Public Listing Choice with Persistent Hidden Information*

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Abstract

How much does firm intangibility amplify CEOs' persistent private information and reduce firms' public listing propensity? We develop a model of competing public and private investors financing firms heterogeneously exposed to persistent private cashflows. Equilibrium financing is driven by information rent differentials in CEO compensation. We validate and structurally estimate the model using firm listing and CEO compensation data. We find private (intangible) cashflows exhibit 63% higher persistence than their tangible counterparts. Further, if firm intangibility levels returned to those of 1980, mean listing propensities would increase 5 percentage points while mean CEO variable pay growth would decrease by 62%.

Keywords: intangible capital, CEO compensation, private equity, dynamic optimal contracts, assignment model, structural estimation
JEL: C78, D86, E22, G32, M12, O33

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1 Introduction

"Our problem – which we can't solve by studying up – is that we have no insights into which participants in the tech field possess a truly durable competitive advantage...Predicting the long-term economics of companies that operate in fast-changing industries is simply far beyond our perimeter."

-Warren Buffett, 1999 Berkshire Hathaway shareholder letter

Many significant firm events, such as legal settlement agreements, new trade secrets or proprietary consumer data, are associated with a firm's intangible assets. Such events have persistent effects on firm cash flows, yet are often not observed by outside investors. Furthermore, even when there is full public disclosure as in the case of newly granted patents, little consensus exists on how investors can appropriately value these individual developments. Firm insiders' persistent private information not only magnifies the lifetime impact of a cash flow shock, but also drastically alters the types of incentives needed for truthful reports. The opacity of intangible assets and the challenges in identifying their resulting cash flows may amplify the persistent private information of insiders. Spurred by the information communication technology (ICT) revolution, firms' aggregate accumulation of intangible assets may have induced a rise in public CEO compensation level and performance sensitivity. To the extent that private investors avoid these information frictions via their expertise and interaction with firm insiders, such increased compensation costs for public financiers may have reduced the net benefit of being publicly listed on stock markets and caused a listing fall.

In this paper, we quantify how much rising intangibility has amplified public CEOs' persistent private information and contributed to the trends in public listings and CEO compensation. Identifying the substantive drivers of these joint patterns is crucial to evaluate the role of potential policy interventions. However, the latent nature of private information precludes direct measurement. To tackle this problem, we build, empirically validate, and structurally estimate a market equilibrium model of firm financing and CEO compensation where CEOs have persistent private information over cash flows generated by intangible assets.

Our model generates variations in firm public listing decisions and CEO compensation packages through heterogeneous firms' exposure to private cash flows. Crucially, the distortions caused by these private cash flows are magnified by their persistence. To test our theory of a common information friction driving both firm listing and CEO pay decisions, we validate our model predictions proxying the exposure to private cash flows using measures of firm intangibility and primarily relying on a large dataset of public and private US firms and their CEOs. Furthermore, we separately structurally estimate the private cash flow persistence parameter identified using non-overlapping moments (and disjoint data) of firm listing choice and CEO compensation and compare their values. We then use our estimates to quantify the amplification effect the ICT revolution had on persistent private information by evaluating the counterfactual where firm intangibility remained at the levels observed in 1980, the approximate start of the technological transformation.

In the model, public investors design optimal compensation contracts to dynamically induce truth-telling akin to Williams (2011) and Bloedel et al. (2020).[1] Despite the manager having no influence over the actual cash flow process, to preclude cash flow diversion the optimal public CEO pay is performance sensitive, with the level of sensitivity increasing in the lifetime size of the private information. The risk built into the contract to incentivize truthtelling is compensated with positive expected growth in pay over time. Private investors have access to a costly monitoring technology which allows them to design first-best efficient contracts. Competition in financing between public and private investors then generates a private equity (PE) premium as the foregone information rents net monitoring costs.[2] Capacity constraints for individual private investors together with competition against the public investors induce a selection effect where more intangible firms are more likely to be privately financed, and, conditional on being publicly financed, typically have higher levels of CEO compensation and performance sensitivity. Aggregate private investment is tied to the average PE premium and in (general) equilibrium is an increasing function of average firm intangibility. Then,

[1]Bloedel et al. (2020) demonstrate with a counterexample that the contract in Williams (2011) is not optimal amongst a general class of contracts, but only amongst stationary contracts satisfying first-order incentive compatibility. In light of this, we implicitly restrict the space of contracts to those ensuring no hidden savings and leverage recent advancements in the stochastic maximum principle literature to assure optimality of the contract amongst all incentive compatible contracts (under mild regularity restrictions).

[2]This PE premium is correlated to the equity share of public CEO compensation, with higher private information persistence mapping to higher equity-based pay, and hence, given the continuously priced equity in public markets, a higher pay-performance sensitivity. While a positive PE premium is generated with permanent shocks (i.e. if cash flows follow a Brownian motion), the gain in theoretical simplicity is diluted by empirical difficulties implied by non-stationary cash flow and compensation processes. Moreover, in our empirical work, we find that reported earnings and compensation dynamics are better captured by a persistent cash flow process.

while an increase in intangibility amongst public firms will lead to higher compensation, an increase in aggregate intangibility can in fact lower average public CEO compensation due to a selection effect with rising PE funds.

This mechanism combined with a relaxation of PE funds by the National Securities Market Improvement Act (NSMIA) in the late 1990s can jointly rationalize the broad historical patterns in aggregate US public listing, CEO pay, PE premium, and new business formation and hence economic growth. Moreover, through the lens of our model, the observed patterns suggest that the increased disclosure requirements of the Sarbanes-Oxley Act (SOX) may not have been effective in increasing transparency in public markets.

We obtain additional support for our theory in supplemental historical data by estimating time-varying elasticities of CEO pay to intangibility. These elasticity estimates rationalize the inflection point in aggregate US public CEO pay trends around 1980 and overall patterns in the past half a century.[3] Furthermore, we use international data and aggregate US time-trends to validate the differential implications of our theory of rising intangibility.

Our estimates of private information persistence across the two structural estimations are statistically indistinguishable from each other. The estimation suggests a 63% higher persistence in private information cash flows than persistence in the tangible cash flows implied by physical investment. The inferred aggregate effects of a secular increase in firm intangibility is large. If US firm intangibility had remained at its 1980 level, listing propensities would be 5 percentage points higher, while the annual growth in average CEO pay would be reduced by 62%.

Related Literature: We quantify a new technological channel driving the decline in publicly listed US firms since 1997 documented by Gao et al. (2013b) and Doidge et al. (2017). Other explanations for the decline have largely focused on US regulatory and institutional changes.[4] However, this phenomenon has recently become apparent across other advanced economies. Our proposed driver of rising information frictions generates joint predictions

[3]In particular, we find that the time-series variation in aggregate firm levels of intangibility helps rationalize the patterns of average public CEO compensation observed over the second half of the 20th century both in level and use of equity grants. This is important as according to Edmans et al. (2017) "[t]he reasons for this evolution are not fully understood."

[4]For example, Leuz (2007) and Iliev (2010) discuss the increase of compliance costs of being public due to SOX, Ewens and Farre-Mensa (2020) and Kwon et al. (2020) the relaxation of private equity funding due to NSMIA, and Davydiuk et al. (2020) a combination of the two.

on public listings, CEO compensation, PE premia, and business dynamism which are consistent with established US and international evidence.[5]

Our theory of a firm's decision to go public is based on lower intangible cash flow risk and involves a competitive matching equilibrium between heterogeneous firms and investors. Classical theories highlight the benefits of going public from a lower cost of capital.[6] Other studies focus on heterogeneity in investment opportunities and costs of going public.[7] These models largely favor sorting in terms of older, bigger, and more productive firms, whereas Doidge et al. (2017) find that listing propensities have declined across all sizes and industries. This suggests that neither the amount of capital nor the type of investments undertaken across industries are core to the decline.

Due to limited data on private firms, empirical examinations of firm listing decisions have been scarce.[8] We contribute to the empirical evidence of firm listing decisions using Capital IQ data on both public and private firms.[9]

Our paper contributes to the literature examining the determinants of CEO compensation. Leading theories on the rise in the level of CEO compensation have tended to focus on size-driven mechanisms like Tervio (2008) and Gabaix and Landier (2008) or size and market power-driven like Bao et al. (2022). Our work complements these stories by examining other firm characteristics which we find not only to have similar levels of explanatory power in the cross-section but also to help rationalize the evolution of US CEO compensation level and composition over the post-war sample documented by Frydman and Saks (2010).[10]

[5]Other works have examined these trends in isolation. See Kahle and Stulz (2017), Caskurlu (2020), Frydman and Saks (2010), Edmans et al. (2017), Moskowitz and Vissing-Jorgensen (2002), Kartashova (2014), Bloom et al. (2020), Pellegrino (2021) and Eckbo and Lithell (2022).

[6]This reduction arises due to (i) a broader pool of financiers in Merton (1987) and Rajan (1992), (ii) diversification of insiders' risk in Levine (1991) and Pagano (1993), (iii) improved monitoring in Holmstrom and Tirole (1993) and Pagano and Roell (1998), or (iv) better guidance from market experts in Maug (2001).

[7]For example, Clementi (2002), Ferreira et al. (2012), and Spiegel and Tookes (2013) focus on firms' productivity, while Ritter (1987) and Gupta and Rust (2017) study regulatory costs, Campbell (1979), Yosha (1995), Maksimovic and Pichler (2001) analyze costs associated with the potential loss of confidentiality, Jensen and Meckling (1976), Leland and Pyle (1977), Jensen (1989), and Chemmanur and Fulghieri (1999) examine asymmetric information costs.

[8]A few exceptions include Lerner (1994), Pagano et al. (1998), and Chemmanur et al. (2010) who have data only on certain industries.

[9]Other studies, such as Gao et al. (2013a), Gao and Li (2015) and Acharya and Xu (2017), have used our main data source, Capital IQ, to study differences between public and private firms, but, to our knowledge, none examine listing status and CEO pay in relation to intangibility.

[10]In perhaps the closest paper empirically examining firm agency-inducing characteristics in the finance industry, Cheng et al. (2015) shows persistent firm-specific risk induces higher levels of CEO pay to compensate the CEO for the magnified pay sensitivity risk faced.

Other papers have examined different facets of exogenous technological change driving compensation trends. Much of this literature focuses on human capital as a facet of intangible assets, which induces increased restricted equity compensation for retention. To the extent that executive human capital is embedded in the private information of the firm, the reduced form predictions on the level of CEO pay are the same. However, in our context, performance sensitivity is an intentional response to, rather than a by-product of, evolving outside options, and rationalizes the initial popularity in the 1980s of option-based compensation rather than simply deferred stock grants.[11]

Beyond the impact on compensation, our paper also examines the implications of contracts on firm dynamics.[12] The closest paper in this literature is Ward (2022) who studies the effect of an intangibility driven agency friction on public firms' investment and market valuation dynamics. The "pure moral hazard" agency friction in his model is tied to hidden effort on intangible asset accumulation while we study a "hybrid moral hazard" framework.[13] As noted by Edmans et al. (2017) small differences in information frictions generally lead to substantively different economic implications. For instance, the "pure moral hazard" friction predicts a positive correlation between intangibility and performance while the "hybrid moral hazard" does not require any type of association.[14] Other quantitative papers, such as Ai et al. (2022), Gayle and Miller (2009) and Gayle et al. (2015), focus on "pure moral hazard" models. In these papers, the estimated size of the private information shock is found to be relatively small and dwarfed by effects based on firm size.[15] To

[11]See Lustig et al. (2011), Sun and Xiaolan (2019), Frydman and Papanikolaou (2018) and Kline et al. (2019). See Murphy (2013) for a survey on explanations about these trends.

[12]Continuous time contracting frameworks have become increasingly popular due to their tractability, beginning with DeMarzo and Sannikov (2006), Biais et al. (2007), and Sannikov (2008). In contrast to our paper, these works largely focus on agency issues with independent and identically distributed private information. Garrett and Pavan (2012) extend this framework to have persistent private information through unknown initial conditions. Bolton et al. (2019) examine how human capital affects debt capacity and CEOs' risk exposure.

[13]Gayle and Miller (2015) define "pure moral hazard" models as those with hidden actions and "hybrid moral hazard" models as those with hidden information and actions. Our choice to study a "hybrid moral hazard" friction is in part motivated by their empirical work which tests the two model classes on data of CEO compensation and performance, and find evidence that the former class is rejected in the data, while the latter class cannot be.

[14]Controlling for selection is important for understanding the fundamental information frictions at play in our dataset, as we find that firm intangibility has a slightly negative correlation with productivity when pooling across public and private firms, as opposed to the positive correlation found in Ward (2022) based on public firms.

[15]In a "hybrid moral hazard" model like ours, for a given level of private information shocks, persistence magnifies the aggregate size of private information and introduces a mixture of moral hazard and adverse selection considerations within the contracting environment which substantially alters the firm financing and compensation structures.

our knowledge, our paper is the first to quantify economic effects of persistent private information arising from intangible assets, as well as provide a credible estimate for the level of persistent private information.

The remainder of the paper is structured as follows. Section 2 presents the model and testable implications. Section 3 describes our dataset. Section 4 uses proxies of firms' private information cash flow characteristics to empirically validate our cross-sectional and time-series predictions. Section 5 structurally estimates the model and conducts counterfactual experiments to infer the quantitative importance of persistent private information to firm listing and CEO compensation decisions. We conclude in Section 6.

2 Model

The model adapts the principal-agent contracting environment of Williams (2011) to a corporate finance setting with cash flows driven by a mixture of publicly and privately observed cash flows, referred to as tangible and intangible cash flows respectively. Taking into account the discussion in Bloedel et al. (2020) we allow the agent to privately save and borrow directly at the risk-free rate.[16] Our characterization of optimal contracts in this setting improves on Williams (2011) and Bloedel et al. (2020) by establishing global incentive compatibility, rather than restricting to contracts which satisfy first-order incentive compatibility.[17] We do so by introducing a stochastic maximum principle (SMP) new to the contracting literature and demonstrating its applicability subject to mild technical restrictions to the contract space.[18]

To study the potential equilibrium effects of intangibility-induced private information, we embed the optimal contracting problem into a market equilibrium setting with two competing principals, representing the public and private equity markets respectively. The principal representing the public

[16]Bloedel et al. (2020) raise concerns about the appropriateness of the stochastic maximum principle applied by Williams (2011) and the numerical observation used to infer the optimal contract by providing a counterexample. However, they also show that, if the agent has the option to engage in private saving, the Williams (2011) contracts are optimal amongst the class of first-order incentive compatible contracts.

[17]Characterizations of sufficient conditions to appeal to the first-order approach in static settings are provided by Mirrlees (1971), Rogerson (1985) and Jewitt (1988). For a discussion of the applicability of the first-order approach in dynamic settings with persistent private information (though in a slightly different environment) see Battaglini and Lamba (2019).

[18]To apply an appropriate SMP we draw on the results in Maslowski and Veverka (2014), Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019). Details on the SMP introduced here and the establishment that its application is amenable to this setting are given in Appendix A.1.

investors is unable to directly observe the intangibility-driven cash flows, but has deep pockets. The other principal has a costly monitoring technology which allows to avoid these intangibility-induced information frictions, but has limited funds.

The remainder of the section is as follows. We begin by describing the contracting problem and the partial equilibrium model environment where the private principal is assumed to be financially constrained. We then endogenize the supply of funds and the PE premium in a general equilibrium, competitive matching model. Finally, we conduct comparative static analysis to evaluate the impact of changing regulation and technology on the public listing and compensation patterns. A formal description of the problem and all the proofs are relegated to Appendix A, as well as to the online appendix.

2.1 Environment

Time is continuous and infinite. All parties share a time discount rate ρ. There is a unit mass of firms, each owned by a risk-averse agent (also called "CEO" or "entrepreneur") with constant absolute risk-aversion (CARA) utility function of consumption c equal to $u(c) = -e^{-\psi c}$, where ψ is the risk-aversion coefficient. At time $t = 0$ an entrepreneur needs one unit of funds to get the firm off the ground. If funding is not obtained, the agent has an outside option of $\underline{q} \in \mathbb{R}_{-}$. If funding is obtained, after the initial financing, the agent, unobserved by the principal, can privately save and borrow at a risk-free rate r.[19] Then, for $t > 0$ a firm of size K produces cash flows $Y_t = y_t K$ where y_t is the firm profitability rate. Since the channel we wish to highlight does not depend on size, we will abstract from size effects and set $K = 1$ throughout the model. Firm profitability y_t can be decomposed into a tangible component x_t and an intangible component z_t, with share $\tau \in [0, 1]$ of tangible profitability dependence, so that

$$y_t = \tau x_t + (1 - \tau) z_t.$$

Both x_t and z_t follow an Ornstein-Uhlenbeck process, which is the continuous time equivalent of an autoregressive of order 1 (AR(1)) process, with persistence λ^i, drift μ^i, volatility σ^i, and initial condition i_0 which is fixed at

[19]We implicitly assume that there are borrowing limits which preclude the agent from self-financing the project through risk-free borrowing. This assumption is consistent with borrowing limits due to exogenous elements, such as lack of reputation or collateral, that could preclude the agent the access to the saving technology.

the steady state long-run average (i.e. $\lambda_i \mu_i$), for $i \in \{x, z\}$, that is

$$di_t = \left(\mu^i - \frac{i_t}{\lambda^i}\right)dt + \sigma^i dW_t^i, \qquad i_0 \text{ fixed }, \qquad i \in \{x, z\}. \tag{1}$$

We assume that the tangible component x_t is publicly observable while the intangible component z_t is observable only by the agent. As such, the agent has persistent private information on the firm profitability y_t. This induces hybrid moral hazard wherein the agent has private information on the current state and can adjust their decisions to misreport accordingly. Finally, firms vary in their cashflow characteristics $\theta = (\{\mu^i, \lambda^i, \sigma^i\}_{i=\{x,z\}}, \tau)$ which are drawn from a distribution with cumulative distribution function (CDF) $G_\theta(\cdot)$.

There are two representative, risk-neutral principals (also called "financiers" or "investors") who compete using compensation contracts to fund the pool of heterogeneous firms. The representative public investor, P, has unlimited funds but cannot observe the intangible cash flows z_t, and so restricts the offered contracts to those that induce truthful reporting. The representative private investor, S, is a specialist and so uses a monitoring technology connected to their expertise to observe the intangible component z_t, avoiding the associated information frictions. This monitoring comes at a lifetime cost ν. The specialist is however financially constrained by a budget $B < \infty$.

2.2 Optimal Compensation Contracts

We restrict our analysis to subgame perfect Nash equilibria (SPNE) which assures that the compensation contracts offered by the representative investors at time 0 are optimally designed at the firm-investor level.

Denote $\omega_t^f(q_0, \theta)$ as the optimal compensation contract offered by a financier of type $f \in \{S, P\}$ to a CEO of a type θ firm with initial promised utility level q_0. A type f investor's expected return from financing a CEO of a firm of type θ offering q_0 initial promised utility is

$$R^f(q_0, \theta) = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\big(y_t - \omega_t^f(q_0, \theta)\big)dt \middle| \theta\right] - \nu \mathbf{1}_{\{f=S\}} - 1.$$

Different exposures to persistent private information generate a firm-specific premium between the private and public investor's returns. Denote $\pi(q_0, \theta)$ as the information premium, that is the expected information cost of the public investor financing a firm of type θ offering promised utility q_0. Since the private investor avoids these information frictions at cost ν, for fixed initial promised utility q_0 and firm type θ, the PE premium takes the form

$$R^S(q_0, \theta) - R^P(q_0, \theta) = \pi(q_0, \theta) - \nu. \tag{2}$$

Theorem 1 presents the optimal compensation contract offered by the two financiers and establishes that the above premium is independent of the level of initial promised utility.

Theorem 1. *Under mild technical restrictions, the optimal contract between the public principal and an agent owning a type θ firm yields an information premium*

$$\pi(\theta) = \frac{\psi}{2}\left(\frac{(1-\tau)\sigma^z}{\rho + \frac{1}{\lambda^z}}\right)^2, \tag{3}$$

with total compensation to the agent financed by a public investor given by

$$\omega_t^P(q_0, \theta) = \omega_{fix}(q_0) + \omega_{gro}(\theta)t + \omega_{per}(\theta)W_t^z, \tag{4}$$

where the first term is a fixed salary compensation component with $\omega_{fix}(q_0) = -\frac{1}{\psi}\log(-\rho q_0)$, which is equal to the total compensation offered by a private investor $\omega_t^S(q_0, \theta) = \omega^S(q_0)$, the second term is a growth compensation component with $\omega_{gro}(\theta) = \rho^2\pi(\theta)$, and the third term is a performance-based compensation component with $\omega_{per}(\theta) = \rho\sqrt{\frac{2}{\psi}\pi(\theta)}$.

By inspection, the optimal contracts offered by the public and private investors generate stark differences in the compensation level, growth rate, and risk. The performance-based compensation component of equation (4) suggests that, in order to incentivize truthful revelation of the entire cash flows, the public investor rewards (penalizes) unexpectedly high (low) reported profitability. Since agents are risk-averse, they must be compensated for this additional risk exposure. Moreover, the possibility for the agent to privately save precludes more complex, non-stationary contracts (including those which result in immiseration), thereby requiring the public investor to steadily increase the expected compensation over time, resulting in a positive growth component.[20] Consequently, for a given level of promised utility, the compensation proposed by the public investor offers positive average compensation growth, but is performance sensitive. In contrast to the model of Ai

[20]This feature implies that, over a sufficiently long time horizon, this investor may have to actually subsidize the CEO compensation from own funds. This can be easily precluded with the introduction of a stochastic Poisson firm destruction rate, η, which yields identical results with an appropriate modification of the discount rate from ρ to $\rho + \eta$.

et al. (2022), the optimal compensation with the uniformed public principal only depends on the realizations of the unobserved cash flows, fully insuring the observable cash flows.[21]

Of course, the complete stabilization of the CEO compensation with private financing is a highly stylized result and abstracts from the illiquid equity tied into private CEO contracts. In Appendix **??**, we extend the model with an ex-ante effort choice which impacts the long-run level of total cash flows. As in Holmstrom and Milgrom (1987), backloaded equity compensation optimally mitigates this pure moral hazard friction. Since equity traded on a public stock exchange has continuously updated valuations, public firm CEOs can rebalance their portfolio of retained equity in the secondary market or use it as collateral for private borrowing, minimizing the effort choice distortion without eliminating incentive provision. On the other hand, illiquid private equity exacerbates this pure moral hazard friction, lowering long-run average performance relative to the same firm if publicly financed (as suggested by Larrain et al. (2021)). Thus, when the intangibility induced information premium is small, public listing provides net economic benefits associated to having a large dispersed ownership with commonly distributed information, but is inefficient when intangibility amplified information frictions are large.

