of time thereafter for the entrepreneur, the promised utility process follows

$$q_t = \begin{cases} \int_t^T u(c_s) \exp(-\rho s) ds, & N_s = 0 \\ q_t^A = \int_t^T u(c^A) \exp(-\rho t) ds, & N_s = 1. \end{cases}$$ [20]

By the Martingale representation theorem,

$$dq = [\rho q - u(c - m_t) + \Delta_t]dt + \gamma \sigma dW - \phi^q(q_t, N_t) dM_t$$

$$M_t = \int_0^t [-\eta_s ds + dN_s]$$

with

$$\phi^q(q_t, N_t) = \begin{cases} 0 & N_t = 0 \\ -q_t + q_t^A A, N_t = 1 \end{cases}.$$

Thus, the entrepreneurs value function is then modified from $V(z, x; c)$ given in the main-body (taking $\eta = 0$ there) to

[19]Note here in an abuse of notation we replaced $f(k)\sigma$ with σ since we later normalize $f(k) = 1$.

$$V(z,x,N;c) = V(z,x;c)E_s\big[\int_s^T \{N_{\tilde{s}}=0\}d\tilde{s}|N_s=0\big]$$

$$+\int_0^T \big[\int_0^{\hat{t}} \exp(-\rho t)u(c_s-m_s)ds + q^A_{\hat{t}}\big]Pr(dN_{\hat{t}}=1)d\hat{t}.$$

By direct calculation, $\mathbb{E}_0[\int_0^T \{N_s=0\}ds] = [\eta(T-0)]^0\exp(-\eta T)$ and so

$$V(z,x,N;c) = E\bigg(\int_0^T \exp(-(\eta+\rho)t)u(c_t-m^y_t)dt + \int_0^T (1-\exp(-\eta t))q^A_t dt$$

$$+\exp(-(\eta+\rho)T)U(c_T-m^y_T)\exp(-\eta t) + (1-\exp(-\eta T))\exp(-\rho T)U(c^A)\bigg).$$

C.2 Proofs for Contracting Results, Theorem 1

HJB for public financier is (re-framing $\tilde{J}=-J$ (so that we are finding the minimum not maximum), where we use that the W^x_t is independent of W^*_t

$$\rho\tilde{J}(z,x,p,q;\theta) = \min_{c,Q,\gamma\geq -p} c - y + J_z\mu(z) + \tilde{J}_x\mu^x(x) + \frac{1}{2}\tilde{J}_{zz}\sigma^2 + \frac{1}{2}J_{xx}\sigma^2$$

$$+J_q[\tilde{\rho}q - u(s)] + \tilde{J}_p[\tilde{\rho}p - \gamma\lambda^{-1} + (1-\tau)u'(c)] + \frac{\sigma^2}{2}\big[\tilde{J}_{qq}\gamma^2 + 2\tilde{J}_{qp}\gamma Q + \tilde{J}_{pp}Q^2\big]$$

$$+\sigma^2\big[\tilde{J}_{zq}\gamma + \tilde{J}_{zp}Q\big].$$

Assuming that the IC constraint is binding everywhere $\gamma=-p$, taking FOCs wrt c, Q, and using the CARA functional form for the entrepreneur

$$1 = (\tilde{J}_q + \tilde{J}_p\psi(1-\tau))u'(c)$$

$$\sigma^2\big[\tilde{J}_{qp}\gamma + Q\tilde{J}_{pp}\big] + \sigma^2\tilde{J}_{zp} = 0$$

We guess that the cost function for the public financier is

$$\tilde{J} = j_0 + (1-\tau)j_1 z + j^x_1 x - j_2 log(-q) + h(k)$$

where $k=\frac{p}{q}$.