Moreover, in the empirical analysis, we relax the assumption of perfect monitoring, implying that the private investor imperfectly observes the intangible cash flows. Nevertheless, the private financier has more accurate information about the realizations of z_t than the public investor, which implies that has to offer a compensation contracts characterized by growth and performance sensitivity, though at lower levels than for the public investor. With these considerations, our first testable empirical prediction is as follows.

CEO Pay Testable Prediction. *All else equal, higher total and performance sensitive pay are associated with (i) higher levels of firm intangibility, (ii) intangible cash flow volatility, and (iii) being publicly listed.*

[21] Ai et al. (2022) study an environment where the agent has a hidden investment activity which influences the long-run level of the project cash flows, as opposed to the splitting of the cash flows in our setting. This feature combined with a constant relative risk-averse agent, which implies non-zero wealth effects, result in an optimal compensation contract which depends on both the realizations of the observable and unobservable stochastic cash flow processes. See Appendix A.1 for details on our results.

2.3 The Market for Firm Financing and Listing Choice

The private and public investors compete to finance the heterogeneous pool of entrepreneurs using the optimal contracts from above. Since CEOs' preferences over financing are summarized by the expected lifetime utility offered under the contract, their financier type selection rule (also referred to as their "listing choice") is, in a SPNE, given by an indicator function $\mathbb{I}^f\left(q_0^f, q_0^{-f}\right) \in \{0,1\}$ which is non-zero for at most one financier and only if q_0^f, the initial promised utility offered by financier f, is (weakly) greater than q_0^{-f}, the rival financier's promised utility, and the resulting compensation gives a higher utility than $\underline{q}$, the CEOs' value of forgoing financing.

Investors maximize their total expected profits from contracting with the various firm types by selecting the firms they will finance through bids of initial promised utility subject to their budget constraint and taking into account the agents' outside option and their own financing type. That is, taking as given both the bidding strategy of the rival financier q_0^{-f} and the financier selection strategy of the agent conditional on the bids, $\mathbb{I}^f\left(q_0^f(\theta), q_0^{-f}(\theta)\right)$, an investor of type $f \in \{P, S\}$ solves the following firm bidding problem:

$$\max_{q_0(\theta)} \int_\theta R^f(q_0(\theta), \theta)\mathbb{I}^f\left(q_0(\theta), q_0^{-f}(\theta)\right) dG_\theta(\theta) \tag{5}$$

s.t.

$$\int_\theta \mathbb{I}^f\left(q_0(\theta), q_0^{-f}(\theta)\right) dG_\theta(\theta) \leq B^f,$$

where the budgets of the private and public investor are $B^S = B < \infty$ and $B^P \to \infty$ respectively.

Using the results from Theorem 1, each representative investor's payoff from financing a given firm of type θ with a bid of initial promised utility q_0 can be decomposed as follows

$$R^f(q_0, \theta) = \Upsilon(\mu) - \Lambda^f(\theta) - X(q_0) \tag{6}$$

where $\Upsilon(\mu) := \frac{\mu - \omega^S(\underline{q})}{\rho} - 1$ is the net present value (NPV) of the investment absent information frictions and paying the agent's outside option, that is the net payoff for an investor with all the market power and full information on the projects cash flows, with firm type θ expected cash flow return $\mu := \mathbb{E}[y_t|\theta]$; $\Lambda^f(\theta)$ is a financier type f-firm type θ specific information cost with

$\Lambda^S(\theta) = \nu$ and $\Lambda^P(\theta) = \pi(\theta)$; and $X(q_0) := \frac{\omega^S(q_0) - \omega^S(\underline{q})}{\rho}$ is a financier's fixed compensation cost in excess of paying the agent's outside option.

Given the different monitoring capabilities and optimal contracts, the set of firms which are individually rational for the private and public financiers to finance are distinct. The minimal bid an agent will accept is $\underline{q}$, so for it to be individually rational for a type f financier to finance a type θ firm, the expected return at the agent's own outside option must be positive, that is

$$\Upsilon(\mu) - \Lambda^f(\theta) \geq 0. \qquad \left(IR^f\right)$$

Thus, private investors are essential for financing projects where the information rent $\pi(\theta)$ required for public financing exceeds the NPV given by $\Upsilon(\mu)$. On the other hand, subject to the fixed monitoring costs ν, low but positive NPV firms cannot be feasibly privately financed, and so must rely on public financing, provided that the information premium $\pi(\theta)$ is not too high.

From equation (2) and Theorem 1, a positive PE premium, $\pi(\theta) - \nu > 0$, reflects a comparative advantage of the private over the public financier. Thus, absent financing constraints, for any project which is individually rational for both investors to compete on and with a positive comparative advantage, competitive bidding results in the public financier's surplus being taken to zero and the private financier's surplus being equal to the PE premium. Given the limited funds, $B < \infty$, the private investor will only finance projects above some information premium cutoff, $\underline{\pi}$, that is with $\pi(\theta) \geq \underline{\pi}$. This results in the public investor funding the residual firms at the agents' outside option $\underline{q}$.

Finally, because of the budget constraint, although the private financier faces no competition to fund firms which are not individually rational for the public financier, this investor will restrict the financing of such firms to those with expected cash flow return, μ, above a return cutoff $\underline{\mu}$, that is with $\mu \geq \underline{\mu}$. Hence, conditional on a distribution of firms' cash flow characteristics, θ and a monitoring cost, ν, both cutoffs, $\underline{\pi}$ and $\underline{\mu}$, are pinned down by the private financier's budget, B.

Combining the above, Appendix A.2 establishes that equilibrium firm listing patterns are characterized by the following theorem.

Theorem 2. *A unique public listing equilibrium exists and is characterized by private financier's minimum cutoffs for the expected cash flow return,* $\underline{\mu}$, *and information premium,* $\underline{\pi}$. *The equilibrium sorting patterns are depicted in Figure 1a.*

Equilibrium firm listing patterns in the expected cash flow return μ and information rent $\pi(\theta)$ space are illustrated in Figure 1a, with equilibrium private investor financing cutoffs given by $(\underline{\mu}, \underline{\pi})$.

The vertical dashed line depicts the binding individual rationality constraint of the private investor, so that any project with expected cash flow return above this level, i.e. with $\Upsilon(\mu) > \nu$, will generate strictly positive expected profits when privately financed. The dot-dashed diagonal line depicts the binding individual rationality constraint of the public investor, so that any project below this line, i.e. with $\Upsilon(\mu) > \pi(\theta)$, will generate strictly positive expected profits when publicly financed. Finally, the horizontal dotted line depicts the set of zero PE premium firms, so that any project above this line, i.e. with $\pi(\theta) > \nu$, has a comparative advantage in being privately financed.

The graph can be divided into three regions which characterize equilibrium firm financing sources conditional on their cash flow characteristics $(\mu, \pi(\theta))$. The unshaded region corresponds to unfinanced firms, the dark shaded region to privately financed firms and the light shaded region to publicly financed firms. The set of unfinanced firms consists of the three regions, (Ia), (Ib) and (Ic). In regions (Ia) and (Ib) it is not individually rational for either investor to finance, independently of the PE premium. This set also includes region (Ic), where it is individually rational solely for the private investor to finance, but, due to financing constraints and preference for higher expected return projects, these projects remain unfinanced.

The set of privately financed firms consists of two regions, (IIa) and (IIb). Region (IIa) consists of firms which the public financier cannot finance, like region (Ic), but yielding an expected return weakly greater than the cutoff, i.e. $\mu \geq \underline{\mu}$. Region (IIb) consists of firms which can be feasibly financed by both investors, but where the private outbids the public due to a sufficiently high PE premium, i.e. $\pi(\theta) \geq \underline{\pi}$.

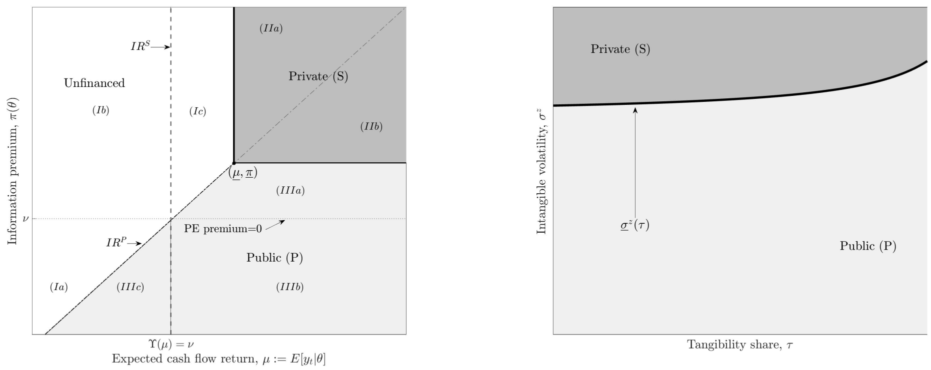
The set of publicly financed firms consists of three regions, (IIIa), (IIIb) and (IIIc). Region (IIIa) consists of firms which both financiers can finance, like region (IIb), but yielding an information premium less than the cutoff, i.e. $\pi(\theta) < \underline{\pi}$. Similarly, region (IIIb) consists of firms for which the public investor faces no competition, as $\pi(\theta) < \underline{\pi}$, but would efficiently fund regardless due to a negative PE premium. Finally, region (IIIc) consists of firms which the private financier cannot finance but the public can.

Thus far we have focused on firm sorting across the expected cash flow return and information premium space. Guided by our data consisting of relatively large and established firms, we next move to examining the equilibrium firm sorting patterns holding fixed a given level of expected cash flows, μ, above the cutoff $\underline{\mu}$ such that sorting is described solely based on the information premium level.

Using the expression of the information premium $\pi(\theta)$ provided by Theorem 1, and conditional on a given level of private financier's funds, B, and of monitoring cost, ν, we obtain that firms will publicly list when their private cash flow volatility is below a cutoff $\underline{\sigma}^z(\tau)$ which is an increasing function of their tangibility level τ. This sorting prediction over tangibility and private cash flow volatility is summarized in Corollary 1.

Figure 1: Equilibrium Firm Sorting

Figure 1a depicts the equilibrium sorting patterns of Theorem 2 based on firm type θ expected cash flow return μ and information premium $\pi(\theta)$. Figure 1b depicts the equilibrium sorting patterns of Corollary 1 based on firm tangibility τ and private information volatility σ^z, holding fixed a given expected cash flow return μ level above the cutoff $\underline{\mu}$. Dark shaded region denotes firm financing by private financiers, lightly shaded denotes public financiers, and no shading denotes no financing.



(a) Theorem 2 Sorting

(b) Corollary 1 Sorting

Corollary 1. *For any $\mu \geq \underline{\mu}$ and a given λ^z, firm sorting is driven by their private cash flow volatility σ^z and tangibility τ as described in Figure 1b with cutoff rule*

$$\underline{\sigma}^z(\tau) := \sqrt{\frac{2\pi}{\psi}\left(\rho + \frac{1}{\lambda^z}\right)\frac{1}{1-\tau}}. \tag{7}$$

From the above results we see that the marginal listed firm has a positive association between intangibility and expected cash flow returns for lower levels of intangibility and expected returns, but this relation disappears for sufficiently high expected cash flow return.

In the region where both financiers compete, all else equal, firms that are more intangible and have higher intangible volatility give more cover for a CEO to hide misbehavior and hence it is more costly to design optimal compensation contracts for the public financier. We emphasize that our theory does not imply that highly intangible firms are necessarily privately financed. The bottom left quadrant of Figure 1b indicates highly intangible firms which are nevertheless publicly listed due to their private cash flows being relatively predictable, that is due to a low σ^z. Hence, intangibility or predictability of cash flows alone are insufficient to predict firm sorting.

To conclude, we summarize below the key testable predictions from this section. Reflecting the data we have available on public and private firms, we restrict attention to firms with sufficient long-run expected profitability levels to be financed by either investor, i.e. with $\mu > \underline{\mu}$. While information premia are jointly driven by σ^z and λ^z, to facilitate testing of our core differentiator from the literature of a persistent private cash flow process, we abstract from firm heterogeneity in the persistence parameter.

Firm Listing Testable Prediction. *All else equal, and conditional on having an expected cash flow return above the minimum cutoff, firms with higher intangibility levels and intangible cash flow volatility will be more likely privately financed.*

2.4 Endogenizing PE Funds

Thus far we have taken the amount of funds available to the representative private investor as exogenous. We now endogenize the aggregate supply of PE funds, B, in a general equilibrium, competitive matching model extension.

To do so, we introduce a continuum of households (investors) of fixed mass $M > 1$, each endowed with a unit of funds and a household-specific monitoring technology with a cost $\nu \geq 0$, drawn from some absolutely continuous distribution with CDF $G_\nu(\cdot)$, and paid should they elect to be private financiers. Households compete with each other to finance a unit measure of heterogeneous projects drawn from a distribution with CDF $G_\theta(\cdot)$ via bids of initial promised utility q_0 and committing to a financing type ex-ante. In particular, relative to the partial equilibrium environment, we add a first stage in which each household makes a financier type choice, denoted as $f(\nu) \in \{P, S\}$, and allow bidding strategies to be contingent on the monitoring cost ν in addition to household's financier type choice f and firm type θ, $q_0^f(\theta, \nu)$. We apply the same notion of equilibrium used in the partial equilibrium setting with these adaptations to construct our general equilibrium.

In lieu of using the monitoring technology at cost ν, a household can invest publicly and pay the expected information rents $\pi(\theta)$. Adapting our partial equilibrium results, a type ν household's return from being a type f financier is $R^f(q_0, \theta, \nu) = \Upsilon(\mu) - \Lambda^f(\theta, \nu) - X(q_0)$ where $\Lambda^S(\theta, \nu) = \nu$ and $\Lambda^P(\theta, \nu) = \pi(\theta)$.[22] We assume each household is capacity constrained and so may only use the monitoring technology on a single firm.

In equilibrium, since the mass of households is greater than the mass of firms, there is an excess supply of aggregate funds which induces a competitive fringe of unmatched public investors. Since all public investors (and their funds) are perfect substitutes for each other (because they do not elect to use the monitoring technology) and in the absence of risk, this competitive fringe drives the equilibrium expected return of public investing to zero, i.e. $R^P = 0$. In contrast, the equilibrium expected payoff from privately investing in a type θ firm after competing out the public investors is given by

$$R^S(\theta, q_0^*(\theta, \nu), \nu) = R^S(\pi', \nu) = \max\left\{\pi' - \nu, 0\right\},$$

where $q_0^*(\theta, \nu)$ is the winning bid and $\pi' := \min\{\Upsilon(\mu), \pi(\theta)\}$ is a modified information premium.[23] As a consequence, a cutoff $\bar{\nu}$ arises whereby households with costs exceeding it invest publicly while the rest privately, and the listing patterns are consistent with the partial equilibrium setting. Further, since $R^S(\pi', \nu)$ is submodular in (π', ν), the equilibrium matching exhibits a sorting pattern where the firm with the highest modified information premium π' is matched with the investor with lowest monitoring cost ν, i.e. it exhibits negative assortative matching (NAM).[24] The general equilibrium results are summarized in the next theorem, and established formally in Appendix A.3.

Theorem 3. *For any household mass $M > 1$, a unique general equilibrium exists, and is characterized by firm public and private sorting status as depicted in Figure 1, aggregate PE funds B given by*

$$B = G_\nu(\bar{\nu})\ M,$$

and NAM between firms indexed by π' and households indexed by monitoring cost ν. In particular, the matching of a type ν household and a firm with modified information

[22]Since households commit ex-ante to use their costly monitoring technology, if unmatched, their payoff is $R^S(\varnothing, \varnothing, \nu) = -\nu$ if they had committed to use the technology, whereas their payoff is $R^P(\varnothing, \varnothing, \nu) = 0$ if they had committed to not use the technology.

[23]Recall from the partial equilibrium setting that if if $\Upsilon(\mu) < \pi(\theta)$, it is not individually rational for public investors to finance a firm, and then a private investor can bid the agent's outside option $\underline{q}$. Instead if $\Upsilon(\mu) \geq \pi(\theta)$ and $\nu < \pi(\theta)$, the private financier can outbid.

[24]For details on supermodularity and equilibrium matching see, for instance, Becker (1974), Tervio (2008), Gabaix and Landier (2008), and the survey by Chade et al. (2017).

premium π', if financing occurs, is given by

$$\pi'_{match}(\nu) := \bar{G}_{\pi'}^{-1}(G_\nu(\nu)\ M),$$

where $G_{\pi'}(\cdot)$ is the CDF of π' and $\bar{G}_{\pi'}(\pi') = 1 - G_{\pi'}(\pi')$ is the survival function.[25]

With the above equilibrium characterization, the average PE premia, Π, public CEO compensation, Ω, and output, O, are given by

$$\Pi := \mathbb{E}[\pi'(\nu) - \nu | \nu \leq \bar{v}],\ \ \Omega := \frac{\omega_{fix}(q)}{\rho} + \mathbb{E}[\pi(\theta)|\theta \in \mathbf{P}],\ \ O := \mathbb{E}[\mu|\theta \in \mathbf{P} \cup \mathbf{S}]$$

where $\mathbf{P} = \{\theta : \pi(\theta) \leq \min\{\Upsilon(\mu), \bar{v}\}\}$ and $\mathbf{S} = \{\theta : \pi'(\theta) \geq \bar{v}\}$ are the set of publicly and privately financed firms, respectively.Finally, we define L the firm listing propensity.

2.5 Comparative Statics and Policy Counterfactuals

In this subsection, we evaluate differential aggregate implications of various policy reforms and technological shifts that have been referenced in the literature. While our testable predictions on firm sorting and CEO compensation up to now have been distribution free, to make predictions on aggregates we must impose some structure on the distributions of firm cash flow characteristics and monitoring costs, $G_\theta(\cdot)$ and $G_\nu(\cdot)$ respectively. We make the following assumptions which facilitate sharp, global monotone comparative static predictions while covering a broad class of distributions.[26]

Assumption 1. The distribution of each component of firm cash flow characteristics is independent, that is $dG_\theta = dG_\mu \cdot dG_\pi$ and $dG_\pi = dG_\tau \cdot dG_\sigma$.

Assumption 2. The survival function $\bar{G}_{\pi'}$ is log convex (concave), and the CDF G_ν is log convex (concave).

Equipped with these primitives, we present comparative static predictions consistent with various policy and environmental changes in Theorem 4. In particular, we study changes on firms' propensity to publicly list, average public CEO compensation, PE premium, and economic output from: (i) an increase in the average firm's intangibility $1 - \tau$, like the one occurred over the

[25]Uniqueness holds up to a re-assignment of unmatched and matched public investors who are all indifferent between the two outcomes in equilibrium.

[26]Note that these assumptions mainly facilitate characterization of the average PE premium Π dynamics. See Bagnoli and Bergstrom (2005) for a discussion on log concavity and its uses.

last fifty years (see Corrado and Hulten (2010)), implemented by an upward scaling of each firm's information premium π; (ii) a relaxation of funding impediments to PE firms, like the one promoted by NSMIA (as studied by Ewens and Farre-Mensa (2020)), implemented by a downward scaling of each investor's monitoring cost ν; (iii) ideas getting harder for new businesses to find (as proposed by Bloom et al. (2020)), implemented by a downward scaling in expected cash flow return μ for new businesses; and (iv) an increase in the public disclosure requirements, like the one caused by SOX (as analyzed by Engel et al. (2007)), implemented by introducing a fixed cost to be public ξ.

Theorem 4. *Suppose that Assumption 1 and 2 hold, and M is sufficiently large. Table 1 presents short-run and long-run impacts on economic aggregates caused by a first-order stochastic shift of (i) G_π to the right, (ii) G_ν to the left, (iii) G_μ to the left, as well as (iv) the introduction of a fixed cost of public listing $\zeta > 0$.*

Table 1: Comparative Statics

This table presents the theoretical comparative static general equilibrium predictions for various economic aggregates across competing theories of secular trends introduced in Theorem 4. Specifically, we study the impact of increasing (i) intangibility, (ii) PE deregulation, (iii) lack of ideas, and (iv) disclosure costs of being public. The considered economic aggregates are the listing propensity (L), the supply of PE funds (B), the average public CEO compensation (Ω), the average PE premium (Π), and the aggregate output (O). For all the experiments, short-run (SR) and long-run (LR) consequences for each aggregate are considered. We define the SR as the case in which the monitoring cost cutoff $\overline{\nu}$ is fixed and so is the supply of PE funds, B, while in the LR case we allow the cutoff to adjust. The symbol $+$ ($-$) denote a rise (fall) relative to the initial level, $\cdot$ indicates not applicable, while ? means that an effect is taken to be ambiguous. See Appendix A.3.2 for details.

	Listing Propensity (L)		PE Funds (B)		Public CEO pay (Ω)		PE Premium (Π)		Aggregate Output (O)	
	SR	LR	SR	LR	SR	LR	SR	LR	SR	LR
(i) Intangibility ($\uparrow G_\pi$)	-	-	·	+	+	?	+	?	-	-
(ii) PE Deregulation ($\downarrow G_\nu$)	·	-	·	+	·	-	·	?	·	+
(iii) Lack of Ideas ($\downarrow G_\mu$)	?	?	·	-	?	?	?	?	-	-
(iv) Disclosure Costs ($\uparrow \zeta$)	-	?	·	+	?	?	+	?	-	-

In Table 1 we consider both short-run (SR) and long-run (LR) consequences. In the SR, the monitoring cost cutoff $\overline{\nu}$ is fixed and so is the supply of PE funds, B, while, in the LR, the cutoff can adjust. In the first row, we consider a rise in firm intangibility implemented through an increase in firm information premia. This has a direct SR effect of raising public CEO pay levels and performance-based shares, and, consequently, magnifying the average

PE premia. Furthermore, higher information premia raise the costs of being public above the expected cash flow returns for marginal public firms causing a fall in publicly listed firms and aggregate firm financing, and thus output. In the LR, the supply of PE funds expands as households take advantage of higher average PE premia. This expansion provides financing not only to previously public firms for whom the private advantage became positive, but also to some previously unfinanced firms, increasing output relative to the SR and exacerbating the decline of public listings. However, on net, output falls in LR due to a first-order attrition of public firms to being unfinanced. In contrast, the competing effects of increased information premia of the surviving public firms and selection of high information premium firms switching to private financing result in ambiguous LR net effects on the average public CEO pay and PE premium.

In the second row, we consider PE deregulation through a decline in monitoring costs. Since we assume that in the SR the household private investing cost cutoff is fixed, our comparative static predictions apply only to the LR. In the the latter case, households switch from public to private financing expanding the supply of PE funds. This expansion of PE funds has unanmbiguous two effects. First, an attrition from public financing of highly intangible firms which results in lower average CEO pay for the remaining public firms. Second, an increase in output due to more high information premium firms that were previously unfinanced receiving private funding.

In the third row, we consider ideas becoming harder to find through a decline in the expected cash flow returns. This reduction makes low productivity firms financed by either investor type no longer profitable to fund. Consequently, privately funded firms and output fall instantaneously to the LR level.[27] The simultaneous fall in both public and private firms results in an ambiguous impact on the listing propensity, average public PE premium and CEO pay due to countervailing selection effects.

In the fourth row, we consider additional public disclosure requirements in the case wherein they don't improve transparency and only impose an added fixed cost from being public. This confers an extra advantage to private financing and forces low profitability public firms to exit. Consequently, in the SR unambiguously the average PE premium rises, and both public

[27]Since private investors can choose to not invest when it is not profitable, the supply of private funds provided to firms immediately drops. In the LR the supply of private funds (i.e. the private investor cutoff) also reduces to match the SR invested level, but has no implications on the economic aggregates considered. As such, there is no substantive distinction in the SR and LR for this comparative static exercise.

listing propensity and output fall. Like in the case of increased intangibility, the LR supply of private funds expands to partially offset the attrition of public firms, but doesn't fully counterbalance the SR decline in output. Again, countervailing forces result in an ambiguous LR net effect on average public CEO pay and PE premium and also on firm listing propensity.

While the previous interpretation of additional public disclosure requirements can be thought of as an unproductive reform, this type of policy intervention could have a beneficial effect of improving transparency reducing information asymmetry. The effects of this productive disclosure are equivalent to a reduction in firm intangibility, and hence information premia, with effects corresponding to reversed signs of those in the first row of Table 1.

3 Data

The data for the empirical analysis come from multiple sources. Our main source is S&P Capital IQ which provides financial and accounting data from 1993 to 2016 as well as CEO compensation data from 2001 to 2016 on US firms. Our initial sample consists of corporations that file with the Securities and Exchange Commission (SEC) either a 10-K, a 10-Q or an S-1 Form for the period considered. This includes firms publicly listed on one of the US historically top stock exchanges, i.e. NYSE, Amex and Nasdaq, and non-listed firms with SEC disclosure requirements due to other reasons.[28] In line with the literature, we consider the first category as public and the latter as private.[29]

We use Compustat Snapshot to obtain information about the listing status of each firm through time as Capital IQ provides only the most recent listing

[28] These reasons are having public debt, securities listed on OTC exchanges, or having more than $10 million in assets and a certain number of shareholders. The latter threshold was 500 shareholders before the 2012 JOBS Act, while it increased to 2000 shareholders after. We exclude from our analysis observations of firms listed on minor stock exchanges following previous works studying differences between public and private firms using Capital IQ.

[29] While to some extent the SEC reporting requirements are similar for both publicly listed and non-listed firms, there are two key distinctions between the two categories. First, publicly listed firms have more comprehensive and frequent reporting requirements to the SEC and receive much more analysts' attention than non-listed firms. Second, non-listed firms are traded on markets that have less depth (and implicitly fewer shareholders) and where the ease of communicating private information to shareholders while avoiding divulging to a broader public should be greater than in top stock exchanges. Moreover, the lower frequency of trade implies that price adjustments of the non-listed firm value should be lower than those of the publicly listed firms, so that the information sensitivity of stock prices and CEO compensation should lie on a continuum between the totally private firms and those publicly listed on a top stock exchange.

status.[30] For similar reasons, we use Execucomp and Capital IQ corporate events data as in Gao et al. (2017) to detect which executive was the CEO of a firm in a given year. Where ambiguity remains (which occurs in 6.5% of the observations), we consider the highest paid executive in terms of total compensation as the CEO of a firm. Capital IQ corporate events data is used also to exclude observations of firms which underwent LBOs and IPOs. We supplement the IPO information with the data on Jay Ritter's website (for details, see Field and Karpoff (2002) and Loughran and Ritter (2004)).

We consider firm-year observations with positive and non-missing book value of total assets. We exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4999) and quasi-governmental firms (SIC codes above 9000). All variables are normalized in 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level. Appendix B provides more details on sample construction and data cleaning.

Besides firm listing status and CEO compensation level, the other key outcome variable of interest is CEO pay sensitivity. To measure the latter, we decompose CEO pay following Frydman and Saks (2010) and Edmans et al. (2017) to have consistency across our empirical analyses. We proxy CEO's fixed compensation component as the sum of salary plus fixed bonuses. Further, for the sum of CEO's growth and performance-based pay components we use two measures. The first is the sum of long-term incentive plans and non-equity incentives and any stock and option compensation while the second is the sum of stock and option compensation. All three proxies are scaled by total CEO compensation.

Having defined our outcome variables, we move to measuring the drivers of firms' private information rents. As before, we decompose these into the exposure to private cash flows, $1 - \tau$, and the volatility of private cash flows, σ^z. We proxy the former with a measure of firm intangibility computed as the fraction of intangible capital over total assets where the numerator is obtained following the approach of Peters and Taylor (2017) and the denominator equals the book asset value plus intangible assets off the balance sheet.[31] To proxy the latter for our reduced form empirical analysis, we compute a firm-

[30] Compustat Snapshot has historical listing information about Compustat firms while standard Compustat has header, that is the latest available, information. We use Compustat Snapshot rather than CRSP due to the fact that it also provides coverage of minor stock exchanges and firms undergoing LBOs, which we exclude from our analysis following the previous literature using Capital IQ.

[31] In particular, our measure combines Research and Development (R&D) expenses, 30% of Sales, General and Administrative (SG&A) expenses, and changes in other intangible assets on the balance sheet and goodwill as an investment flow into an intangible capital stock.

year proxy using the three year standard deviation of a firm's profitability, measured by EBITDA scaled by total assets. We defer to Section 5 a discussion of our measurement of these different private information components via structural estimation, and to Appendix B a more careful description of how the variables were constructed.

Table 2 reports descriptive statistics for public and private firms. Public firms are larger, both in terms of book assets and sales (both reported in millions of US dollars), and older than private firm. They are also more tangible than their private counterparts and have less volatile earnings, but they are more profitable on average. Public and private firms carry a similar fraction of gross physical property, plants and equipment (PPEGT) on their book assets and have a similar investment level, computed as capital expenditure (CAPEX) divided by PPEGT. Private firms conduct more R&D, have higher SG&A expenditures and hold more goodwill as a fraction of their book assets than public firms. Public CEO pay (reported in millions) is substantially higher than that of private CEOs. The last three rows of the table present the ratio of salary plus fixed bonuses, long-term incentive plans and non-equity incentives plus any stock and option compensation, and stock and option compensation divided by total pay, respectively. For private CEOs the fixed part of their compensation part accounts for a substantially larger portion than for public CEOs. At the same time approximately two thirds of private CEOs do not report any performance-based pay and, even when they do, the fraction of this compensation is smaller than that of public CEOs.

In addition to our main empirical analysis, we evaluate the empirical content of our theory on historical and international data. For the historical analysis we chiefly use the data about intangible capital stocks of Peters and Taylor (2017), Execucomp, and the historical CEO compensation data of Frydman and Saks (2010). We apply to this data the same filters used to clean our shorter Capital IQ panel data. For the international analysis we use the World Development Indicators (WDI) data provided by the World Bank, and OECD Structural and Demographic Business Statistics database (see OECD (2019)), and restrict our attention to mainly G7 and OECD member countries.

4 Testing the Theory

Our theoretical results suggest that firm listing choice as well as CEO pay level and sensitivity are driven by a firm-level information premium, $\pi(\theta)$, which is

Table 2: Descriptive Statistics

This table reports descriptive statistics for firm characteristics from 1993 to 2016 as well as CEO compensation characteristics from 2001 to 2016 for companies in our sample. The table is divided between observations related to public and private firms. All the scaled variables without clear upper or lower bounds are annually winsorized at the 1% and 99% level. All nominal values are adjusted to 2016 US dollars.

	Public				*Private*			
	Mean	Median	Std	N	Mean	Median	Std	N
Firm Characteristics								
Book Assets	3182.36	358.33	15075.44	74374	973.94	34.51	5338.11	45468
Firm Age	34.08	25.00	25.44	74374	20.31	12.00	21.74	45468
Tangibility	0.64	0.64	0.20	74374	0.61	0.63	0.25	45468
Volatility	0.03	0.02	0.04	65489	0.06	0.03	0.08	25677
Profitability	0.07	0.09	0.12	74237	0.01	0.05	0.18	43968
Sales	2834.78	342.33	13374.08	74374	713.14	33.02	3423.33	45468
PPEGT	0.47	0.40	0.32	74374	0.50	0.43	0.36	45468
Capital Expenditure	0.14	0.10	0.14	72579	0.19	0.11	0.22	40103
R&D	0.09	0.07	0.09	28049	0.13	0.09	0.14	12675
SG&A	0.22	0.17	0.20	73351	0.36	0.22	0.47	44715
Goodwill	0.16	0.12	0.15	42345	0.22	0.17	0.19	15999
CEO Pay Characteristics								
CEO Pay	4.18	1.66	9.53	45294	1.20	0.37	4.07	19580
Fixed Share	0.52	0.44	0.34	45294	0.79	0.96	0.29	19580
Incentive Share	0.61	0.67	0.26	32793	0.45	0.43	0.28	6752
Equity Share	0.51	0.53	0.24	30750	0.41	0.37	0.29	5586

increasing in firm intangibility level, $1 - \tau$, and private cash flow volatility, σ_z. We validate firm listing predictions in Subsection 4.1, and CEO compensation predictions in Subsection 4.2. We proceed by considering historical evidence in Subsection 4.3, and international evidence in Subsection 4.4.

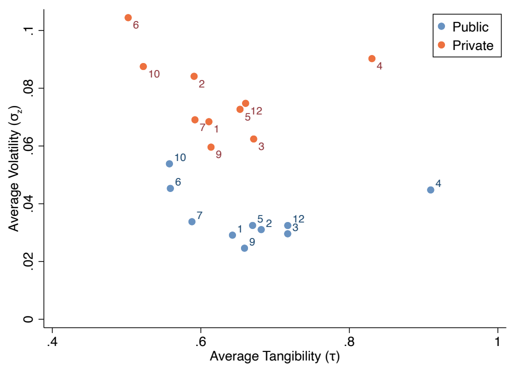
4.1 Testing Cross-Sectional Firm Listing Predictions

In Figure 2 we compute the average tangibility and earnings volatility within Fama-French 12 industries separately for public and private firms. The within-industry analysis falls along the patterns predicted by our sorting theory. That is, private firms exhibit higher volatility and lower tangibility than public companies. The only departure is the telecommunication industry (Fama-French Industry 7). Here the joint sorting prediction still holds, but tangibility levels are similar across the two sides.

In Table 3 we present the results from regressing a dummy that takes value 1 if a firm is publicly listed and 0 otherwise on our proxies of τ and σ_z. We control also for the age of a firm and its size, proxied by the total book asset value, and we include industry and year fixed effects in the first and third columns and industry times year fixed effect in the second one. All the independent variables are in logs and lagged by 1 year. Standard errors are heteroskedasticity-robust and clustered at the firm level. The first two

Figure 2: Empirical Firm Listing Patterns

Within-industry average tangibility and 3-year earnings volatility conditional on firm listing status (classified by in-sample median). The 12 Fama-French industries are 1 non durables, 2 durables, 3 manufacturing, 4 energy, 5 chemicals, 6 business equipment, 7 telecommunications, 8 utilities, 9 shops, 10 health, 11 money, and 12 other. Given our data cleaning filters, the Fama-French industries 8 and 11 are not considered.



columns display the results from a linear regression model while the third one displays the results from a logistic regression.

Our empirical results are consistent with the theoretical listing predictions across all the specifications. A 1% increase in firm tangibility corresponds to a 9% increase in the probability of being publicly listed. Similarly, we find a one to one correspondence of volatility to a reduction in the probability of a firm being listed. All these coefficients are economically and statistically significant at the 1% level and substantive notwithstanding the inclusion of firm age and size (which themselves have the expected effects meaning that older and larger firms are more likely to be public). Similar results are obtained using the logistic regression approach. Alternative specifications (e.g. including other controls) are presented in the Online Appendix and are wholly consistent with our testable predictions.

4.2 Testing Cross-Sectional CEO Pay Predictions

In Table 4 we present the results of empirical tests of our theoretical predictions for the relation of the level and composition of CEO pay and firm characteristics. The regression specification is similar to the listing one with dependent variables the logs of the CEO pay, the fixed share, the incentive share, and the equity share, respectively. In addition, we also control for the firm listing status.

The results are again consistent with our theory and both statistically and economically significant. In the first column, a 10% increase in firm

Table 3: Firm Listing Regressions

This table shows the results of regressing a dummy variable which takes value 1 if a firm is public and 0 otherwise on one year lagged firm tangibility, profitability volatility, age, and size. All non-indicator variables are in logs. Industry and year fixed effects are included in the regressions of the first and third columns while industry times year fixed effects are included in the regression of the second column. The regressions in the first two columns are linear regressions while the regression in the third column is a logistic one. Standard errors are heteroskedasticity-robust, clustered at the firm level and reported in brackets. All variables are in 2016 US dollars. All continuous variables are annually winsorized at the 1% and 99% levels. Superscript *, **, *** correspond to statistical significance at the 1%, 5%, and 10% levels, respectively.

	OLS (1)	OLS (2)	Logistic (3)
Tangibility ($\hat{\tau}_{t-1}$)	0.089***	0.090***	0.442***
	(0.010)	(0.010)	(0.063)
Volatility ($\hat{\sigma}_{z,t-1}$)	-0.011***	-0.011***	-0.049**
	(0.003)	(0.003)	(0.019)
Age	0.049***	0.048***	0.337***
	(0.005)	(0.005)	(0.034)
Size	0.068***	0.069***	0.456***
	(0.002)	(0.002)	(0.017)
Industry and Year FE	Yes	No	Yes
Industry x Year FE	No	Yes	No
Observations	77332	77332	77332
R^2	0.198	0.197	
Pseudo R^2			0.192

tangibility is associated with a 3% decrease in total CEO pay, while a 10% increase in firm volatility corresponds to a predicted 0.25% increase in CEO total compensation. As has been found previously in the literature, listed, older, and larger firms pay their CEOs more on average. In the second column of Table 4, we find that a 10% increase in firm intangibility is linked to a 1% increase in the fixed share of the CEO pay whereas a 10% increase in firm volatility corresponds to a 0.3% fall of the same fraction. Older firms tend to pay their CEOs with a larger fixed share while listed and larger ones with a smaller one. Finally, in the last two columns we assess the predictions for CEO performance based pay proxied by the incentive and equity shares of CEO pay. A 10% increase in firm tangibility is associated with a roughly 0.6% (0.8%) rise in the incentive (equity) share of CEO pay while a 10% increase in firm volatility is linked to a slightly milder rise of 0.3% (0.4%) in the incentive (equity) share. Again, similarly to the listing regressions, the Online Appendix contains various alternative specifications with additional controls validating the robustness of these empirical regularities.

Table 4: CEO Pay Regressions

This table shows the results of regressing the logs of the CEO pay, the fixed share, the incentive share, and the equity share on one year lagged firm tangibility, profitability volatility, age, size, and listing status. All non-indicator variables are in logs. Industry and year fixed effects are included in all the regressions, which are all linear. Standard errors are heteroskedasticity-robust, clustered at the firm level and reported in brackets. All variables are in 2016 US dollars. All continuous variables are annually winsorized at the 1% and 99% levels. Superscript *, **, *** correspond to statistical significance at the 1%, 5%, and 10% levels, respectively.

	CEO Pay (1)	Fixed Share (2)	Incentive Share (3)	Equity Share (4)
Tangibility ($\hat{\tau}_{t-1}$)	-0.313***	0.106***	-0.058***	-0.081***
	(0.022)	(0.016)	(0.021)	(0.023)
Volatility ($\hat{\sigma}_{t-1}$)	0.025***	-0.030***	0.033***	0.044***
	(0.008)	(0.006)	(0.006)	(0.007)
Age	0.092***	0.046***	-0.096***	-0.149***
	(0.013)	(0.009)	(0.010)	(0.011)
Size	0.452***	-0.197***	0.173***	0.169***
	(0.006)	(0.004)	(0.005)	(0.005)
Publicly Listed	0.307***	-0.156***	0.307***	0.261***
	(0.021)	(0.015)	(0.028)	(0.033)
Industry and Year FE	Yes	Yes	Yes	Yes
Observations	53811	53811	34837	32293
R^2	0.546	0.275	0.209	0.179

4.3 Evaluating Time-Series Trends

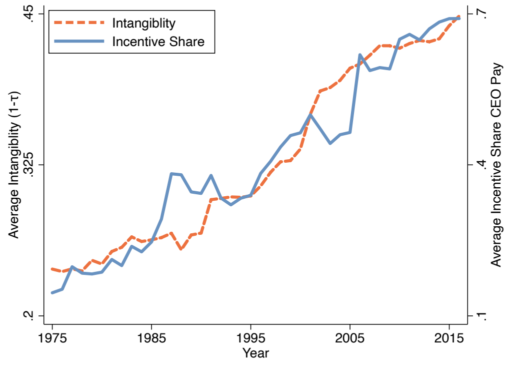
We now turn to study some of the longer-term historical trends in CEO compensation and how they relate to different policy interventions and other structural changes previously discussed. Since our primary data source, Capital IQ, has CEO compensation data extending back to 2000 and firm data to 1993, we use a combination of Compustat, Peters and Taylor (2017) data, Execucomp and Frydman and Saks (2010) data to evaluate the longer historical trends (extending back to the 1975). Details on variable definitions and construction are provided in Appendix B.

Figure 3 depicts (solid line) the average share of annual CEO compensation paid in terms of long term incentive, non equity incentive, stocks and options from 1975 to 2016 as well as (dashed line) the average intangibility of publicly listed firms implied by the accumulation of R&D and SG&A expenses as well as goodwill and other intangible assets on the balance sheet.

We can see that the two time series moved fairly consistently in lockstep upwards. Indeed, a Dickey-Fuller test statistic of - 3.743 (p-value of 0.2%) and a correlation above 95% suggest a co-integrated relationship between the two series. Moreover, since the average annual CEO pay has an 84% correlation with the incentive share, and a Dickey-Fuller statistic of -2.433 (p-

Figure 3: Historical CEO Pay Performance Sensitivity and Firm Intangibility

This figure depicts the average CEO pay incentive share on the right axes and the average share of intangible assets from 1975 to 2016. The CEO compensation data pre 1991 is taken from Frydman and Saks (2010) while post 1991 is from Execucomp. The average intangibility share is constructed using Compustat and Peters and Taylor (2017) data.



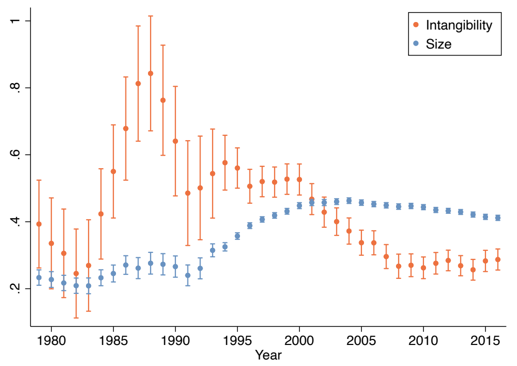
value of 0.9%), the data suggests that the level of CEO pay and the incentive share move in virtual lockstep. Thus, the broad compensation trends for CEO compensation from 1975 to 2016 are in line with our theory that the information sensitive component of pay (and, through risk-compensation, the level) is tied to the degree of firm intangibility.

We test how our theory of the influence of firm intangibility on CEO pay holds over the longer historical time-series and cross-section in Figure 4. Leading alternative explanations of the rise in CEO pay seen since the 1970s are based on size as in Gabaix and Landier (2008) and so we examine the relative contributions of size and intangibility over the historical time-series. To do so, we run similar regression exercises as in the earlier subsection, regressing the logarithm of total compensation on the logarithms of firm tangibility, volatility and firm size on a five year rolling window (from year $t-4$ to year t). We plot the estimated coefficients tied to firm intangibility (negative of the tangibility coefficient) and size in Figure 4.

We see substantial time-variation in the magnitudes of association between intangibility and size with CEO pay. In particular, intangibility had a mild impact in the 1970s, which then substantially jumped in the 1980s to elasticities about four times higher than those of size, before attenuating in the 21st century. In contrast, the inferred impact of size was relatively flat until the 1990s suggesting intangibility may better explain the inflection point, documented by Frydman and Saks (2010), in the rise in level and equity-based CEO pay which occurred around 1980. The implied information asymmetry stemming from intangible assets broadly declines to the end of the sample consistent with the relative flattening of average public CEO pay over this time window.

Figure 4: Annual Estimated Elasticities of Intangibility and Size on Pay

This figure depicts the point estimate and standard error bands of the elasticities of public CEO compensation to intangibility and size over fiver year rolling windows. The sample begins in 1975 due to the unreliability of Peters and Taylor (2017) intangibility measure in the preceding years. CEO compensation data prior to 1991 consists of on average 69 firms from Frydman and Saks (2010), while post 1991 consists of on average 1,350 firms from Execucomp.



Finally, we examine the empirical evidence regarding the CEO pay, PE premia and listing trends from the differential predictions of various policy and environmental shifts given in Table 1. Harris et al. (2014) document a substantial rise in venture capital premia relative to public markets (as measured by the public market equivalent) between 1984 - 1996. Following the relaxation of PE markets by NSMIA in 1996, the average VC premia sharply dipped, while the right tail VC premia remained positive throughout the 2000s. These dynamics are consistent with a rising intangibility-driven comparative advantage with a relatively inelastic PE supply prior to 1996, followed by a substantial decline in both listings and VC premia as aggregate PE funds expanded. The relatively flat dynamics of PE / VC premia, combined with the moderate drop in listings and CEO pay-intangibility exposure suggests SOX moderately amplified the attrition of public firms but was not the fundamental driver of the exodus.[32] Similarly, the timing of private firm exodus and lack of shift in private equity premia during the aggregate US productivity slowdown from 2004 onwards, amidst a rise in aggregate PE funds runs counter to the hypothesis that ideas being harder to find is a primary driver. Thus, overall the dynamics of listings, CEO pay and VC premia between the 1980s and the

[32]Moreover, analyzing estimated annual intangibility listing elasticities over the 1996 - 2016 time-window in a similar manner to the CEO pay, we find a reduction of intangibilities association to public listing around the introduction of SOX, which almost immediately returns to the previous elasticity level.

first half of the 2010s are most consistent with a secular rise in intangibility combined with a tight constraint on PE funds and sharp relaxation in the late 1990s.