With this guess, the optimal solutions for c and Q are given by

$$c = \frac{\log\psi}{\psi} + \log\big(j_2 + h'(k)(k-\psi(1-\tau))\big) - \frac{\log(-q)}{\psi}$$

$$Q = p\frac{\tilde{J}_{pq}}{\tilde{J}_{pp}} = -qk\big(\frac{h'(k)}{h''(k)} + k\big).$$

Taking derivatives of the guess, we have $\tilde{J}_z = j_1(1-\tau), \tilde{J}_x = \tau j_1^x, \tilde{J}_q = -\frac{1}{q}[j_2 + h'(k)k], \tilde{J}_p = \frac{h'(k)}{q}, \tilde{J}_{pp} = \frac{h''(k)}{q^2}, \tilde{J}_{qq} = \frac{1}{q^2}\big[j_2 + 2h'(k)k + h''(k)k^2\big], J_{pq} = -\frac{1}{q^2}(h'(k) + kh''(k))$.

Combining this with the FOC/solution for Q we have

$$\frac{\sigma^2}{2}\big[\tilde{J}_{qq}\gamma^2 + 2\tilde{J}_{qp}\gamma Q + \tilde{J}_{pp}Q^2\big] = \frac{\sigma^2 k^2}{2}\big[j_2 - \frac{(h'(k))^2}{h''(k)}\big].$$

Hence, substituting all of the above into the HJB and matching coefficients we get

$$\tilde{\rho} j_0 = \frac{\log\psi}{\psi} + j_1(1-\tau)\mu_0 + j_1^x\tau\mu_0^x \tag{31}$$

$$\tilde{\rho} j_1 = -(1 + \frac{j_1}{\lambda}) \tag{32}$$

$$\tilde{\rho} j_1^x = -(1 + \frac{j_1}{\lambda^x}) \tag{33}$$

$$\tilde{\rho} j_2 = \frac{1}{\psi} \tag{34}$$

$$\tilde{\rho} h(k) = \frac{h'(k)k}{\lambda} + \frac{1}{\psi}\log\big(j_2 + h'(k)(k - \psi(1-\tau))\big) + \frac{\sigma^2 k^2}{2}\big[j_2 - \frac{(h'(k))^2}{h''(k)}\big]. \tag{35}$$

The last equation is a second order ODE, with p_0 fixed. The final solution for the public financier takes q_0 as fixed (as well as x_0, z_0 and solves for the optimal p_0. From the definition fo $k_t = \frac{p_t}{q_t}$ and the solved forms of p_t, q_t as promised marginal utility and promised utility processes given in (12), (11) respectively, that when $\lambda \to \infty$ (intangible TFP is permanent), $k_t = (1-\tau)\psi = k^*_{\lambda\to\infty}$ for all t. Referring to Williams (2011), who has the same ODE except with $\tau = 0$, and finds (via numerical methods) that for not perfectly persistent processes the optimal initial condition is $k_0 = \frac{\tilde{\rho}}{\tilde{\rho}+\frac{1}{\lambda}} k^*_{\lambda\to\infty}$. This k_0 is simply the ratio of the discount factors for the promised utility and marginal utility process (the wedge coming from the degree of persistence in the private information cash-flow component). In our case, with $\tau > 0$,

$$k_0 = \frac{\tilde{\rho}}{\tilde{\rho} + \frac{1}{\lambda}}(1-\tau)\psi. \tag{36}$$

Solving the above system of equations and plugging in the optimal k_0 in (36) yields the solution for $J^P = -\tilde{J}^P$ given in Theorem 1, (19), where we use the fact that $h'(k_0) = 0$ and so

$$\tilde{\rho}h(k_0) = \frac{1}{\psi}\log(\frac{1}{\tilde{\rho}\psi}) + \frac{\sigma^2}{2\tilde{\rho}\psi}k_0^2. \tag{37}$$

With the above, optimal compensation under the contract is given by

$$c(z,x,q,p) = \frac{1}{\psi}\Bigg(\log(\frac{1}{\tilde{\rho}} + \psi h'(k)(k - \psi(1-\tau)) - \log(-q)\Bigg) \equiv -\frac{\log(-q\hat{c}(k))}{\psi}. \tag{38}$$

In other words, compensation is independent of the levels / history of x and z conditional on the level of promised utility q and the ratio of promised marginal utility to promised utility k.