4.4 Evaluating Cross-Country Evidence

Our proposed channel of US public listings decline and rising CEO compensation is a technological one. Consequently, we provide some stylized descriptives of international patterns of public listing patterns and their associations with other economic aggregates in support of this common driver.

Using WDI data in Figure 5 we present the number of domestic publicly listed firms and number of R&D researchers per million of people scaled by their respective 1996 levels. We see that with the exception of Japan and Canada, domestic public listings have declined across all G7 countries.[33] Moreover, the level of country intangibility proxied by their R&D researchers as in Bloom et al. (2020) have increased across all countries but Japan. When we extend our international comparisons to all OECD countries with available data, we find a correlation of -17% between the growth of intangibility and number of listings from 1996 to 2014.[34] Overall, we see that the decline of public listings accompanied by rising intangibility is not only a US phenomenon but it is also exhibited in many other advanced economies. The main difference being that the US decline preceded that in other nations.

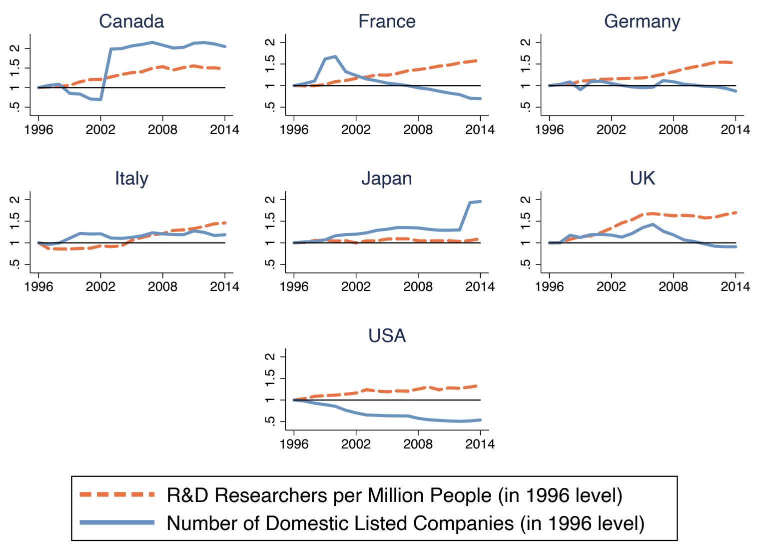
Our predictions for public CEO compensation association with intangibility are also supported by international evidence. While significant differences in US and non-US CEO pay were documented in the 20th century, Fernandes et al. (2013) find that systemic differences in US and non-US CEO pay levels and equity-pay have largely vanished in the 2000s after adjusting for corporate governance differences. The timing of this convergence aligns with the delayed rise in firm intangibility for EU as documented by Corrado et al. (2016) and Corrado et al. (2022). In contrast, Pan and Zhou (2018) document that pay differences between US and Japanese CEOs have not disappeared, consistent with the opposing intangibility trends in Figure 5.

[33]Note that in contrast to our analysis on US firms, public listing data here include firms in all industries. In addition, while total listings in Canada have increased, the increase seems to be comprised largely of financial vehicles (so called "frankenstocks"), and in fact corporate listings have instead dropped by roughly one third (see Tingle and Pandes (2021) and the Maclean's article here).

[34]We find similar findings using R&D scaled by GDP rather than the number of R&D researchers as our proxy of country intangibility.

Figure 5: International Public Listings and Intangibility

This figure depicts the number of domestic publicly listed firms and the number of R&D researchers per million of people scaled by their respective 1996 levels for the G7 countries from 1996 to 2014 using the World Bank WDI data.



Finally, we examine the international associations of listing patterns, VC funding, GDP growth in relation Theorem 4. To do so, we link the World Bank WDI data to the OECD data on annual total VC investments. Sorting firms by their relative intensity of VC financing to the market capitalization of publicly listed domestic companies in the previous year, we present in Table 5 the correlation between GDP growth and lagged growth in number of public listings. Consistent with the previous literature, we find a positive correlation between public listings and GDP growth on aggregate, however but also that this positive relationship diminishes and completely reverses for countries with high VC intensity.[35] Countries populating the high VC intensity category include US, Israel, and Canada, the mid categories include France, Germany and UK, and the low category include Poland, and Italy. As such, the sorting isn't based solely on size or level of development. The results suggest that for economies more supportive of VC, public markets are no longer the engines of economic growth in line with the PE deregulation predictions of Table 1.

[35]These qualitative patterns are robust to the grouping chosen. We also find a negative relation between the bottom and top deciles when we sort countries by VC investments over GDP.

Table 5: Conditional Correlations between Listings and GDP Growth

This table presents the correlations between the time t GDP growth and the time $t-1$ number of public domestic listings growth conditional on time $t-1$ quartiles of VC intensity. The latter is computed as the ratio of VC investments over market capitalization of listed domestic companies. To compute this table we use World Bank WDI data and OECD VC data.

Quartile	1st	2nd	3rd	4th
Correlation	0.08	0.04	0.04	-0.23

5 Structural Estimation

In this section we structurally estimate the model and study counterfactual experiments. In subsection 5.1 we describe the estimation method which we apply to firm listing data in subsection 5.2 and to public CEO compensation data in subsection 5.3 in order to compare the private persistence estimates across the two distinct estimations. Finally, in subsection 5.4 we evaluate the counterfactuals where firm intangibility levels returned to their previous years values.

5.1 Methodology

We estimate the model using the generalized method of moments (GMM) developed by Hansen and Singleton (1982). Since our model implies a common underlying information premium governing both the firm listing decision and CEO compensation packages, we are interested in estimating the underlying parameters governing the size and distribution of this premium, $\pi(\theta)$, as well as the private monitoring cost ν. We assume all heterogeneity in the information premium is due to dispersion in the loading on intangible cash-flows $1-\tau$ and the volatility of these private cash-flows σ_z^2. We again treat the loading on private cash-flows, $1-\tau$, as observable and given by our proxy of firm intangibility, and assume the distribution of σ_z^2 to be $\Gamma(\alpha_0, \alpha_1)$, which is the same parametric form as the one of the earnings volatility, and independent from $1-\tau$.[36] All other parameters are assumed to be common across firms. To keep the estimation tractable, we fix the CEO preference parameter governing risk-aversion to be $\psi = 5$ as in He (2011) and the discount rate to $\rho = 2.5\%$ which equals the 3-month T-bill interest rate during our sample

[36]We assume independence to facilitate closed-form moment expressions. Testing for independence of a copula with beta and gamma marginal distributions is non-trivial. However, the low Kendall statistic (which is a sufficient statistic for the dependence for some common copulas) of 0.0003 and non statistically significant computed using the earnings volatility proxy suggests a tiny distortion.

period. Further, we take the CEO outside option q_0 conditional on the listing status as constant.[37] Let δ denote the vector of parameters to be estimated and $h(\delta, v_{it})$ the vector of moment conditions as a function of the parameters δ and the data v_{it}. We estimate the model parameters by minimizing the objective function

$$\hat{\delta} = \underset{\delta}{argmin} \quad \bar{h}(\delta)' \, W \, \bar{h}(\delta), \tag{8}$$

where $\bar{h}(\delta) = N^{-1}T^{-1}\sum_i \sum_t h(\delta, v_{it})$ is the sample average of the vector of moment conditions for a sample of N firms and T periods.

5.2 Listing Structural Estimation

Guided by our model results on firm listing, we use firm listing moments which are informative on the information premium as well as cash-flow moments which are informative on the productivity parameters. To perform this estimation we collapse the panel data set into a single cross-section with listing status given by a firm median value (dropping observations which are exactly public half the time). Our key listing moments consider our model implied listing choice, $L_i = \{\nu - \pi(\theta) \geq 0\}$. Since our model abstracts from other considerations dictating a firm's listing choice, we assume that the listing choice is given by $L_i^* = \{\nu - \pi(\theta) + \varepsilon_i \geq 0\}$ where ε_i is an unobserved preference shock which is independently and identically distributed following a logistic distribution. This implies that the log odds ratio of listed versus non-listed is given by

$$log\left(\frac{Pr(L_i = 1)}{1 - Pr(L_i = 1)}\right) = \nu + \beta \, (1 - \tau)^2, \tag{9}$$

where $\beta = \frac{-\sigma_z^2 \psi}{2(\rho + \lambda_z^{-1})^2}$. Since we assume that σ_z^2 is firm specific and latent in this exercise, taking the average over σ_z^2 to obtain $\hat{\beta} = \mathbb{E}[\beta] = \frac{-\mathbb{E}[\sigma_z^2]\psi}{2(\rho+\lambda_z^{-1})^2} = \frac{-\alpha_0 \alpha_1 \psi}{2(\rho+\lambda_z^{-1})^2}$ which is independent of the firm i and yields the logistic regression

$$\mathbb{E}\left[log\left(\frac{Pr(L_i = 1)}{1 - Pr(L_i = 1)}\right)\middle| \tau\right] = \nu + \hat{\beta} \, (1 - \tau)^2. \tag{10}$$

[37] The moments utilized to estimate the key parameters of interest are independent of the level of q_0, and hence we do not estimate the outside option.

Taking the appropriate GMM moment conditions for a logistic regression provides us with two moments that identify the monitoring cost ν from the constant and $\frac{-\alpha_0\alpha_1\psi}{2(\rho+\lambda_z^{-1})^2}$ from the slope on $(1-\tau)^2$.

To pin down the volatility and persistence of the observable cash-flows we follow the arguments of Olley and Pakes (1996) used to estimate the TFP of a neoclassical production function, and use a version of physical capital investment intensity adapted to our model, that is we scale a firm CAPEX by its tangible assets.[38] To this extent we compute the firm level variance of this flow to identify σ_x^2 and its autocorrelation using the method of Han and Phillips (2010) to identify λ_x. To identify the parameters governing the Gamma distribution of σ_z^2, that is α_0 and α_1, and λ_z, besides the slope coefficient of the logistic regression, we use the variance of the variance of a firm cash-flows and average residual variance of cash-flows, computed in the data as the difference between the firm-level variance of cash-flows and the product of the firm-level variance of physical investment intensity and the square of the firm-level tangibility level. In total we have a just-identified system of 6 moments to identify 6 parameters. Standard errors are bootstrapped using 10,000 resampling draws. Further details are provided in Appendix B.

The results of the listing-based structural estimation are given in the first column of Table 6. We find that the average size (σ_z^2) of the private information component of cash-flows is just under 5 times larger than the tangible component of cash-flows as proxied by physical investment intensity volatility (σ_x^2). Moreover, the annual private information persistence is estimated to be 63% higher than the persistence implied by physical investment.[39] Together these estimates suggest that in the sample firm risk exposure is more driven by intangible assets than their tangible counterparts. The estimated monitoring cost ν is positive. All estimated coefficients are statistically significant at the 1% level.

5.3 CEO Pay Structural Estimation

We now test our main theoretical result that the same information premium governs both the cross-sectional listing decisions of firms and the CEO compensation packages. Since in our theory the information premium appears only in the compensation of the public CEOs, the selection of firms into be-

[38] Olley and Pakes (1996) reason that the physical investment intensity identifies innovations in productivity on tangible assets.

[39] We refer to the persistence here as the discrete time AR(1) coefficient $= e^{-\frac{1}{\lambda_i}}, i \in \{x, z\}$.

ing public makes it difficult to compare our estimates of the distribution of information premia from public CEO compensation data with the estimates obtained before. Instead, we seek to estimate the private information persistence parameter λ_z, which is assumed to be common to all firms and examine how closely our estimate identified off of CEO compensation data coheres with our results using moments implied by the firm listing decision. To ensure consistency of the sample across estimations we use the Capital IQ compensation data rather than the richer Execucomp for this estimation, although we note that our compensation sample only begins in 2001. To identify the private information persistence parameter λ_z, we use a measure of pay performance sensitivity of CEO contracts

$$\frac{cov\left(\omega_t, \frac{y_t}{dt}\right)}{Var\left(\frac{y_t}{dt}\right)} = \mathbb{E}\left[\frac{\rho(1-\tau)^2\sigma_z^2}{\rho + \lambda_z^{-1}}\right], \tag{11}$$

which in the appendix we show that can be rewritten as

$$\frac{\mathbb{E}[(1-\tau)^2\sigma_z^2]}{\mathbb{E}[(1-\tau)^2\sigma_z^2 + \tau^2\sigma_x^2]}\frac{\rho}{\rho + \lambda_z^{-1}}. \tag{12}$$

Since we want to focus on λ_z, the other parameters are nuisance parameters. Furthermore, according to our theory CEO pay packages are independent of the observable cash-flows, and so we cannot verify σ_x from CEO compensation moments without diluting the exercise. To avoid this issue, we make the simplifying assumption that $\sigma_x^2 \approx 0$ which further simplifies our target moment condition to

$$\frac{\rho}{\rho + \lambda_z^{-1}} \tag{13}$$

which depends only on λ_z and the fixed parameter ρ. To compute the above moment, we compute a panel regression of CEO compensation on the yearly change of EBITDA and we scale both variables by their initial value in the sample in which a firm is publicly listed.

The results from this moment estimation are given in Column 2 of Table 6. We find that using CEO compensation pay sensitivity our estimate for the persistence λ_z is 5.38 which falls slightly below the firm listing estimate of 7.63. In discrete time, this corresponds to a relatively high AR(1) coefficient of 0.83. Using bootstrapped standard errors on the difference between these two estimates, we find that there is no statistically significant difference between our persistence estimate from the firm listing or CEO compensation moments. Now to the extent that σ_x^2 is strictly positive, our estimate of λ_z is downward biased. In column 3, we provide a bias-corrected estimate using our parameter estimates from the firm-listing side on σ_x^2 and the observed distribution of firm tangibility (τ).

Table 6: Structural Estimation

Parameter	Firm Value	CEO Value	Bias Adjusted CEO Value
λ_z - Persistence of private cash flows	7.663	5.377	8.241
	(1.174)	(1.101)	
ν - Monitoring cost	0.664		
	(0.036)		
α_0 - Shape of distribution of volatility of private cash flows	0.512		
	(0.012)		
α_1 - Scale of distribution of volatility of private cash flows	0.027		
	(0.004)		
σ_x^2 - Volatility of tangible cash flows	0.003		
	(0.000)		
λ_x - Persistence of tangible cash flows	1.623		
	(0.110)		

5.4 Counterfactuals

To quantify the importance of intangibility-induced persistent private information, we consider a counterfactual world where firm intangibility levels remained at the level observed in 1980. We compute that average firm intangibility increased 61% between 1980 and 2016, going from 9.5% to 24.7%. Using this change along with our firm-listing based structural estimates implies that returning intangibility levels to those in 1980 would increase the listing probability from 58% to 63%, that is a 5 percentage point increase. Further, using the implied information premium our public firms only, we find that substituting in the implied tangibility levels of the 1980s leads to a fall in information premium, $E[\pi(\theta)_{1980}]$, from 0.350 to 0.134. Since this percent fall in information premium is equal to that of the average annual variable pay growth, we conclude that annual public CEO variable pay growth would be 62% lower without the increase in the exposure to persistent private information from intangible assets. The magnitude of these effects suggests that the proliferation of information asymmetries can account for jointly a sizable, but partial fraction, of public CEO pay and listing trends.

6 Conclusion

Firm's reliance on intangible assets is a significant source of cross-sectional and time-series variation in firm public listing choice and public CEO compensation. Public CEOs in relatively intangible firms are paid more than CEOs of public firms with more tangible cash-flows, yet highly intangible firms are typically privately financed and have lower levels of CEO pay (even for the comparable levels of size, age and profitability). Public CEO pay began growing in the US at the same time that patents (an important intangible asset) and their contribution to firm value exploded around 1980. Since the dot com bubble collapse, public CEO pay growth has been less monotonic as a selection effect of high intangibility firms reduced their public listing propensities.

Our equilibrium framework generates these sorting and compensation patterns through a hidden information agency conflict and monitoring advantage of private investors. Our mechanism provides a micro-foundation to Glover and Levine (2017) who find that the size of agency issues implied by observed compensation contracts correlate strongly with firm intangibility. The hidden information friction is important to understand these patterns and account for findings by Gayle and Miller (2015) that CEOs are paid for luck in cash-flows whose risk profile they have no control over. Persistence in the hidden information, not only magnifies the agency friction, but generates a growing expected share of profits accruing to the CEO through the duration of the contract independent of the expected profitability evolution of the firm. Our estimates of private information from intangible sources are larger than those of Ai et al. (2022) identified from physical investment in a hidden action framework.

Our explanation for the rise in CEO compensation complements the arguments of the rising pay inequality in the labour force (e.g. Garicano and Rossi-Hansberg (2006), Lustig et al. (2011) and Frydman and Papanikolaou (2018)) that the ICT revolution has increased the growth opportunities from heterogeneous labourer ability to leverage new information to expand the production set of the firm. Such explanations predict increased levels of pay and feature performance sensitivity as a by-product of increasing outside options. Cziraki and Jenter (2021) provide evidence that only a small fraction of CEOs are substituted across firms and that the majority are promoted internally which motivates examination of complementary channels like ours of increasing private information.

A number of recent papers have examined the effects of rising firm intangibility on the declining labour share and physical investment, or rising firm cash holdings and markups (Karabarbounis and Neiman (2014), Crouzet and Eberly (2018), Ward (2022), Covarrubias et al. (2020), Falato et al. (2020), Hartman-Glaser et al. (2019), Autor et al. (2020), Kehrig and Vincent (2021)). These studies have largely focused on US public firms and typically associate higher intangibility with greater markups, productivity and profitability. In our data encompassing public and large private firms we do not find this overall positive association of intangibility and either productivity or profits suggesting the selection of high intangibility firms to be public have some other characteristics that distinguish them from the private intangible firms. Alternatives to private financing like large institutional shareholders may be able to reduce the size of the information frictions for high intangibility public firms similar to private investors as suggested by Aghion et al. (2013) with the positive association between innovative activity and institutional ownership.

For transparency in the model mechanism and identification in our structural estimation we made some simplifying assumptions that may limit the generalizability of our results. First, we abstracted from dynamic investment and financing opportunities for a given firm (akin to the static structural estimation of block pricing by Albuquerque and Schroth (2010)), taking the firm level of intangibility as an exogenous, permanent characteristic. Allowing an exogenous evolution of intangibility for a given firm should not affect the main thrust of our channel, simply adding variability in the growth and pay-performance sensitivity under the optimal contract. Endogenizing existing firms level of intangibility is likely important for capturing the evolution and distribution of firm size (Ward (2022)), and consequently capture lifecycle determinants in firms capital structure (e.g. Falato et al. (2020)) and timing of going-public or going private decisions (e.g. Ferreira et al. (2012)), but due to conflicting effects in firm size and age for listing likelihood it is unclear of the size or direction of the bias introduced by our static sorting mechanism. Second, in our contracting environment we restricted attention to CARA utility and precluded the CEO from over-reporting. Relaxation of either one or both of these restrictions will modify and complicate the equilibrium evolution of optimal compensation (see for instance Edmans et al. (2012) and Di Tella and Sannikov (2021)) but shouldn't change the the qualitative links between firm intangibility, performance sensitivity and the level of CEO pay. Third, we abstract from career concerns and heterogeneity in the outside options of potential CEOs which has been found to be important considerations in the cross-sectional variation in public executive pay. To the extent that these career concerns haven't changed substantially over time this abstraction is innocuous.

Our framework suggests the decline of US public firms is the efficient market equilibrium response to rising informational asymmetry between firm insiders and the general public. A richer model with welfare costs to wealth inequality and advantages to broader access to financial makers may (e.g. with highly concentrated wealth) entirely reverse the efficiency of private financing. In our static financing decision, differential returns between the private investors and the public investors have no dynamic effects on the future selection of firms. A simple dynamic extension of the model would imply that the highest ability private investors obtain the highest net return and accumulate ever increasing shares of aggregate wealth over time, crowding out public investors from a widening segment of the economy and leading to a declining correlation between US stock market performance and domestic economic indicators (as found by Greenwald et al. (2022)). We leave such extensions and examinations for future work.

To increase public listings and reduce public CEO pay our model suggests improving accounting, general understanding and disclosure of intangible assets is key. However, transparency in intangible assets may be impossible without diluting their value or exposing them to increased litigation risk. Consequently, providing broader access to private investments through new investment vehicles for the broader public, or making alternative disclosure requirements may mitigate the diminishing value of public markets without unduly distorting market efficiencies.

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Appendix A Theory

A.1 Optimal Contracts

We consider a principal-agent contracting problem in continuous time between an uninformed financier (or investor) and an entrepreneur (or CEO) with a project of type $\theta = (\mu^x, \mu^z, \lambda^x, \lambda^z, \sigma^x, \sigma^z, \tau)$. The setting is that of Williams (2011) (hereon W11) with the addition of a mixture of publicly and privately observed cash flows, and hidden savings, as analyzed in Bloedel et al. (2020) (hereon BKS20).[40]

In addition to extending and adapting the arguments of W11 and BKS20 to this mixture of publicly observed and unobserved cash flows, we appeal to a new, meaning not previously referenced in the literature, discounted infinite horizon stochastic maximum principle (SMP) to establish that the set of first-order incentive compatible (FO-IC) contracts contains the set of incentive compatible (IC) contracts in this setting, thereby ensuring that the contract is in fact the optimal contract.

The remainder of this section is structured as follows. We first provide a formal description of the contracting environment, the agent's reporting problem with hidden savings (given a contract), and the principal's optimal contract design problem. We then break the solution of the principal's problem into three parts: first, we solve for the agent's optimal reports given no hidden savings and conditional on a given contract; second, we solve for the agent's optimal consumption-savings decision under truthful reporting; third, we solve for the optimal contract specified by the principal. In particular, to solve for the optimal contract we:

1. Characterize the set of IC contracts by
   (a) Using a change of variables to tractably reformulate the agent's reporting problem into a tractable controlled diffusion problem on the infinite horizon (as in W11 and Cvitanić and Zhang (2013));
   (b) Establishing that the infinite horizon problem conforms to the primitives needed to invoke the results from Haadem et al. (2012), Haa-

[40] W11 claims to characterize the optimal (insurance) contract with persistent private information and without hidden savings. BKS20 provide a counterexample to establish the generic sub-optimality of the relevant contract of W11, demonstrating an issue with the reliance of W11 on a numerical observation for his solution. However, BKS20 show that the contract of W11 is indeed optimal for the class of stationary contracts amongst the class of first-order incentive-compatible contracts.

dem et al. (2013), Maslowski and Veverka (2014), and Øksendal and Sulem (2019) of an appropriate (necessary) SMP.

2. Characterize the set of No-Savings compatible (NS-compatible) contracts.

3. Use a stochastic variant of the dynamic programming principle to guess and verify the solution of the optimal contract.

4. Provide a verification argument that the resulting contract does induce truthtelling and no-hidden savings.

A.1.1 Contracting Environment

Time is continuous and infinite. Let $\mathbf{W} = (W_t)_{[0,\infty)}$ be a bivariate Wiener process on a filtered probability space $(\Omega, \mathscr{F}, \mathbb{F} = \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P})$ as presented in Øksendal and Sulem (2019) and Pham (2009).

The agent receives a random endowment $\mathbf{y} = (y_t)_{[0,\infty)}$, with $y_t \in \mathbb{R}$, of cash flows adapted to the filtration $\mathbb{F}$ as a mixture of two univariate Ornstein-Uhlenbeck processes, $\mathbf{x} = (x_t)_{[0,\infty)}$ and $\mathbf{z} = (z_t)_{[0,\infty)}$, described by

$$di_t = \mu^i(i_t)dt + \sigma^i dW_t^i,$$

where $\mu^i(i_t) = \mu^i - \frac{i_t}{\lambda^i}$, with $\mu^i, \lambda^i, \sigma^i > 0$ with $i \in \{x, z\}$.

The principal designs and commits to a compensation contract $\boldsymbol{\omega} : C[0,\infty)^2 \to C[0,\infty)$, where $C[0,\infty)$ is the set of continuous sample paths (functions). In return the principal receives the reported cash flows $\widehat{y}_t := y_t + m_t^y$, where m_t^y is the $(\mathbf{x}, \mathbf{z})$-adapted misreporting process. Thus, $-m_t^y$ is the amount of the cash flows which the agent diverts for their own consumption. For convenience, define $m_t := \frac{m_t^y}{1-\tau}$ as the misreports in terms of intangible cash flows (rather than total cash flows).

The principal observes jointly the realizations of the tangible cash flow process and the reports about the total cash flow $(\mathbf{x}, \hat{\mathbf{y}})$, but, besides the initial value, z_0, not the realizations of the private cash flow process $\mathbf{z}$, and hence of the total cash flows $\mathbf{y}$. Hybrid moral hazard arises as the principal can fully commit to a contract, while the agent cannot commit to a set of actions post-contracting and has the opportunity to misreport.

In addition to reporting the cash flows, we allow the agent to privately save and borrow at a risk-free rate r. Denote A_t as the agent's assets at time

$t \geq 0$, then **A** evolves according to

$$dA_t = (rA_t + \omega_t - m_t^y - c_t)dt, \ \lim_{t\to\infty} e^{-rt} A_t \geq 0 \text{ almost surely, } \quad \text{and } A_0 \text{ fixed.} \tag{14}$$

The agent has instantaneous exponential constant absolute risk-aversion (CARA) utility of consumption c given by $u(c) = e^{-\psi c}$, where $\psi > 0$ is the risk-aversion coefficient, while the principal is risk-neutral. Both parties have the same discount rate ρ.[41]

A.1.2 Agent's Problem

Given the contracted compensation process, $\boldsymbol{\omega}$, taking values $\omega(t, x_{[0,t]}, \widehat{y}_{[0,t]}) \in \mathbb{R}$, where $\mathbf{x}_t := x_{[0,t]}$ and $\widehat{\mathbf{y}}_t := \widehat{y}_{[0,t]}$ are sample paths of x_t and $\widehat{y}_t$ over the interval $[0, t]$, and agent's initial assets A_0, the agent's reporting-consumption problem is given by

$$V(\boldsymbol{\omega}) := \sup_{\mathbf{m}\in\mathscr{M}} \widetilde{V}(\boldsymbol{\omega}, \mathbf{m}) := \sup_{\mathbf{c}\in\mathscr{A}(\boldsymbol{\omega},\mathbf{m})} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right] \tag{15}$$

where expectations are taken over the sample paths of the processes $\mathbf{x}$ and $\mathbf{z}$ implied by the probability measure $\mathbb{P}$, and the admissible space $\mathscr{A}(\boldsymbol{\omega}, \mathbf{m})$ is given by

$$\mathscr{A}(\boldsymbol{\omega}, \mathbf{m}) = \{\mathbf{c} : \mathbf{v} = (\mathbf{c}, \mathbf{m}) \text{ which is } (\mathbf{x}, \mathbf{z}) \text{ adapted, and implies } \mathbf{A}^{\mathbf{v}} \text{ given by } (14)\}.$$

As the uninformed principal knows the fundamental parameters governing the cash flow process $\mathbf{y}$, i.e. θ, and the initial conditions, x_0 and z_0, and observes the realizations of the processes $(\mathbf{x}, \hat{\mathbf{y}})$, the admissible (undetectable) misreporting strategies must generate the same stochastic law as the total cash flows and exhibit the same statistical properties of $(\mathbf{x}, \mathbf{y})$.

Specifically, $\mathscr{M}$ is the set of feasible misreports given by $\mathbf{m}$ which

1. is adapted to $(\mathbf{x}, \mathbf{z})$,
2. has continuous sample paths, i.e. $m_t = \int_0^t \Delta_s ds$ for some process $\boldsymbol{\Delta}$,
3. has square integrable stochastic exponential martingale and is bounded, i.e. $\exists K_m > 0 : |m| \leq K_m$,[42]

[41] Since both the agents and principals share the same discount rate, in a closed economy, via standard arguments, the risk-free asset must have a rate of return equal to the discount rate, that is $r = \rho$, and so hereon we will not distinguish between the two.

[42] See **?** Chapter 8.8 for a discussion of stochastic exponential martingales. This technical restriction ensures that misreports can be equivalently characterized via the Girsanov's

4. is uncorrelated with $\mathbf{x}$.[43]

A contract $\boldsymbol{\omega}$ is truthful revelation IC if

$$\widetilde{V}(\boldsymbol{\omega}, \mathbf{0}) \geq \widetilde{V}(\boldsymbol{\omega}, \mathbf{m}) \qquad \forall \mathbf{m} \in \mathscr{M} \qquad (IC)$$

and NS-compatible, for a given misreporting process $\mathbf{m}$, if

$$\widehat{\mathbf{c}}(\boldsymbol{\omega}, \mathbf{m}) \in \arg \sup_{\mathbf{c} \in \mathscr{A}(\boldsymbol{\omega}, \mathbf{m})} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right] \quad s.t. \quad \widehat{c}_t = \omega_t - m_t^y \ \ \forall t. \qquad (NS)$$

Given the outside option of the agent $q_0 \in \mathbb{R}_-$, a contract is individually rational (IR) for the agent if $\widetilde{V}(\boldsymbol{\omega}) \geq q_0$.[44]

A.1.3 Principal's Problem

Given outside option q_0, the principal solves

$$J(q_0) := \sup_{\widehat{\mathbf{m}} \in \mathscr{M}} J(q_0, \widehat{\mathbf{m}}) := \sup_{\widehat{\mathbf{m}} \in \mathscr{M}} \sup_{\boldsymbol{\omega} \in \mathbb{S}(q_0)} \mathbb{E}\left[\int_0^\infty e^{-\rho t}(y_t - \omega_t + \widehat{m}_t^y) dt\right] \quad (16)$$

where $\widehat{\mathbf{m}}$ is the recommended misreporting strategy and the feasible space of contracts is given by

$$\mathbb{S}(q_0) = \left\{ \begin{array}{c} \omega : C([0,\infty)^2) \to C[0,\infty), \mathscr{F}_t\text{-predictable}, \\ \mathbb{E}[\int_0^\infty e^{-\rho t} u(\omega_t)^2 dt] < \infty \text{ and satisfies (IC), (NS) and (IR)} \end{array} \right\}.$$

That is, feasible contracts are any which is incentive and no-savings compatible, individually rational for the agent, and yields a well-defined (discounted) square-integrable expected lifetime under truthful reporting.

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transformation. Note that the restriction imposed here is similar but slightly stronger than that of BKS20, but is used to appeal to an appropriate SMP and ensure, in contrast to BKS20, that the first-order approach to IC constraints is globally optimal.

[43] Since x is commonly observed and independent of private cash flows z, a principal can detect existence of misreports if there is a non-zero correlation of the residual cash flow reports and observable cash flows $\mathbf{x}$. Specifically, observing x, the principal can deduce $\widetilde{y}_t := \widehat{y}_t - \tau x_t = (1-\tau)\big(z_t + m_t\big)$, so that the stochastic law (distribution) governing $\widetilde{y}$ must be equal to that of z to avoid detection of misreports.

[44] Recall the agent's instantaneous payoff is $u(c) = -\exp(-\psi c) \in (-\infty, 0)$, so $q_0 = \int_0^\infty e^{-\rho t} u(c_t^0) dt \leq 0$ for any deterministic, real-valued process $\mathbf{c^0}$. Notice that q_0 can correspond either to $\underline{q}$ if there is no financing competition or to the initial promised utility offered by the competiting principal.

A.1.4 Transforming the Agent's Problem

Fix a path of the observable cash flows $\mathbf{x}$ and an admissible reporting strategy $\mathbf{m}^z := \mathbf{m}(\mathbf{x})$, which, conditional on $\mathbf{x}$, is adapted to the filtration generated by the private cash flow process z. That is, $\mathbf{m}$ is adapted to the filtration of x and z, $\mathbb{F} = \mathbb{F}^{x,z}$, while $\mathbf{m}^z$ is adapted to the private cash flow filtration, $\mathbb{F}^z$. With this, the stochastic exponential (likelihood) process for an admissible misreport strategy given x, m^z, is given by

$$\log \Gamma_t := \int_0^t a_s^z dW_s^z - \frac{1}{2} \int_0^t (a_s^z)^2 ds,$$

where

$$a_t^z := \frac{\mu^z(-m_t^z) + \Delta_t}{\sigma^z}.$$

Conditioning on a fixed path x, the evolution of private cash flow reports, $\tilde{y}_t := \widehat{y}_t - \tau x_t$, is described by $\frac{d\widetilde{y}_t}{1-\tau} = dz_t + dm_t$. Hence, by an application of Girsanov's Theorem, the likelihood process of observing private cash flow reports $\widetilde{y}$ under a misreport strategy m is

$$\frac{dP^{z,m}}{dP^{z,*}}(\mathbf{z}) = \Gamma_\infty \tag{17}$$

where $\mathbb{P}^{z,*}$ denotes the distribution of $\tilde{y}$ conditional on a path of x under truthful revelation, while $\mathbb{P}^{z,m}$ is the probability measure over the path of $\tilde{y}$ induced by m.

Then the evolution of private cash flow reports must satisfy

$$\frac{d\widetilde{y}_t}{1-\tau} \overset{d}{=} \frac{d\widetilde{y}_t^m}{1-\tau} = \big(\mu^z(z_t - m_t^z) + \Delta_t\big)dt + \sigma^z dW_t^{z,m}$$

where $\overset{d}{=}$ denotes equality in distribution and $dW_t^{z,m} = dW_t^z - a_t^z dt$ is the Brownian motion implied by the misreports. In this way, $\widetilde{y}^m$ is a weak solution to the (realized) private cash flow report SDE $\widetilde{y}$. Restricting attention to weak solutions of $\widetilde{y}$, we may then without further loss of generality use a change of variables $\widetilde{y}$ for Γ which, as first illustrated by Bismut (1978), makes the path $\widetilde{\mathbf{y}}$ deterministic (but only known up to time t).

The problem for the agent is then

$$V(\boldsymbol{\omega}) = \sup_{\mathbf{m}\in\mathscr{M}} \mathbb{E}^x \left[\mathbb{E}^{m^z}\left[\int_0^\infty e^{-\rho t} u(c_t)dt \big| \mathbf{x}\right]\right] = \sup_{\mathbf{m}\in\mathscr{M}} \mathbb{E}^x \left[\mathbb{E}^z\left[\int_0^\infty e^{-\rho t}\Gamma_t u(c_t)dt\right]\right]$$

where $c_t = \omega_t(\mathbf{x}, \widehat{\mathbf{y}}) - (1-\tau)m_t$, $\mathbb{E}^x[\cdot]$ is the expectation over the distribution of tangible cash flow paths $\mathbf{x}$, $\mathbb{E}^{m^z}[\cdot]$ is the expectation over $\tilde{y}$ under the measure $\mathbb{P}^{m^z}$, and $\mathbb{E}^z[\cdot]$ is the expectation under the (true) "P" measure for z, $\mathbb{P}^z$. The first equality is simply the law of iterated expectations, while the second equality follows from the independence of z and x.[45]

Noting that the evolution of x_t satisfies the Markov property and the agent's choice of controls does not affect its diffusion, we will conjecture that in any optimal contract $\omega_t(\mathbf{x}_t, \widehat{\mathbf{y}}_t) = \omega_t(x_t, \widetilde{\mathbf{y}}_t)$. Moreover, we will require $\omega_t(x_t, \widetilde{\mathbf{y}}_t)$ to be continuous in its first order derivative and its derivative with respect to x_t to be bounded.[46] Finally, for the agent's problem we will take $\omega_t = \omega_t(x_t)$ to be a deterministic function of x_t, taking as given a particular path $\widehat{y}_t$ which are impacted by the agent's cash flow reports, as is standard in the contracting literature (see, for instance, Cvitanić and Zhang (2013)). Thus, the agent's problem is transformed to a controlled diffusion problem with random coefficients.[47]

For convenience, we use an additional change of variables of m to $\tilde{m} = \Gamma m$, so that the agent's controlled state processes (using stochastic integration by parts) are

$$d\Gamma_t = \Gamma_t \frac{\mu^z(-\frac{\tilde{m}_t}{\Gamma_t}) + \Delta_t}{\sigma^z} dW_t^{z,m}, \qquad \Gamma_0 = 1,^{48}$$

$$d\tilde{m}_t = \Gamma_t \Delta_t dt + \frac{1}{\Gamma_t}\tilde{m}_t d\Gamma_t, \qquad \tilde{m}_0 = 0,$$
$$dx_t = \mu^x(x_t)dt + \sigma^x dW_t^x, \qquad x_0 = \lambda^x \mu^x,$$

with $c_t = \omega_t(x_t) - (1-\tau)\frac{\tilde{m}_t}{\Gamma_t}$, and $dW_t^m = (dW_t^{z,m}, dW_t^x)$ are such that the private cash flow evolves as a martingale, i.e. $\frac{d\tilde{y}_t}{1-\tau} = \sigma^z dW_t^{z,m}$.

[45] Notice that the admissibility of misreports requires the tangible cash flows to be independent of the private cash flow reports, since under truthtelling x and z are both uncorrelated Gaussian processes, and so x and $\tilde{y}$ must also be uncorrelated to avoid detection, which given that uncorrelated multivariate Gaussian implies independence, yields the result. Consequently, $m = m^z$. See Lemma 1 of Szydlowski and Yoon (2022) for a formal discussion of the result of independence of separate change of measures of bivariate brownian motions.

[46] Solving the principal's optimal contracting problem, we will verify that indeed ω_t takes this form. In fact, we'll find that it is independent of x consistent with the full information case.

[47] Observe that if we were to restrict our attention to weak solutions of x_t and do a similar change of variables (in the same way as we did for z_t), then taking Γ^x to be the stochastic exponential and given that the principal perfectly observes x_t, we would require $\Gamma^x = 1$ everywhere and so m is automatically independent of x in this weak formulation.

[48] Again note that here we've dropped the dependence on the state of the sample space $(z_{[0,\infty)} + m_{[0,\infty)})$ which is taken as fixed.

A.1.5 Agent's Optimal Reports Without Hidden Savings

Let $X_t = (\Gamma_t, \tilde{m}_t, x_t)'$ denote the state vector process with evolution summarized by

$$dX_t = b(X_t, \Delta_t)dt + \Sigma(X_t, \Delta_t)dW_t^m$$

where $b(X_t, \Delta_t) = \begin{pmatrix} 0 \\ \Gamma_t \Delta_t \\ \mu^x(x_t) \end{pmatrix}$, $\Sigma(X_t, \Delta_t) = \begin{pmatrix} \Gamma_t a_t^z & 0 \\ \tilde{m}_t a_t^z & 0 \\ 0 & \sigma^x \end{pmatrix}$, $dW_t^m = \begin{pmatrix} dW_t^{z,m} \\ dW_t^x \end{pmatrix}$,
and $a_t^z = a_t^z(\frac{\tilde{m}_t}{\Gamma_t})$.

The associated generalized (current value) Hamiltonian to the above control problem is

$$H(X, \Delta, Y, Z) = b(X, \Delta)'Y + Tr(\Sigma(X, \Delta)'Z) + \Gamma u(X, \Delta) - \rho X'Y$$

where Y is a 3×1 vector and Z is a 3×2 matrix. The adjoint process (Y_t, Z_t) of the Hamiltonian then evolves according to the backward stochastic diffusion equation (SDE)

$$-dY_t = \nabla_X H(X_t, \Delta_t, Y_t, Z_t)dt - Z_t dW_t^m.$$

By inspection, the above problem now matches the setting of a discounted infinite horizon optimal control with controlled diffusions. The SMP invoked by W11 is for finite horizon and, is extended to the infinite horizon case by taking the limit $T \to \infty$. However, potential issues with this approach can arise as pointed out by Halkin (1974). Moreover, BKS20 raise concerns about the lack of an appropriate SMP known in the literature for payoff functions which are not bounded below (e.g. exponential utility). We leverage the works of Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) for a necessary SMP applied to Ito-Levy processes in discounted infinite horizon settings based on the satisfaction of a terminal transversality condition and restricted to payoff functions which are (discounted) integrable and have squared discounted integrable growth, i.e.

$$\mathbb{E}\left[\int_0^\infty |f_t(X_t, \Delta_t)| + ||\nabla_X f_t(X_t, \Delta_t)||^2 dt\right] < \infty \tag{18}$$

for any admissible controls Δ_t, where $f_t(X_t, \Delta_t) = e^{-\rho t}\Gamma_t u(c_t)$. A difficulty with the application of this SMP however is the verification of their transversality condition. In a restricted setting of controlled diffusions, that is, without jumps, Maslowski and Veverka (2014) provide a sufficient SMP as well as sufficient conditions to ensure the transversality condition holds.[49] In section **??** of this appendix, we formally verify our problem is amenable to an application of the results of Maslowski and Veverka (2014), Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) in order to establish that all IC contracts are necessarily FO-IC.

Having established the necessity of the Hamiltonian optimality conditions for incentive compatibility, we now characterize the IC constraint as done by W11 and BKS20. To keep the notation close to W11, we'll drop the z superscripts and keep just x ones. We take $Y = (q, p, p^x)'$, where, as demonstrated in section A.1.6, q is the promised utility, p is the promised marginal utility with respect to z, p^x is the promised marginal utility with respect to x. Moreover, we also take the diffusion of the adjoint, $Z = \begin{pmatrix} \sigma\gamma & \sigma^x\gamma^x \\ \sigma\iota & \sigma^x\iota^x \\ \sigma\zeta & \sigma^x\zeta^x \end{pmatrix}$, where $\sigma := \sigma^z$ (σ^x) is the volatility of the private (public) cash flows, γ (γ^x) is the sensitivity of the promised utility to private (public) cash flows, ι (ι^x) is the sensitivity of the promised marginal utility with respect to z to private (public) cash flows, ζ (ζ^x) is the sensitivity of the promised marginal utility with respect to x to private (public) cash flows. Suppressing the arguments, the generalized Hamiltonian simplifies to

$$H = \Gamma\Delta p + \mu^x(x)p^x + \sum_{i,j}\Sigma_{ij}Z_{ij} + \Gamma u(c) - \rho(\Gamma q + \tilde{m}p + xp^x) \tag{19}$$

with $\sum_{i,j}\Sigma_{ij}Z_{ij} = \Gamma a^z\sigma\gamma + \tilde{m}a^z\sigma\iota + \left(\sigma^x\right)^2\zeta^x$.

Recalling $a^z = \frac{\mu^z\left(-\frac{\tilde{m}}{\Gamma}\right)+\Delta}{\sigma}$ the agent's reporting FOC is given by

$$\Gamma p + \Gamma\gamma + \tilde{m}\iota = 0. \tag{20}$$

Invoking the Revelation Principle, we have under truthful revelation $\tilde{m} = \Delta = 0$, which combined with $\Gamma > 0$, yields the FO-IC condition

$$p + \gamma = 0. \tag{21}$$

Finally, the requisite evolution of the adjoint process is by direct calculation (using that under truthtelling $\Gamma = 1$ and $\tilde{m} = \Delta = 0$) is given by

[49]Unfortunately, we cannot directly appeal to the Maslowski and Veverka (2014) sufficient SMP since, as is common for sufficient SMPs, it requires the Hamiltonian to be concave.

$$dY_t \equiv \begin{pmatrix} dq_t \\ dp_t \\ dp_t^x \end{pmatrix} = \begin{pmatrix} \rho q_t - u(c_t) - \mu_t^z \gamma \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} - \mu_t^z \iota \\ \rho p_t^x - u'(c_t)\partial_x \omega_t + \frac{p^x}{\lambda^x} \end{pmatrix} dt + Z_t dW_t^m \tag{22}$$

Using the change of measure, $dW_t^z = dW_t^{z,*} = dW_t^{z,m} - a_t^z dt = dW_t^{z,m} - \frac{\mu_t^z}{\sigma^z} dt$, $dW_t^{x,m} = dW_t^x$, and recalling that $W_t = (W_t^z, W_t^x)'$, we have the evolutions of the elements of Y under truthtelling probability measure are given by

$$dY_t = \begin{pmatrix} \rho q_t - u(c_t) \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} \\ \rho p_t^x - u'(c_t)\partial_x \omega_t + \frac{p^x}{\lambda^x} + \zeta_t \mu^z \end{pmatrix} dt + Z_t dW_t, \tag{23}$$

with random initial conditions $q_0, p_0, p_0^x \in \mathbb{R}_-$, and terminal conditions

$$\lim_{t\to\infty} \mathbb{E}[e^{-\rho t} Y_t] = 0.$$

A.1.6 Solving for Optimal Contracts

First, by the Revelation Principle, we can without loss of generality restrict attention to optimal contracts inducing truthtelling, $\widehat{m}^y = 0_{[0,\infty)}$.

From the necessary optimality conditions solved for the agent's reporting problem, any incentive compatible contract must have promised utility and marginal utility processes (q, p, p^x) as described above and satisfy the FO-IC condition $\gamma_t + p_t = 0$.

The principal's problem can then be reformulated as

$$J(q_0) = \sup_{p_0, p_0^x} J^*(q_0, p_0, p_0^x) = \sup_{p_0, p_0^x} \sup_{\omega, \gamma, \gamma^x, \iota, \iota^x, \zeta, \zeta^x} \mathbb{E}\left[\int_0^\infty e^{-\rho t}(y_t - \omega_t)dt\right] \tag{24}$$

subject to the IC constraint $\gamma_t = -p_t$, the NS constraint, the stochastic evolution of y, x, z and adjoint processes dp_t, dq_t, dp_t^x.