Using these results, $u(c_t) = \hat{c}(k_t)q_t$ and so the dynamics of q_t and p_t can be written as

$$dq_t = [\tilde{\rho} - \hat{c}(k_t)]q_t dt - \sigma p_t dW_t$$

$$dp_t = [(\tilde{\rho} + \frac{1}{\lambda})p_t - \psi\hat{c}(k_t)q_t]dt - \sigma\hat{Q}(k_t)q_t dW_t$$

where $\hat{Q}(k) = k\left(\frac{h'(k)}{h''(k)} + k\right)$ and $W_t = W_t^*$.

At the optimal k_0, applying Ito's lemma, direct calculation gives the ratio $k_t = \frac{p_t}{q_t}$ remains constant.

At k_0, $\hat{c}(k_0) = \tilde{\rho}$ and since $p_t = k_0 q_t$, q_t is a martingale:

$$dq_t = -\sigma k_0 q_t dW_t^*. \tag{39}$$

Solving this directly we obtain

$$q_t = q_0 \exp\Bigg(-\frac{\sigma^2 k_0^2}{2}t - k_0 W_t\Bigg) \tag{40}$$

or, in terms of consumption,

$$c_t = \bar{c}(q_0) + \frac{\sigma^2 k_0^2}{2\psi} + \exp\Bigg(-\frac{k_0^2}{2}t - k_0 W_t\Bigg). \tag{41}$$

C.3 Verifying Incentive Compatibility

With the adjustment for $\tau > 0$, the proof of incentive compatibility follows those of Williams (2011).

More specifically, from the above, we have $Q_t = (-k_0)^2 q_t = -(1-\tau)^2\psi^2(\frac{\tilde{\rho}}{\tilde{\rho}+\frac{1}{\lambda}})q_t$.

Let Q_t^W denote the Q_t with $\tau = 0$ (which was studied in Williams (2011)), then $Q_t = (1-\tau)^2 Q_t^W$ and hence the verification of sufficient conditions in Theorem 4.1 of Williams (2011) follows immediately from A3.2 of his paper.

C.4 Proof of Theorem 3

Proof: The proof follows the logic of Bertrand competition with heterogeneous costs across firms. As q_0 is a sufficient statistic for the entrepreneur in his utility under either financier's contract, the entrepreneur's best-response is to select the financing offer which offers the highest q_0.

Notice that the public financier faces no fixed cost of financing and has no financing constraint and so provided the projects under the individual rationality assumption for the public financier in funding all projects, the dominant strategy to bid $q_0 > 0$ for all θ. On the other hand, this is not the case for the private specialist given $\nu > 0$.

Fix a given θ and suppose $q_0^S = q_0^P \geq V^A(\theta)$. With equal levels of promised utility and cost of injecting capital, from the contracting results of the earlier section the specialist's surplus above that of the public financier is $\pi(\theta) - \nu$.

First case: $\pi(\theta) - \nu > 0$

First, if $J^P(q_0^P; \theta) - [p_0^k k - M_0] > 0$. Assuming the entrepreneur puts some positive weight on accepting the specialist offer the public financier can deviate and offer $q_0^P + \varepsilon$ and win the bid with probability one. On the other hand, if all weight is put on the entrepreneur selecting the public financier's offer, then the specialist can offer $q_0 + \varepsilon$. Taking $\varepsilon \to 0$, by continuity of the specialist's contract, the net surplus of this deviation is $\pi(\theta) - \nu > 0, w[\pi - \nu]$ where $w \in (0,1)$ is the entrepreneurs mixing strategy hence also not an equilibrium.

Now, if $J^P(q_0^P; \theta) - [p_0^k k - M_0] < 0$ the public financier will always do at least weakly better by reducing q_0^P to the point $J^P(q_0^P; \theta) - p_0^k k \geq 0$. Hence such a q_0^P cannot occur in equilibrium.