Fixing p_0 and p_0^x and ignoring the NS constraint for now, we take a dynamic programming approach (as in W11 and BKS20). That is, we assume that the principal's value function $J_t : [0, \infty) \times \mathbb{R}^5 \to \mathbb{R}$ is twice continuously differentiable in its arguments, and we redefine the state X as $(z, x, q, p, p^x)'$, and the controls as (p_0, p_0^x, α), where $\alpha = (\omega, \iota, \iota^x, \gamma, \gamma^x, \zeta, \zeta^x)$. Then the associated HJB is given by

$$0 = \sup \mathscr{L}_t J_t + y - \omega, \qquad \limsup_{t\to\infty} \mathbb{E}[J_t] = 0,$$

$$\text{where } \mathscr{L}_t J_t = \tfrac{\partial}{\partial t} J_t dt + \tfrac{\partial}{\partial X'} J_t \mathbb{E}_t[dX_t] + \tfrac{\partial^2}{\partial X' \partial X} J_t \mathbb{E}_t[dX_t' dX_t],$$

$$dX_t = \widetilde{b}(X_t, \alpha_t)dt + \widetilde{\Sigma}(X_t, \alpha_t)dW_t$$

$$\widetilde{b}(X_t, \alpha_t) = \begin{pmatrix} \mu^z(z_t) \\ \mu^x(x_t) \\ \rho q_t - u(c_t) \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} \\ \rho p_t^x - u'(c_t)\partial_x \omega_t + \frac{p^x}{\lambda^x} + \zeta_t \mu^z \end{pmatrix}, \qquad \widetilde{\Sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma^x \\ Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \\ Z_{31} & Z_{32} \end{pmatrix},$$

where Z_{ij} is (ij)th entry of the Z matrix.

By inspection, taking $J_t(z, x, q, p, p^x) = e^{-\rho t} J(z, x, q, p, p^x)$, this problem is of controlled diffusion with principal's payoff satisfying a quadratic growth condition.[50] Hence, appealing to Theorem 3.5.3 of Pham (2009), if the principal's value function J is twice continuously differentiable, satisfies a quadratic growth condition and a transversality condition, $\lim_{t\to\infty} e^{-\rho t}\mathbb{E}[J(X_t^\alpha, \alpha_t)] \leq 0$, solves the HJB above with a measurable and feasible control function α and admits a unique solution to the state SDE, then J is the value function and α the optimal Markovian control.[51] Then the HJB simplifies to

$$\rho J = \max_{\omega, \iota, \iota^x, \gamma^x, \zeta, \zeta^x} y - \omega + \frac{\partial}{\partial X'} J \widetilde{b}(X, \alpha) + \frac{1}{2} Tr\left(\widetilde{\Sigma}\widetilde{\Sigma}' \frac{\partial^2}{\partial X' \partial X} J\right)$$

Using the binding IC constraint $\gamma = -p$, by direct computation we get

$$tr\left(\widetilde{\Sigma}\widetilde{\Sigma}' \frac{\partial^2}{\partial X' \partial X} J\right) = J_{zz}\sigma^2 + J_{xx}\left(\sigma^x\right)^2 + J_{qq}[Z_{11}^2 + Z_{12}^2] + J_{pp}[Z_{21}^2 + Z_{22}^2] + J_{p^x p^x}[Z_{31}^2 + Z_{32}^2]$$

$$+2\left[J_{xq}\left(Z_{12}\sigma^x\right) + J_{xp}\left(Z_{22}\sigma^x\right) + J_{xp^x}\left(Z_{32}\sigma^x\right)\right]$$

$$+2\left[J_{zq}\left(Z_{11}\sigma\right) + J_{zp}\left(Z_{21}\sigma\right) + J_{zp^x}\left(Z_{31}\sigma\right)\right]$$

[50] In particular, note that the instantaneous payoff of the principal is $y - \omega$, where $y = \tau x + (1-\tau)z$ is linear in x and z, and hence trivially satisfies a quadratic growth condition, $|y| \leq C(1 + |y|^2)$ for $C = 1$, and ω is a control.

[51] The growth condition can be generalized to any N degree polynomial growth condition with suitable restriction on the discount rate. See Fabbri et al. (2018).

$$+2\left[J_{qp}\left(Z_{11}Z_{21}+Z_{12}Z_{22}\right)+J_{qp^x}\left(Z_{11}Z_{31}+Z_{12}Z_{32}\right)\right]$$

$$+2\left[J_{pp^x}\left(Z_{21}Z_{31}+Z_{22}Z_{32}\right)\right].$$

For any J, the optimization of the HJB is separable with respect to ω and the other controls, and using $u'(c)=-\psi u(c)$, we get the following optimization problem for ω:

$$\max_{\omega} -\omega - u(c)J_q - \psi u(c)(1-\tau)J_p + \psi u(c)\partial_x \omega J_{p^x}.$$

We now guess that the value function takes the form

$$J = J^x(x) + J^{p^x}(p^x) + J^z(z,q,p) \tag{25}$$

where we further conjecture that

$$J^z(z,q,p) = j_0 + j^z(z) - j^z_q \log(-q) + h\left(\frac{p}{q}\right). \tag{26}$$

With this guess and using the agent's consumption c is linear in ω and $u(c)$ is strictly concave, the resulting FOC is:

$$u(c)\psi\left[J_q + (1-\tau)\psi J_p\right] = 1 \tag{27}$$

which is necessary and sufficient provided $J_q + \psi(1-\tau)J_p < 0$. Plugging in the guess of the value function into the HJB results in the remaining optimizations separating into individual problems:

$$\sigma^2 \max_{\iota}\left\{\frac{J_{pp}}{2}\iota^2 + J_{qp}\iota\gamma\right\}+$$

$$\max_{\zeta}\left\{\frac{J_{p^xp^x}}{2}\zeta^2\sigma^2 + J_{p^x}\zeta\mu^z(0)\right\},$$

and

$$(\sigma^x)^2 \max_{\zeta^x}\left\{\frac{J_{p^xp^x}}{2}(\zeta^x)^2 + J_{xp^x}\zeta^x\right\}+$$

$$(\sigma^x)^2 \max_{\iota^x,\gamma^x}\left\{\frac{J_{qq}}{2}(\gamma^x)^2 + \frac{J_{pp}}{2}(\iota^x)^2 + J_{qp}\iota^x\gamma^x\right\}.$$

Recalling that with the IC constraint $\gamma=-p$ is fixed, the first two problems are (strictly) concave provided that $J_{pp}\le 0$ ($J_{pp}<0$) and $J_{p^xp^x}\le 0$ ($J_{p^xp^x}<0$), respectively. In this case, defining $k:=\frac{p}{q}$, the FOCs yield

$$\iota = p\frac{J_{qp}}{J_{pp}} = -qk\left[k + \frac{h'(k)}{h''(k)}\right],$$

$$\zeta = -\frac{\mu^z(0)J_{p^x}}{\sigma^2 J_{p^x p^x}},$$

and

$$\zeta^x = -\frac{J_{xp^x}}{J_{p^x p^x}}.$$

The last problem of choosing jointly ι^x and γ^x is strictly concave if, in addition to the above, we have that $J_{qq}J_{pp} > J_{qp}^2$, which we will verify ex-post that holds. In this case, the FOCs yield $\gamma_x = \iota_x = 0$.

By direct computation, the derivatives of the guessed value function with respect to q and p are:

$$J_q = -\frac{1}{q}[j_q + h'(k)k], J_p = \frac{1}{q}h'(k) \tag{28}$$

$$J_{qq} = \frac{1}{q^2}\Big([j_q + h'(k)k] + [h''(k)k^2 + h'(k)k]\Big) \tag{29}$$

$$J_{pp} = \frac{1}{q^2}h''(k) \tag{30}$$

$$J_{qp} = -\frac{1}{q^2}\Big(h''(k)k + h'(k)\Big) \tag{31}$$

so that $\frac{J_{qp}}{J_{pp}} = -k - \frac{h'(k)}{h''(k)}$.

Hence the sufficient condition for an interior optimal solution of ι is $J_{pp} = \frac{h''(k)}{q^2} < 0$, and the additional sufficient conditions required for $\iota^x = \gamma^x = 0$ are (a) $J_{qq} < 0$, and (b) $J_{pp}J_{qq} - J_{qp}^2 = h''(k)j_q - h'(k)^2 > 0$.

Conjecturing that the principal provides full insurance of the public cash flows x and concentrates all carrots and sticks to those associated with the private cash flows z, we guess that $J_{p^x} = 0$.

So, from the FOC with respect to ω, the contracted utility is given by

$$u(c) = \frac{q}{\psi}U(k)$$

where $U(k) := \Big(j_q + h'(k)\big[k - (1-\tau)\psi\big]\Big)^{-1}$ and so compensation is given by $\omega = u^{-1}(\frac{q}{\psi}U(k)) = -\frac{1}{\psi}\log\Big(-\frac{q}{\psi}U(k)\Big)$. Observe that this satisfies our assumptions on ω for the agent's reporting problem, namely that ω is Markov. From this, we have that the principal's instantaneous payoff is

$$y - \omega = y + \frac{1}{\psi}\log\Big(-\frac{q}{\psi}U(k)\Big) = y - \frac{1}{\psi}\log\psi + \frac{1}{\psi}\log(-q) + \frac{1}{\psi}\log(U(k)).$$

Then using equation (27), using the fact that from the guesses we obtain that $-qJ_q = j_q + h'(k)k$ and $J_p p = kh'(k)$, the gradient terms of the HJB in (q,p) (of $\nabla_X J dX/dt$) is given by:

$$J_p\left[\rho p - (1-\tau)u'(c) + \frac{p}{\lambda}\right] + J_q[\rho q - u(c)]$$

$$= -u(c)[J_q - (1-\tau)\psi J_p] + \rho[J_p p\left(1 + \frac{1}{\rho}\frac{1}{\lambda}\right) + J_q q]$$

$$= -\frac{1}{\psi} - \rho j_q + \frac{kh'(k)}{\lambda}.$$

Similarly, using the FOC for ι, the relation of J_{qq} to J_{qp}, the diffusion correction terms can be written as:

$$\sigma^2\left(\frac{1}{2}J_{qq}[p^2] + \frac{1}{2}J_{pp}[\iota^2] + J_{qp}(-p\iota)\right) = \frac{\sigma^2 k^2}{2}\left(j_q - \frac{h'(k)^2}{h''(k)}\right).$$

Thus, plugging in the guess in the HJB on both sides and matching the coefficients we have

$$\rho j_x(x) = \tau x + j_x'(x)\mu^x(x) + j_x''(x)\frac{(\sigma^x)^2}{2} \tag{32}$$

$$\rho j_z(z) = (1-\tau)z + j_z'(z)\mu^z(z) + j_z''(z)\frac{\sigma^2}{2} \tag{33}$$

$$-\rho j_q \log(-q) = \frac{1}{\psi}\log(-q) \tag{34}$$

$$\rho h(k) = \frac{1}{\psi}\log(U(k)) + \frac{kh'(k)}{\lambda} + \frac{\sigma^2 k^2}{2}\left(j_q - \frac{h'(k)^2}{h''(k)}\right) \tag{35}$$

$$\rho j_0 := \rho(j_0^x + j_0^z) = -\frac{\log\psi}{\psi}. \tag{36}$$

The first two are second order linear differential equations with initial conditions of $x_0 = \lambda^x \mu^x$ and $z_0 = \lambda \mu^z$ (whose existence and uniqueness is guaranteed), and by guess of $j^z(z) = j_0^z + j_1^z z$ and $j^x(x) = j_0^x + j_1^x x$ have solution $j_0^z = \frac{1}{\rho}\mu^z j_1^z, j_1^z = \frac{1-\tau}{\rho+\frac{1}{\lambda}}$ and $j_0^x = \frac{1}{\rho}\mu^x j_1^x, j_1^x = \frac{\tau}{\rho+\frac{1}{\lambda^x}}$. The third and fifth are solved directly as $j_q = -\frac{1}{\rho\psi}$, and $j_0 = -\frac{\log\psi}{\rho\psi}$. The fourth is a second order, non-linear homogeneous ordinary differential equation for some initial condition, $h(k_0) = h_0, h'(k_0) = h'_0, k_0 = \frac{p_0}{q_0}$ (which itself is chosen optimally by the principal in a first stage), provided that $j_q + h'(k)(k - (1-\tau)\psi) > 0$. Using a change of variables $H(k) = h'(k)$ this second order ODE can be reframed as a system of first order ODEs which by Picard-Lindelof theorem a solution exists and is unique.

Given a solution $h(k)$, we must then verify that J satisfies the concavity restrictions necessary for the utilized FOC of ι to be optimal, and that the resulting value function satisfies a transversality condition.

The solution $-h(k)$ is plotted in W11 Figure A.1 for a fixed ψ and λ. Recalling W11 frames the principal's problem as a minimization of costs so his h(k) is the negative of ours, by inspection we have that $h''(k) < 0$ in our framing as well as $h'(k) \geq 0$ for any k not too large. Noting that taking $\rho\psi \to 0$ the sufficient conditions for interior solutions can be assured to be satisfied for a given k_0.

All of this takes as given a particular set of initial conditions for $h(k_0), h'(k_0)$. These are itself free-variables for the principal, and so the optimal contract (without no savings constraints) is then pinned down by solving for the optimal initial condition k_0 (assuming an interior optimum exists). W11 found by numerical solutions a local optimum where $h'(k_0) = 0$, so that in our context $k_0 = (1-\tau)\frac{\rho\psi}{\rho+\lambda}$.

However, given the counterexample of BKS20 we know that at least without the restriction of no hidden savings, this contract is in fact not optimal.

Rather than attempting to solve for the unconstrained optimal initial conditions, we will now move to the restricted optimal contract in the case where the agent has access to hidden savings.

A.1.7 Optimal Contracts without the No Hidden Savings Restriction

In Proposition 3.1 of BKS20 they characterize the agent's (self-insurance) problem for a single Ornstein-Uhlenbeck endowment process and exponential utility and establish the solution satisfies the standard consumption intertemporal Euler equation as

$$u'(c_t) = E_t[u'(c_{t+\tau})], \forall \tau \geq 0 \qquad (NoHiddenSavingsConstraint) \qquad (37)$$

Consequently, to preclude hidden savings, a contract must ensure that the agent's associated consumption satisfies this condition.

From the previous section, we have that the contracted compensation fully stabilizes the observable cash flow component, x_t, so that the consumption and (hidden) savings problem under the contract only depends on the unobservalb cash flow process, z_t. Thus we can directly apply the results of Theorem 4 of BKS20.

Specifically, given the exponential utility, we have $-\psi u(c_t) = -\psi \mathbb{E}_t^*[u(c_\tau)]$, so that by cancelling $-\psi$ and integrating from t to ∞, we have

$$q_t = \int_t^\infty e^{-\rho(\tau-t)} E_t^*[u(c_\tau)]d\tau = \frac{u(c_t)}{\rho}$$

.

Similarly, by direct computation

$$p_t = \int_t^\infty e^{-(\rho+\frac{1}{\lambda})(\tau-t)} E_t^*[-(1-\tau)u'(c_\tau)]d\tau = (1-\tau)\frac{\psi}{\rho\lambda}q_t$$

so that the ratio of (negative) promised marginal utility to the level of promised utility, $k_t = \frac{p_t}{q_t} = k_0^* = (1-\tau)\frac{\rho\psi}{\rho+\lambda}$.

Thus, introducing the no hidden savings restriction, in order to preclude hidden savings, the initial ratio of marginal utility to promised utility must satisfy $h'(k_0) = 0$. Plugging this into the differential equation for $h()$ implies that $h(k_0) = \frac{-1}{\psi}\log(j_q) + \frac{\sigma^2 k_0^2}{2} j_q$. With this, the W11 contract (adjusted for τx_t cash flows) is the unique contract which satisfies the required optimality conditions.

Now using the numerical observation that $h''(k_0) < 0$ (see figure A.1 in the W11 appendix, and multiply the function by -1), and so the needed concavity for the FOCs used when solving for the optimal contract are both necessary and sufficient, e.g. $J_{pp} < 0, J_{qq} = \frac{1}{q^2}(j_q + h''(k_0)k_0^2) < 0, h''(k_0)j_q > 0$ (required for $\gamma_x = \iota_x = 0$, ι interior), thus, this is indeed a solution here in our context as well. By inspection, J is finite and hence $e^{-\rho t}J \leq 0$ for $t \to \infty$ so that it indeed satisfies the requisite transversality condition for the principal's problem.

A.1.8 Verifying Agent's Optimal Reports Given the Contract

As the last point of the argument, since we only utilized a necessary SMP for the agent's problem, we must make a verification argument. That is, we verify that the agent, with the contract in hand, finds indeed it optimal to truth-tell and to not privately save.

First, observe that with $\tau = 0$, then we are in the setting of W11 and BKS20, where from BKS20 Lemma D.2 and D.3 the value function of the agent with hidden savings and private reports (indirectly implemented by solving BKS20's agent's self-insurance problem in section 3.1 of BKS20) is given by

$$V(A,z) = V_0 \exp\left(-\rho\psi\left(A + \frac{z}{\rho + \frac{1}{\lambda}}\right)\right)$$

where $V_0 = -\exp\left(\rho\psi\left[\frac{\mu^z(0)}{\rho(\frac{1}{\lambda}+\rho)} + \frac{\log\rho}{\rho\psi} - \frac{1}{2\psi\rho^2}\left(f(\rho;\frac{1}{\lambda})\sigma\right)^2\right)\right)$ with $f = \frac{\psi\rho}{\rho+\frac{1}{\lambda}}$ as in BKS20, and with optimal assets (given by BKS20 eq. 3.7) equal to

$$A_t^* = A_0 + \frac{1}{2\psi\rho}\left(\sigma f(r;\frac{1}{\lambda})\right)^2 t - \int_0^t \frac{\mu^z(z_t)}{\rho + \frac{1}{\lambda}} dt$$

for optimal initial assets $A_0(z_0, q) = \frac{\bar{c}(q)}{\rho} - \frac{\mu^z(0)}{\rho(\rho+\frac{1}{\lambda})} - \frac{z}{\rho+\frac{1}{\lambda}} + \frac{\left(\sigma f(r;\frac{1}{\lambda})\right)^2}{2\psi\rho^2}$ (given by BKS20 eq. 3.11), $\bar{c}(q) = -\frac{\log(-q)}{\psi\rho}$.

Second, observe that with $\tau = 1$, the optimal contract is equal to the full information insurance contract (complete stabilization) so that agent utility is $V^x(q) = \bar{c}(q)$ and can be implemented via agent self-insurance contract by simply giving the agent $A_0^x = \bar{c}(q)$.

Finally, by similar logic, for $\tau \in (0,1)$, the optimal contract indirectly implemented via a self-insurance contract with

$$A_0(q;\tau) = \frac{\bar{c}(q)}{\rho} - \frac{(1-\tau)\mu^z(0)}{\rho(\rho+\frac{1}{\lambda})} - \frac{(1-\tau)z}{\rho+\frac{1}{\lambda}} + \frac{\left((1-\tau)\sigma f(r;\frac{1}{\lambda})\right)^2}{2\psi\rho^2}.$$

A.2 Firm Listing Equilibrium

A.2.1 Partial Equilibrium Definition

For any $\underline{q} < 0$, $B^f > 0$ with $f \in \{P, S\}$, $\nu > 0$ and absolutely continuous CDF $G_\theta(\cdot)$, we define a public listing market equilibrium as a collection of financiers' bidding strategies $q^f : \mathscr{Q} \times \Theta \rightarrow \mathbb{R}_-$ and financier selection rules of the entrepreneurs $\mathbb{I}^f : \mathscr{Q}^2 \times \Theta \rightarrow \{0,1\}$ such that

1. $q^f, q^{-f}, \mathbb{I}^f$ is an extensive form trembling perfect equilibrium. That is, for each $\theta \in \Theta$
   (a) q^f is a best-response to $q^{-f}, \mathbb{I}^f$ and any sequence of trembles to q^{-f}, $\mathbb{I}^f$
   (b) $\mathbb{I}^f \in \{0,1\}$ is a best-response to q^{-f}, q^f and any sequence of trembles to q^{-f}, q^f
2. firm listing choice is feasible, that is $\mathbb{I}^f + \mathbb{I}^{-f} \leq 1$, as well as equilibrium financing is feasible for both financiers, that is

$$\int_\theta \mathbb{I}^f(q^f(\theta), q^{-f}(\theta))dG(\theta) \leq B^f$$

where $\Theta \subseteq \mathbb{R}^4_+$, $\theta = (\mu, \sigma^z, \tau)$ and $\mathscr{Q} = \{-\infty\} \cup [\underline{q}, 0]$.

A.2.2 Proof of Theorem 2

First note that, given the agents' preferences over optimal contracts solved for in Theorem 1, the agent utility varies over contracts solely based on the initial promised utility q_0. Hence, in any equilibrium, a type θ agent's best-response to the financiers' bids of initial promised utilities (q^f, q^{-f}) is

$$(\mathbb{I}^f, \mathbb{I}^{-f}) = \begin{cases} (1,0) & \text{if } q^f > \max\{q^{-f}, \underline{q}\}, \\ (0,1) & \text{if } q^{-f} > \max\{q^f, \underline{q}\}, \\ (0,0) & \text{if } \underline{q} > \max\{q^f, q^{-f}\}, \\ (a,b), a+b \in [0,1], a,b \in \{0,1\} & \text{else.} \end{cases}$$

Assuming a successful bid, we can rearrange the payoff of a type f financier financing a type θ firm with a bid of initial promised utility q_0 as

$$R^f(q_0, \theta) = \Upsilon(\mu) - \Lambda^f(\theta) - X(q_0).$$

Notice that $X(\underline{q}) = 0$, $X'(q_0) > 0$, $\Upsilon'(\mu) > 0$, $\Lambda^S(\theta) = \nu$, and $\Lambda^P(\theta) = \pi(\theta)$ $\forall \theta \in \Theta$. Then, absent competition, so that $X(\underline{q}) = 0$, it is individually rational (IR) for a type f financier to finance a type θ firm if

$$\Upsilon(\mu) - \Lambda^f(\theta) \geq 0 \qquad \left(IR^f\right).$$

This implies that the two representative financiers have two distinct IR sets. As a result, three regions arise where it is IR for neither financier, it is IR for either one or the other financier, and it is IR for both financier. In the first region, trivially no financing occurs in equilibrium as both financiers bid less than $\underline{q}$ (or do not bid at all) and $\mathbb{I}^f = \mathbb{I}^{-f} = 0$ is the unique equilibrium.

In the second region, only one financier can earn a positive payoff from financing a type θ firm. In this situation, a given type f financier can act as a monopolist and bid the agent's outside option $\underline{q}$ so to extract full surplus.

In the third region, both financiers can earn a positive payoff at the agent's outside option, $\underline{q}$. Denote $\bar{q}(\theta) := \min\{\bar{q}^S(\theta), \bar{q}^P(\theta)\}$ where $\bar{q}^f(\theta)$ solves $R^f(\theta, \bar{q}^f(\theta)) = 0$ for $f \in \{S, P\}$. Since a type θ agent's best-response is to choose the higher bid of initial promised utility, standard arguments imply that the best response for a type f financier in this region is to weakly outbid the competitor as long as the payoff from financing are strictly positive, that is

$$q^f(q^{-f}, \theta) = \begin{cases} q^{-f} + \epsilon, & \text{if } R^f(\theta, q^f(q^{-f}, \theta)) > 0, \\ [\underline{q}, q^{-f}] & \text{else} \end{cases}$$

for $\epsilon \to 0$. This implies that any $q^{-f} < \bar{q}(\theta)$ cannot be an equilibrium. Without loss of generality suppose that $\bar{q}(\theta) = \bar{q}^{-f}(\theta)$. If the firm selection rule $\mathbb{I}^f(\bar{q}(\theta), \bar{q}(\theta)) < 1$, f can profitably deviate to $\bar{q}(\theta) + \epsilon$, and hence it is not an equilibrium. If $\mathbb{I}^f(\bar{q}(\theta), \bar{q}(\theta)) = 1$, then from the best-response functions neither financier has incentive to deviate and the firm is indifferent. For a given θ, by the financier decomposition, and since $X'(q_0) > 0$, this intersection point is unique.

Define Region (A) as the region of Θ where $\bar{q}^S \geq \bar{q}^P$, that is the private financier has a comparative advantage over the public financier. Combining the above arguments and solving for the cutoffs demarcating these regions yields the following lemma.

Lemma 1 (No BC version)**.** *Denote* $\Upsilon := \frac{\mu - \omega^S(\underline{q})}{\rho} - 1$, *where* $\mu = \mathbb{E}[y_t|\theta]$. *In a public listing equilibrium with B not binding, we have*

1. *if* $\pi(\theta) \leq \Upsilon < \nu$, θ *publicly financed at bid* $\underline{q}$
2. *if* $\nu \leq \Upsilon < \pi(\theta)$, θ *privately financed at bid* $\underline{q}$
3. *if* $\Upsilon \geq \max\{\nu, \pi(\theta)\}$
   (a) *if* $\pi(\theta) < \nu$, θ *publicly financed at bid* $\underline{q}$

(b) *if* $\pi(\theta) \geq \nu$, θ *privately financed at bid* $\bar{q}_0^P(\theta)$

4. *if none of the above,* θ *not financed.*

Introducing the limited financing constraint, B for the specialist financier, there are two cases. If unconstrained the equilibrium is as specified in the lemma above. Now suppose it is constrained and consider $\theta \in$ Region (A) so that the specialist has a comparative advantage.

If θ also in Region (I), then the specialists payoff from financing the firm is $Y(\theta) - C^S - \Omega(\underline{q})$ and zero otherwise. In contrast, for $\hat{\theta}$ in Region (A) $\cap$ Region (III), then the specialists payoff from financing the firm is $Y(\hat{\theta}) - C^S - \Omega(\bar{q}(\hat{\theta}))$. By the definition of $\bar{q}(\theta)$, we have $\Omega(\bar{q}(\hat{\theta})) = Y(\hat{\theta}) - C^P(\theta)$ so that $Y(\hat{\theta}) - C^S - \Omega(\bar{q}(\hat{\theta})) = \pi(\theta) - \nu$ (consistent with Theorem 1). It follows that within Region (A) $\cap$ Region (III), the specialist is indifferent over μ and strictly increasing preference in $\pi(\theta)$, while within Region (A) $\cap$ Region (I), the specialist is indifferent over $\pi(\theta)$ and strictly increasing preference in μ. The specialist financier is indifferent between θ in Region (A) $\cap$ Region (I) and $\hat{\theta}$ in Region (A) $\cap$ Region (III) when $\pi(\hat{\theta}) - \nu = Y(\theta) - \nu - \Omega(\underline{q})$ which simplifies to

$$\pi(\hat{\theta}) = Y(\mu). \tag{38}$$

As an immediate consequence of the above (including that specialist financier equilibrium payoffs, without financing constraints, are non-decreasing in π and μ), subject to rationing, the specialist financier will impose a cutoff rule $(\underline{\mu}, \underline{\pi})$ satisfying (38) such that

$$\int_{\mu \geq \underline{\mu}, \pi \geq \underline{\pi}} dG(\mu, \pi) = B.$$

For all $(\mu, \pi(\theta))$ in Region (A) $\cap$ (Region (II)) but where $\mu < \underline{\pi}$ or $\pi < \underline{\pi}$, the public financier faces no competition in financing these firms and can earn profitable returns from financing, therefore in equilibrium bids $\underline{q}$ and finances these firms.[52]

Finally, for all $(\mu, \pi(\theta))$ in Region (A) $\cap$ (Region (I)) but where $\mu < \underline{\pi}$ or $\pi < \underline{\pi}$, the public financier faces no competition in financing these firms but cannot profit from financing, thus, this set of firms is un-financed.

[52]Again, note trembling perfection rules out alternative bidding equilibria of the non-financing financier (the specialist) in this region. Specifically, imposing any (accidental) violation of the budget constraint results in an unbounded penalty, any alternative bidding strategy $q > \underline{q}$ by the specialist in this region

With this we have the result, Theorem 2.

A.3 General Equilibrium and Comparative Statics

A.3.1 Proof of Theorem 3

First, since $M > 1$ there is an excess of financial resources over financing needs so that at least some measure of investors will not be in equilibrium matched to a firm. For such (unmatched) investors, $R^S(\nu, \varnothing, \varnothing) = -\nu < 0 = R^P(\nu, \varnothing, \varnothing)$ so that all investors electing to use the monitoring technology is not sustainable in equilibrium. Thus, in any equilibrium there is a positive mass of un-matched public investors.

Since the returns to public investing are not contingent on the investor type, these unmatched public investors can generate the same total surplus as any matched public investor, so that Bertrand competition ensues yielding $q^P(\theta)$ such $R^P(\cdot, \theta, q^P) = 0$ for any θ where it is individually rational for a public investor to invest (i.e. $R^P(\cdot, \theta, \underline{q}) \geq 0$). With this, we have a zero profit condition for public investors, that is, the equilibrium investor returns on any publicly financed firm is zero.

Combining this result with the slot constraints on using the monitoring technology, so that each private investor will bid $q > -\infty$ on at most one firm, and no two private investors will bid on the same firm, there are two distinct regions competition for private investors (just as in the partial equilibrium setting).

In one region, public investors do not find it individually rational to bid, whereas it is individually rational for a private investor with $\nu < \pi(\theta)$ to do so (for a given investor with ν, this is Region (IIa) and Region (I) in Figure 1a). In this case, as found in the partial equilibrium setting, a private financier is an effective monopolist, and earns $\Upsilon(\mu) - \nu$.

In the other region, both public and private investors find it individually rational to compete for the firm, and so, a given private investors return to outbid the public investors is $R^S(\nu, \theta, q^P) = \pi(\theta) - \nu$ (see Region (IIb) and Region (III) of Figure 1a).

Consequently, a given private investor is indifferent between financing a firm in one or the other region for $\Upsilon(\hat{\mu}) = \pi(\theta)$ where $\hat{\theta}$ in the first region and θ in the second. Re-indexing firms, by a change of variables $\pi'(\theta) := \min\{\Upsilon(\mu), \pi(\theta)\}$ and noting the free-exit of the private investor (switch to public), the return generation of a prospective private investor financing a given firm is

$$R(\nu, \theta, q) = [\pi'(\theta) - \nu]_+ - [\Omega(q) - \Omega(q^*(\theta))]_+$$

where $q^*(\theta) = \max\{q^P(\theta), \underline{q}\}$.

By inspection, the surplus (production) function $\mathbb{S}(\nu, \pi') := R(\nu, \pi', \underline{q}) = [\pi' - \nu]_+$ is submodular. Thus, negative assortative matching (NAM) is an equilibrium (pairwise stable) sorting. Denote $m(\nu)$ as the equilibrium matching of investor with cost ν to type π' firm. Equilibrium firm matching with NAM satisfies $\bar{G}_{\pi'}(\pi') = M \cdot G_\nu(\nu)$, so $m(\nu) = \bar{G}_{\pi'}^{-1}(M \cdot G_\nu(\nu))$.

With this, equilibrium private investor returns are given by $R^*(\nu) = m(\nu) - \nu$. By direct computation, $m'(\nu) < 0$, $m(0) > 0$ and $\lim_{\nu \to \infty} R^*(\nu) < 0$, by the Intermediate Value Theorem, there exists a unique fixed point, $\bar{\nu} > 0$, $R^*(\bar{\nu}) = 0$. Noting that $B = M \cdot G_\nu(\bar{\nu})$, and all sorting patterns are the same as in the partial equilibrium setting, we have established that the result is an equilibrium.

Finally, we move to uniqueness. We will show that without trembles other matching functions of $\nu \leq \bar{\nu}$ and $\pi' \geq \bar{\nu}$ can also constitute an equilibrium (e.g. PAM), but with trembling perfection, only NAM matching function survives.

To see this, define $q^S(\nu, \pi')$ to be full surplus transfer of financier of type ν to firm of type π' (so $\Omega(q^S(\nu, \pi')) = \pi' - \nu$). Observe that for any $\hat{\nu} < \nu$, and $\hat{\pi}' > \pi'$,

$$\begin{aligned} & R(\hat{\nu}, \hat{\pi}', q^S(\nu, \hat{\pi}')) - R(\hat{\nu}, \pi, q^S(\nu, \pi')) \\ &= [\hat{\pi}' - \hat{\nu}]_+ - [\pi' - \hat{\nu}]_+ + [\pi' - \nu]_+ - [\hat{\pi}' - \nu]_+ \geq 0 \end{aligned} \tag{39}$$

with equality holding iff $\pi' \geq \nu$ and strict inequality otherwise. As this inequality is the same as condition (7) in Chade et al. (2017), we have generalized decreasing differences in the bidder's surplus globally over the full support of ν, π'. However, restricting to $\nu \leq \bar{\nu}$ and $\pi' \geq \bar{\nu}$, (39) is identically zero so that any matching between types is an equilibrium. With trembles of investor financing type choice, (39) holds with strict inequality for any $\hat{\nu} < \bar{\nu}$ and $\nu = \bar{\nu} + \epsilon$, violating optimality of the matching of any non-NAM matching function. QED

A.3.2 Introduction to Comparative Statics

As established in the previous section, the GE equilibrium is fully characterized by the two equations, the equilibrium matching condition (40) and the equilibrium cutoff condition (41)

$$\pi'(\nu) : \bar{G}_{\pi'}(\pi'(\nu)) = G_\nu(\nu) \cdot M \tag{40}$$

and

$$\bar{\nu} : \pi'(\bar{\nu}) - \bar{\nu} = 0. \tag{41}$$

We now move to characterizing comparative statics of various economic aggregates of interest. To do so, we distinguish between the short and long-run, where in the short-run we hold fixed the set of (potential) private financiers, $\{\nu : \nu \leq \bar{\nu}\}$, but allow adjustment of the matching of this fixed set of financiers to firms, denoted $\tilde{\pi}'(\nu)$. We allow for free-exit, so that any (potential) private financier ($\nu \leq \bar{\nu}$) with negative profits may exit and earn zero return instead, but preclude free-entry of new private financiers (so that (40) holds in the short-run, but only $\tilde{\pi}'(\bar{\nu}^{SR}) - \bar{\nu}^{SR} \geq 0$ component of (41) is imposed. In the long-run, free-entry is allowed, imposing $\tilde{\pi}'(\bar{\nu}^{LR}) - \bar{\nu}^{LR} \leq 0$.

To assist with some of the comparative statics, consider $\Pi(\bar{\nu})$. Observe that $\Pi(\bar{\nu}) = E[\pi'(\nu) - \nu | \nu \leq \bar{\nu}] = E[\pi' | \pi' \geq \pi'(\bar{\nu})] - E[\nu | \nu \leq \bar{\nu}]$ where the second equality follows from the bijective, monotonic matching function / equality of the sets $\{\theta : \pi'(\theta) \geq \pi'(\bar{\nu})\}$ and $\{\theta : \pi'(\nu) = \pi'(\theta), \nu \leq \bar{\nu}\}$.

By an application of the inverse function theorem, the matching function of firms of type π' given financier type ν, $\pi'(\nu)$, is strictly decreasing in ν for any $\nu \geq 0$ (since $\frac{\partial G_{\pi'}(\pi')}{\partial \pi'} > 0$ for any π' in support by definition of PDF).

Adding and subtracting $\bar{\nu}$ and using the equality (41) yields

$$\Pi(\bar{\nu}) = MRL_{\pi'}(\bar{\nu}) + MAI_\nu(\bar{\nu})$$

where $MRL_{\pi'}(\bar{\nu}) := E[\pi' | \pi' \geq \pi'(\bar{\nu})] - \pi'(\bar{\nu})$ is the mean residual lifetime function and $MAI_\nu(\bar{\nu}) := \nu - E[\nu | \nu \leq \bar{\nu}]$ is the mean advantage over inferiors function.

Drawing from the above and results from Bagnoli and Bergstrom (2005) we have the next two lemmas.

Lemma 2. *If Assumption 2(i) (converse) holds, then the mean residual lifetime of* π' $MRL_{\pi'}(\bar{\nu})$ *is strictly increasing (decreasing) in* $\pi'(\bar{\nu})$ *and decreasing (increasing) in* $\bar{\nu}$*.*

Lemma 3. *If Assumption 2(ii) (converse) holds, then the mean advantage over inferiors of* ν *at* $\bar{\nu}$ $MAI_\nu(\bar{\nu})$ *is strictly decreasing (increasing) in* $\bar{\nu}$*.*

Proof of Lemma 2. Taking $\pi'(\bar{\nu}) = \underline{\pi}$ fixed, then if log convex (concave) $\bar{G}_{\pi'}$ appealing to Theorem 6 of Bagnoli and Bergstrom (2005), $MRL_{\pi'}(\bar{\nu})$ is strictly increasing (decreasing) in $\underline{\pi}$.[53] Combining this result with $\pi'(\nu)$ a strictly decreasing function of ν gives the result (MRL decreasing (increasing) in $\bar{\nu}$ if $\bar{G}_{\pi'}$ log convex (concave).

□

Proof of Lemma 3. An application of Theorem 5 of Bagnoli and Bergstrom (2005) directly yields $MAI_\nu(\bar{\nu})$ is strictly decreasing (increasing) in $\bar{\nu}$ if $G_\nu(\nu)$ is log convex (concave). □

Putting these two lemma's together we have that if Assumption 2 holds then $\Pi(\bar{\nu})$ is strictly decreasing in $\bar{\nu}$ (which implies that an increase in PE funds B leads to a decrease in the aggregate PE premium Π in GE). We summarize this and other immediate implications in the next lemma.

Lemma 4. *If Assumption 2 holds, with an increase in $\bar{\nu}$, total PE funds B increase, PE premium Π decreases, output O increases, public CEO pay Ω decreases and public listing propensity decreases.*

Proof. Total PE funds increase by definition of $B(\bar{\nu})$ implied by (40) and (41). Previous arguments established Π and by extension yields Ω decreasing and public listing propensity decreasing. Finally, O increases by assumption of full support and so positive weight on region (I) in Figure 1a. □

A.3.3 Proof of Theorem 4 - (i) Intangibility

We consider an increase in firm intangibility culminating in first-order stochastic increase in the information premium, π, so

$$\tilde{\pi}(\theta) = (1 + \epsilon(\theta))\pi \text{ for } \epsilon(\theta) > 0, \forall \theta.^{54} \tag{42}$$

For simplicity, let $\epsilon > 0$ be constant. Moreover, a first-order stochastic increase in firm intangibility by Assumption 1 corresponds to a first order stochastic increase in π' the (modified) information premium.[55] Denote $G^{SR}_{\pi'}$ as the

[53] Observe that Bagnoli and Bergstrom (2005) has a typo in Table 3 where the MRL(x) for both the Weibull and Gamma (with $c \in (0, 1)$) is proven to be increasing rather than decreasing.

[54] Recall $\pi \propto (1-\tau)^2$, so a transformation of $(1-\tau)^2 \mapsto (1-\tau)^2(1+\tau c)$ for some constant $c > 0$ results in $\tilde{\pi} = (1+\epsilon(\theta))\pi$ with $\epsilon(\theta) = \tau c$. Moreover, $\widetilde{(1-\tau)^2} \in (0,1) \forall \tau \in (0,1)$ and $\widetilde{(1-\tau)^2} > (1-\tau)^2$ for any $\tau \in (0,1)$, thus this transformation maintains the same support for τ as the original.

[55] By definition $\pi' = \min\{\Upsilon(\mu), \pi\}$, so $Pr(\tilde{\pi}' \leq \underline{\pi}) = Pr(\Upsilon(\mu) \leq \underline{\pi}) \cdot Pr(\epsilon\pi \leq \underline{\pi}) = G_\mu(\Upsilon^{-1}(\underline{\pi}))G_\pi(\frac{\underline{\pi}}{1+\epsilon}) < G_\mu(\Upsilon^{-1}(\underline{\pi}))G_\pi(\underline{\pi}) = Pr(\pi' \leq \underline{\pi})$.

CDF of π' after the increase in intangibility. By the definition of first-order stochastic dominance (FOSD):

$$G^{SR}_{\pi'}(\pi') \leq G_{\pi'} \forall \pi' \Rightarrow \bar{G}^{SR}_{\pi'} \geq \bar{G}_{\pi'}.$$

Let $\underline{\pi} = \pi'(\bar{v})$ prior to the shock and $\underline{\pi}^{SR}$ denote the interim cutoff (prior to extensive margin adjustment of $\bar{v}/B$). Then from (40) at the cutoff we have

$$\bar{G}^{SR}_{\pi'}(\underline{\pi}) > \bar{G}_{\pi'}(\underline{\pi}) = G_{\nu}(\bar{v}).$$

Since $\bar{G}$ strictly decreasing, $\underline{\pi}^{SR} > \underline{\pi}$ (and this also holds for any equilibrium matching of financiers to firms, $\tilde{\pi}'(\bar{v}) > \pi'(\bar{v})$ where $\tilde{\pi}'(\cdot)$ is the short-run matching function following the increase in intangibility and $\pi'(\cdot)$ the original matching function.

In the long-run, the set of financiers (and so financier cutoff $\bar{v}$) can adjust to ensure (41) holds. Since $\tilde{\pi}'(\bar{v}) > \pi'(\bar{v})$ and the matching function $\tilde{\pi}'(\cdot)$ is monotonically decreasing, it follows immediately that $\bar{v}^{LR} > \bar{v}$ and so (by (41)) $\underline{\pi}'^{LR} > \underline{\pi}'$. Thus, in summary,

$$\underline{\pi}' < \underline{\pi}'^{LR} < \underline{\pi}'^{SR} \tag{43}$$

Since $\pi' = \min\{\pi, \Upsilon(\mu)\}$, $\Upsilon(\underline{\mu}) = \underline{\pi}$ and so

$$\underline{\mu} < \underline{\mu}^{LR} < \underline{\mu}^{SR} \tag{44}$$

Combining these results with the definitions of the equilibrium economic aggregate definitions, we have the following results.

In the short-run (holding fixed B in partial equilibrium):

1. Π increases

   - Proof: the mass of privately financed firms **S** remains fixed and the financier cutoff type $\bar{v}$ is unaffected (since B is constant), so MAI is unchanged and MRL is increasing in π' by FOSD. i.e.

$$\Pi^{SR} = E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - E[\nu|\nu \leq \bar{v}^{SR}]$$

$$= E[\tilde{\pi}'(\theta)|\{\theta : \pi'(\theta) \geq \underline{\pi}\}] - E[\nu|\nu \leq \bar{v}^{SR}]$$

$$\geq E[\pi'(\theta)|\{\theta : \pi'(\theta) \geq \underline{\pi}\}] - E[\nu|\nu \leq \bar{v}^{SR}] = \Pi$$

2. Output O decreases

- Proof: Re-expressing O in terms of the total potential output - un-financed (and noting by independence $E[\mu]$ is invariant to transformation of π) we have

$$O^{SR}(\underline{\mu}^{SR}) = E[\mu] - \int_0^{\underline{\mu}^{SR}} \int_{\Upsilon(\mu)}^{\infty} \mu d\tilde{G}_\pi dG_\mu = E[\mu] - \int_0^{\underline{\mu}^{SR}} \mu \bar{\tilde{G}}_\pi(\Upsilon(\mu)) dG_\mu$$

$$\leq E[\mu] - \int_0^{\underline{\mu}^{SR}} \mu \bar{G}_\pi(\Upsilon(\mu)) dG_\mu$$

$$\leq E[\mu] - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} \mu d\tilde{G}_\pi dG_\mu = O(\underline{\mu})$$

where the first inequality follows from FOSD of $\tilde{G}_\pi$, the second inequality from $\underline{\mu} < \underline{\mu}^{SR}$ (given in (44)). QED

3. the mass of publicly financed firms $\mathbf{P}$ and public listing propensity $\frac{|\mathbf{P}|}{|\mathbf{P}|+|\mathbf{S}|}$ decreases

- Proof: Observe $|P \cup S| = 1 - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu$ so that

$$|P| = |P \cup S \setminus S| = 1 - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu - \int_{\underline{\mu}}^{\infty} \int_{\underline{\pi}}^{\infty} dG_\pi dG_\mu$$

By definition of short-run $|S|$ (and hence third term) is constant, while using the same argument as for output O, $|P \cup S|$ falls in short-run, yielding the result. The change in public listing propensity follows immediately.