Finally if $J^P(q_0^P; \theta) - [p_0^k k - M_0] = 0$, and the entrepreneur is mixing in their selection between the two financiers then the public financier cannot deviate to a higher promised utility to the entrepreneur without doing worse than autarky for themselves. On the other hand, the specialist can again make an arbitrarily small higher bid and earn $\pi(\theta) - \nu > w[\pi - \nu]$ where $w \in (0,1)$ is the entrepreneurs mixing strategy. If instead the entrepreneur selects financing solely by the specialist given these bids, then the specialist will lose the bid if he bids any lower (with $q_0^S = q_0^P$) and win but with surplus less than $\pi(\theta) - \nu$ for any $q_0 > q_0^P$. Thus, $q_0^P = q_0^S$ s.t. $J^P(q_0^P) = 0$ with the entrepreneur being financed by the specialist is the unique, symmetric bid equilibrium for this case where the specialist has the comparative advantage.

Second case: $\pi(\theta) - \nu < 0$

If the entrepreneur is mixing with weight w then the specialist is better off reducing his bid $q_0^S < q_0^P$ and thus losing on θ. Now if $J^P(q_0^P) > 0$ and $q_0^P > V^A(\theta)$ then the public financier always has a positive deviation until $q_0^P = V^A(\theta)$. At this level, (with $q_0^S = q_0^P$), assuming the entrepreneur chooses the public financier in a tie, the payoff to the specialist is negative if he tries to weakly outbid the public financier and zero otherwise. The public financier cannot do any better while satisfying individual rationality for the entrepreneur and hence $q_0^P = V^A(\theta) = q_0^S$ with the entrepreneur financed by the public financier is the unique, symmetric bid equilibrium in this case.

Asymmetric bid equilibria:

Now we have shown that if bids are equal to each other what the equilibrium strategies must be (ie which levels of q_0 result in fixed points). It remains to pin down the asymmetric bid equilibria.

First if $\pi(\theta)-\nu > 0$ with $q_0^P = q_0^S$, suppose an equilibria exists with $q^P < q_0^P$. In this case, the specialist wins with certainty but earns $< \pi(\theta) - \nu$ (since from the contracting solution his payoff is strictly decreasing in q_0 and his payoff exactly equals $\pi(\theta) - \nu$ at $q^S = q^P$) and so can strictly increase his payoff by reducing his bid to $q^S \in (q^P, q_0^P)$.

Second, if $q^P > q_0^P = q_0^S$ then the Public financier wins the bid, resulting in zero for the specialist. But then the specialist could increase his bid to $q^S = q^P + \varepsilon$ and receive $\pi(\theta) - \nu > 0$. Thus no asymmetric bidding equilibrium exists when the specialist has a comparative advantage ($\pi(\theta) - \nu > 0$).

On the other hand, $\pi(\theta) - \nu \leq 0$ then we claim that any $q_0^S < V^A(\theta) = q_0^P$ with the entrepreneur selecting the Public financier is an equilibrium. As reasoned above, the entrepreneur will simply take his outside option for any downward deviation in q_0^P resulting in a payoff of zero for the public financier, while raising q_0^P increases the payment to the entrepreneur without increasing the winning probability. Finally the specialist strictly prefers to not bid than bid weakly higher than V^A (whereby he receives surplus $< \pi(\theta) - \nu < 0$).

To conclude the characterization of the equilibria set, it is sufficient to note that with a sufficiently low outside option for the entrepreneurs and insufficient internal funds to start the project and $\mu > 0, z_0 \geq 0$, an equilibrium where an entrepreneur receives no financing cannot occur.

To map to the specialists problem stated in the theorem, bidding on a firm is individually rational for the specialist only when $\pi(\theta) - \nu \geq 0$ and in the case he bids, the equilibrium must be as solved above. However, the set of $\theta : \pi(\theta) - \nu \geq 0$ may require more funding than the specialist is endowed with, thus the financing problem for the specialist must include the budget constraint.

The Public problem is even simpler. On the set of projects for which the specialist doesn't choose to bid above the entrepreneur's outside option, the public financier has monopoly power and so their optimal bid is to take the entrepreneur to her outside option, that is autarky. For the other projects, not bidding on the projects cannot be an equilibrium given the NPV of funding the project is positive.

□