4. CEO pay Ω increases

- Proof: By definition of Ω, $\Omega^{SR} > \Omega$ iff $\tilde{E}[\pi(\theta)|\theta \in P(\underline{\mu}^{SR})] > E[\pi(\theta)|\theta \in P(\underline{\mu})]$ By direct computation, for the short-run:

$$\tilde{E}[\pi(\theta)|\theta \in P(\underline{\mu}^{SR})] = \frac{1}{|P^{SR}|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi d\tilde{G}_\pi dG_\mu + \int_{\underline{\mu}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} \pi d\tilde{G}_\pi dG_\mu + \int_{\underline{\mu}^{SR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{SR})} \pi d\tilde{G}_\pi dG_\mu\right]$$

$$> \frac{1}{|P|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi d\tilde{G}_\pi dG_\mu + \int_{\underline{\mu}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} \pi d\tilde{G}_\pi dG_\mu + \int_{\underline{\mu}^{SR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{SR})} \pi d\tilde{G}_\pi dG_\mu\right] \tag{45}$$

where the inequality follows from $|P^{SR}| < |P|$ (shown above) Similarly, prior to the shock:

$$E[\pi(\theta)|\theta \in P(\underline{\mu})] = \frac{1}{|P|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi dG_\pi dG_\mu + 0 + \int_{\underline{\mu}}^{\infty} \int_0^{\Upsilon(\underline{\mu})} \pi dG_\pi dG_\mu\right] \tag{46}$$

The first term of (45) exceeds that of (46) by (42) (i.e. $\tilde{\pi} = \pi\varepsilon, \varepsilon > 1$)

The second term of (45) exceeds that of (46) since $\underline{\mu}^{SR} > \underline{\mu}$

The difference in the third terms of (45) -(46) is

$$\geq \bar{G}_\mu(\underline{\mu}^{SR})\left[\int_0^{\Upsilon(\underline{\mu}^{SR})} \pi d\tilde{G}_\pi - \int_0^{\underline{\mu}} \pi dG_\pi\right]$$

$$\geq \bar{G}_\mu(\underline{\mu}^{SR})\left[\int_0^{\mathrm{Y}(\underline{\mu})} \pi d\tilde{G}_\pi - \int_0^{\underline{\mu}} \pi dG_\pi\right]$$

$$\geq \bar{G}_\mu(\underline{\mu}^{SR})\left[\int_0^{\mathrm{Y}(\underline{\mu})} \pi dG_\pi - \int_0^{\underline{\mu}} \pi dG_\pi\right] = 0$$

where the first inequality follows from $\underline{\mu}^{SR} > \underline{\mu}$, and so $\bar{G}_\mu(\underline{\mu}^{SR}) < \bar{G}_\mu(\underline{\mu})$, the second inequality follows from $\mathrm{Y}(\underline{\mu}^{SR}) > \mathrm{Y}(\underline{\mu})$, and the third from the transformation of π (42). QED

In the long-run:

1. PE funds B expands (already shown above) and so $|S^{LR}| > |S| = |S^{SR}|$
2. Public listing mass declines relative to short-run (and hence declines even more relative to initial), $|P^{LR}| < |P^{SR}| < |P|$
   - Proof:

$$|P^{SR}| - |P^{LR}| = \int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \int_0^{\mathrm{Y}(\mu)} d\tilde{G}_\pi dG_\mu + \left[\tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{SR}))\bar{G}_\mu(\underline{\mu}^{SR}) - \tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{LR}))\bar{G}_\mu(\underline{\mu}^{LR})\right]$$

Observe that

$$\int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \int_0^{\mathrm{Y}(\mu)} d\tilde{G}_\pi dG_\mu \geq \int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{LR}))dG_\mu$$

$$= \tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{LR}))\left(\bar{G}_\mu(\underline{\mu}^{LR}) - \bar{G}_\mu(\underline{\mu}^{SR})\right)$$

where the inequality follows from $\tilde{G}_\pi$ non-decreasing and $\mathrm{Y}(\mu) \geq \mathrm{Y}(\underline{\mu}^{LR})$ for $\mu \geq \underline{\mu}^{LR}$, and the last equality follows from adding and subtracting $\tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{LR}))$ and using the definition $\bar{G}_\mu(\underline{\mu}^{LR}) = 1 - G_\mu(\underline{\mu}^{LR})$. Using this result, $|P^{SR}| - |P^{LR}| \geq \left[\tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{SR})) - \tilde{G}_\pi(\mathrm{Y}(\underline{\mu}^{LR}))\right]\bar{G}_\mu(\underline{\mu}^{SR}) > 0$ where the last inequality follows from $\underline{\mu}^{SR} > \underline{\mu}^{LR}$. QED

3. public listing propensity decreases relative to short-run and decreases even more in long-run
   - since $|S^{LR}| > |S^{SR}| = |S|$and $|P^{LR}| < |P^{SR}| < |P|$,

$$\frac{|P|}{|P| + |S|} = \frac{1}{1 + \frac{|S|}{|P}} > \frac{1}{1 + \frac{|S|}{|P^{SR}|}} = \frac{1}{1 + \frac{|S^{SR}|}{|P^{SR}|}} > \frac{1}{1 + \frac{|S^{SR}|}{|P^{LR}|}} > \frac{1}{1 + \frac{|S^{LR}|}{|P^{LR}|}}$$

4. $O^{SR} < O^{LR} < O$, output increases relative to short-run but is lower than initial,
   - Proof: of $O^{LR} > O^{SR}$

$$O^{LR}(\underline{\mu}) = \int_0^{\underline{\mu}} \int_0^{\mathrm{Y}(\mu)} \mu dG_\pi dG_\mu + \int_{\underline{\mu}}^{\infty} \int_0^{\infty} \mu dG_\pi dG_\mu = E[\mu] - \int_0^{\underline{\mu}} \mu \bar{\tilde{G}}_\pi(\mathrm{Y}(\mu))dG_\mu$$

Thus, $O^{LR} - O^{SR} = \int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \mu \bar{\tilde{G}}_\pi(\Upsilon(\mu))dG_\mu > 0$.

- Proof: of $O^{LR} < O$

$$O - O^{LR} = \left[\int_0^{\underline{\mu}} \mu G_\pi(\Upsilon(\mu))dG_\mu - \int_0^{\underline{\mu}^{LR}} \mu \tilde{G}_\pi(\Upsilon(\mu))dG_\mu \right] + \int_{\underline{\mu}}^{\underline{\mu}^{LR}} \mu dG_\mu$$

$$= \left[\int_0^{\underline{\mu}} \mu\big(G_\pi(\Upsilon(\mu)) - \tilde{G}_\pi(\Upsilon(\mu))\big)dG_\mu \right] + \left[\int_{\underline{\mu}}^{\underline{\mu}^{LR}} \mu\big(1 - \tilde{G}_\pi(\Upsilon(\mu))\big)dG_\mu \right] > 0$$

where the inequality follows from $\underline{\mu}^{LR} > \underline{\mu}$, $\tilde{G}_\pi \leq 1$ and FOSD of $\tilde{\pi}$. QED

5. PE premium Π (i) falls relative to short-run, and (ii) net long-run affect ambiguous (if Assumption 2 holds)

- Proof of $\Pi^{LR} < \Pi^{SR}$:

$$\Pi^{LR} = E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{LR}] - E[\nu|\nu \leq \bar{\nu}^{LR}]$$

$$= E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{LR}] - \underline{\pi}^{LR} + MAI_\nu(\bar{\nu}^{LR})$$

$$\leq E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - \underline{\pi}^{SR} + MAI_\nu(\bar{\nu}^{LR})$$

$$\leq E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - \underline{\pi}^{SR} + MAI_\nu(\bar{\nu}^{SR}) = \Pi^{SR}$$

where the second equality uses long-run condition (41), the first inequality uses Lemma 2 and $\underline{\pi}^{LR} < \underline{\pi}^{SR}$, and fourth uses Lemma 3 and $\bar{\nu}^{SR} < \bar{\nu}^{LR}$. QED

- Proof of $\Pi^{LR} - \Pi$ ambiguous

By Leibniz rule,

$$\frac{\partial \Pi(\bar{\nu})}{\partial \epsilon} = \frac{\partial E[\pi'(\nu) - \nu|\nu \leq \bar{\nu}]}{\partial \epsilon} = \int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon} \frac{dG_\nu}{G_\nu(\bar{\nu})} + \int_0^{\bar{\nu}} (-1)G'_\nu(\bar{\nu})\frac{\bar{\nu}}{\partial \epsilon}\pi'(\nu)\frac{dG_\nu}{G_\nu(\bar{\nu})^2} + \frac{\bar{\nu}}{\partial \epsilon}[\pi'(\bar{\nu}) - \bar{\nu}]\frac{dG_\nu(\bar{\nu})}{G_\nu(\bar{\nu})}$$

$$= \int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon} \frac{dG_\nu}{G_\nu(\bar{\nu})} + \int_0^{\bar{\nu}} (-1)G'_\nu(\bar{\nu})\frac{\bar{\nu}}{\partial \epsilon}\pi'(\nu)\frac{dG_\nu}{G_\nu(\bar{\nu})^2}$$

$$= \int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon}[1 - \pi'(\nu)\frac{G'_\nu(\bar{\nu})}{G_\nu(\bar{\nu})}]\frac{dG_\nu(\nu)}{G_\nu(\bar{\nu})}$$

since $\pi'(0) \to \infty$ for $\bar{\nu} \to 0$ $\frac{\partial \Pi}{\partial \epsilon} < 0$, while for $\bar{\nu} \to \infty$ $\frac{\partial \Pi}{\partial \epsilon} > 0$ (given MAI_ν decreasing $\iff$ $\frac{G_\nu(\nu)}{G'_\nu(\nu))}$ increasing - see proof of Lemma 1 of Bagnoli and Bergstrom (2005)) QED

6. Public CEO pay has net increase long-run, change relative to short-run ambiguous

- Proof of $\Omega^{LR} > \Omega$ Observe that since $|P^{LR}| < |P|$ and $\underline{\mu}^{LR} > \underline{\mu}$, the same proof used in the short-run applies here.

A.3.4 Proof of Theorem 4 - (ii) PE deregulation

Define the PE de-regulation transformation as $\tilde{\nu} = \frac{\nu}{\epsilon}, \epsilon > 1$ so that each private financier's costs are lower, thereby implying ν FOSD $\tilde{\nu}$ (i.e. $G_{\tilde{\nu}}(x) = G_\nu(\epsilon x) > G_\nu(x)$ for any x).

In the short-run:

As this transformation only affects private financier costs, allocations do not change in the short-run an the only impact is a direct increase in the PE premium Π, $\Pi^{SR} = E[\pi|\pi \geq \underline{\pi}] - E[\tilde{v}|\tilde{v} \leq \bar{v}] > \Pi$.

In the long-run:

1. aggregate PE funds B increase while $\bar{v}^{LR} < \bar{v} < \bar{v}^{LR}\epsilon$

   - Proof $B^{LR} > B$:
   $$B \equiv G_v(\bar{v}) \cdot M = \bar{G}_\pi(\bar{v})$$
   (By contradiction), suppose that $B \geq B^{LR}$ then $B \equiv G_v(\bar{v}) \cdot M \geq B^{LR} \equiv G_{\tilde{v}}(\bar{v}^{LR}) \cdot M = G_v(\epsilon\bar{v}^{LR})M$ (where the last equality uses the definition of the transformation). By monotonicity of G_v it follows that $\bar{v} \geq \epsilon\bar{v}^{LR}$. From (40) and (41)) we have
   $$B^{LR} \equiv G_{\tilde{v}}(\bar{v}^{LR}) \cdot M = \bar{G}_\pi(\bar{v}^{LR})$$
   so $B^{LR} = \bar{G}_\pi(\bar{v}^{LR}) \geq \bar{G}_\pi(\epsilon\bar{v}^{LR})$ given $\bar{G}_\pi$ decreasing Using $\bar{v} \geq \epsilon\bar{v}^{LR}$, $\bar{G}_\pi(\epsilon\bar{v}^{LR}) > \bar{G}_\pi(\bar{v}) = B$ where the last equality follows from (40) and (41)) at $\bar{v}$. Contradiction. Thus, $B^{LR} > B$.QED

   - Proof $\bar{v}^{LR} < \bar{v}$: By FOSD of ν and (40)
   $$\bar{G}_{\pi'}(\pi'(\nu)) = G_\nu(\nu) \cdot M < G_{\tilde{v}}(\nu) \cdot M = \bar{G}_{\pi'}(\tilde{\pi}'(\nu)$$
   thus, by $\bar{G}$ decreasing, $\tilde{\pi}'(\nu) < \pi'(\nu)$. Substituting this into the equilibrium financier cutoff condition (41)) we have immediately $\bar{v}^{LR} < \bar{v}$. QED

2. Public listing mass P and public listing propensity declines

   - Proof: since $B^{LR} > B$, we have immediately from (40) and (41)) and definition of $\underline{\pi}, \underline{\mu}$ that $\underline{\pi}^{LR} < \underline{\pi}$ and $\underline{\mu}^{LR} < \underline{\mu}$.

   Finally,
   $$|P^{LR}| = \int_0^{\underline{\mu}^{LR}} \int_0^{\Upsilon(\mu)} dG_\pi dG_\mu + \int_{\underline{\mu}^{LR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{LR})} dG_\pi dG_\mu$$
   $$< \int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} dG_\pi dG_\mu + \int_{\underline{\mu}^{LR}}^{\infty} \int_0^{\Upsilon(\underline{\mu})} dG_\pi dG_\mu = |P|$$

- As $|S^{LR}| > |S|$ and $|P| > |P^{LR}$ we have immediately the listing propensity drops

3. Output increases

- Proof: since $\underline{\mu} > \underline{\mu}^{LR}$

$$O^{LR} = E[\mu] - \int_0^{\underline{\mu}^{LR}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi dG_\mu > E[\mu] - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi dG_\mu = O$$

4. CEO pay ambiguous

- Consider any degenerate distribution of dG_μ[56]
- if MAI_π is decreasing[57] then $\Omega^{LR} < \Omega$
- Proof:

$$\Omega - \Omega^{LR} = E[\pi|\pi \leq \underline{\pi}] - E[\pi|\pi \leq \underline{\pi}^{LR}]$$
$$= MAI_\pi(\underline{\pi}^{LR}) - MAI_\pi(\underline{\pi}) + \underline{\pi} - \underline{\pi}^{LR} > 0$$

  since $\underline{\pi}^{LR} < \underline{\pi}$ and by assumption on MAI_π decreasing.
- if $\underline{\pi}$ large then $\Omega^{LR} > \Omega$, while if $g_\pi(\underline{\pi}) > 1$ then $\Omega^{LR} < \Omega$
  Proof: by Leibnitz rule, and an integration of parts

$$\frac{\partial E[\pi|\pi \leq \underline{\pi}]}{\partial \underline{\pi}} = \frac{\underline{\pi} G_\pi(\underline{\pi})(g_\pi(\underline{\pi}) - 1) + \int_0^{\underline{\pi}} G_\pi(\pi) d\pi}{G_\pi(\underline{\pi})^2}$$

  if $g_\pi(\underline{\pi}) > 1$ then given $\underline{\pi} > \underline{\pi}^{LR}$ result follows immediately. if $\underline{\pi}$ large then noting $\int_0^{\underline{\pi}} G_\pi(\pi) d\pi < \underline{\pi} G_\pi(\underline{\pi})$ and given that $g_\pi(\underline{\pi}) \to 0$ is necessary for any well defined distribution, the result of $\Omega^{LR} > \Omega$ follows.

5. $\Pi^{LR} < \Pi$, PE premium decreases (if Assumption 2 holds)

- Proof:

$$\Pi = MRL_\pi(\underline{\pi}) + MAI_\nu(\bar{\nu})$$
$$\Pi^{LR} = MRL_\pi(\underline{\pi}^{LR}) + MAI_{\tilde{\nu}}(\bar{\nu}^{LR})$$

  Given our first result that $\epsilon\bar{\nu}^{LR} > \bar{\nu} > \bar{\nu}^{LR}$, and using the transformation, we have $MAI_{\tilde{\nu}}(\bar{\nu}^{LR}) = MAI_\nu(\bar{\nu}^{LR}\epsilon)$, yielding the result

[56] For simplicity, will illustrate with this assumption, which is sufficient to establish the ambiguity
[57] No 'named' distribution is known with this property

A.3.5 Proof of Theorem 4 - (iii) Ideas harder to find

Ideas harder to find is first-order stochastic leftward shift in G_μ, so the new distribution $\tilde{G}_\mu > G_\mu$ (i.e. expected μ is lower). Since $G_{\pi'}(\underline{\pi}) = G_\mu(\Upsilon^{-1}(\underline{\pi})) \cdot G_\pi(\underline{\pi}))$, this maps to the modified info premium distribution being FOSD after the transformation (so π' is lowered).

By equilibrium matching function, the post-shock financier-firm matching is given by

$$\bar{G}_{\tilde{\pi}'}(\tilde{\pi}'(\nu)) = G_\nu(\nu) \cdot M = \bar{G}_\pi(\pi'(\nu))$$

Since $\bar{G}_{\tilde{\pi}]}(\tilde{\pi}'(\nu)) = \bar{G}_\pi(\epsilon\tilde{\pi}'(\nu)) > \bar{G}_\pi(\pi'(\nu))$ we have that $\tilde{\pi}'(\nu) < \pi'(\nu)$. That is, each private financier is matched with a lower info premium firm than prior to the shock. Evaluating at the marginal (potential) private financier profit, we have $\tilde{\pi}'(\bar{\nu}) < \bar{\nu}$ and hence, since $\tilde{\pi}'$ a decreasing function in ν, we have (by free-exit) that for any ν such that $\tilde{\pi}'(\nu) < \nu$, this financier will switch out from being a private financier. Let $\bar{\nu}^{SR}$ denote the maximal $\nu < \bar{\nu}$ such that the zero profit condition holds with equality, $\tilde{\pi}'(\bar{\nu}^{SR}) = \bar{\nu}^{SR}$, which given above, we know exists. But this satisfies the long-run GE equilibrium definition, so we have immediately that the short-run equals the long-run in this counterfactual. In light of this, $\underline{\pi}^{LR} < \underline{\pi}$, $\underline{\mu}^{LR} < \underline{\mu}$ and $B > B^{LR}$.

In the short-run = long-run,

1. PE funds fall, $B > B^{LR}$

   - Follows directly from above

2. Output falls, $O > O^{LR}$

   - Proof: $|P \cup S| = 1 - \int_0^{\underline{\pi}} \int_0^{\Upsilon^{-1}(\pi)} dG_\mu dG_\pi$ and

   $$|P \cup S| - |P \cup S|^{SR} = -\int_{\underline{\pi}^{LR}}^{\underline{\pi}} [\bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi)) - \bar{G}_\mu(\Upsilon^{-1}(\pi))] dG_\pi > 0$$

   where the inequality follows from $\bar{G}_\mu > G_{\tilde{\mu}}$. Thus, the mass of financed firms falls.

   Observe that by definition aggregate output is $O = \int_{\theta:\theta \in P \cup S} \mu dG_\mu dG_\pi$. Since $P \cup S \subseteq (P^{SR} \cup S^{SR})$ we have

   $$O = \int_{\theta:\theta \in P \cup S \setminus (P^{SR} \cup S^{SR})} \mu dG_\mu dG_\pi + \int_{\theta:\theta \in P^{SR} \cup S^{SR}} \mu dG_\mu dG_\pi > O^{SR}.$$

3. Public financed firms and public listing propensity ambiguous

- Proof:

$$|P| - |P^{LR}| = |P \cup S| - |P \cup S|^{SR}| - (|S| - |S^{LR}|) > 0$$

$$\iff \frac{\int_{\underline{\pi}^{LR}}^{\underline{\pi}} [\bar{G}_{\mu}(\Upsilon^{-1}(\pi)) - \bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi))] dG_{\pi}}{B - B^{LR}} > 1$$

where $B - B^{LR} = M \cdot (G_{\nu}(\bar{v}) - G_{\nu}(\bar{v}^{LR}))$. That is, public listings can increase if the substitution away from private towards public dominates the expansion of un-financed firms, this depends on the relative curvature of G_{ν} vs $\bar{G}_{\mu}$ around the cutoffs. Since $|S|$ falls and $|P|$ may either (a) increase or (b) fall, the listing propensity will rise in the case of (a) and could rise or fall with (b) depending on the magnitudes.

4. Average public CEO pay ambiguous

- Proof: Suppose $|P| < |P|^{LR}$, then

$$\Omega - \Omega^{LR} = \frac{1}{|P|} E[\pi|\theta \in P] - \frac{1}{|P^{LR}|} E[\pi|\theta \in P^{LR}]$$

$$\geq \frac{1}{|P^{LR}|} \left(\int_{\underline{\pi}^{LR}}^{\underline{\pi}} \pi [\bar{G}_{\mu}(\Upsilon^{-1}(\pi)) - \bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi))] dG_{\pi} \right) > 0$$

where the inequality follows from $\Upsilon(\cdot)$ monotonic increasing, $\bar{G}_{\mu} > \bar{G}_{\tilde{\mu}}$. On the other hand, if $|P| > |P^{LR}|$ then sign is ambigous

5. PE premium ambiguous

- Proof: Note that, under Assumption 2, MRL_{π} decreases ($\underline{\pi} > \underline{\pi}^{LR}$ and MAI increases since ($\bar{v} > \bar{v}^{LR}$).

A.3.6 Proof of Theorem 4 - (iv) Costly (unproductive) disclosure

We model an increase in unproductive disclosure as a fixed cost $\zeta > 0$ to being publicly listed, so $\tilde{J}^P = J^P - \zeta$. This results in a downward shift in the IR region for public financiers, $\{\Upsilon(\mu) \geq \pi + \zeta\}$ and an upward shift in the premium earned by private financier competing with the public, $\{\pi + \zeta : \Upsilon(\mu) \geq \pi + \zeta\}$. That is, the modified info premium indifference curve for the private financier is now $\tilde{\pi}' = \min\{\Upsilon(\mu) - \zeta, \pi\}$ (i.e. rightward shift in the indifference curves for private). In the short-run, private financier funding remains fixed (no free-entry in short-run binds), so it must be $\underline{\mu}^{SR} > \underline{\mu}$ while $\underline{\pi}^{SR} < \underline{\pi}$,[58]

Short-run

1. Set of financed firms, and aggregate output falls

   - Proof: By definition $|P \cup S|^{SR} = 1 - \int_0^{\underline{\mu}^{SR}} \int_{\Upsilon(\mu)-\zeta}^{\infty} dG_\pi dG_\mu$, thus,

   $$|P \cup S| - |P \cup S|^{SR} \geq \int_0^{\underline{\mu}} [\bar{G}_\pi(\Upsilon(\mu - \zeta)) - \bar{G}_\pi(\Upsilon(\mu))] dG_\mu > 0$$

   where the first inequality follows from $\underline{\mu}^{SR} > \underline{\mu}$ and the second from $\bar{G}$ decreasing and Υ increasing. By similar logic, $O = E[\mu] - \int_0^{\underline{\mu}} \mu \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu$ so

   $$O - O^{SR} \geq \int_0^{\underline{\mu}^{SR}} \mu [\bar{G}_\pi(\Upsilon(\mu - \zeta)) - \bar{G}_\pi(\Upsilon(\mu))] dG_\mu > 0.$$

2. Publicly listed firms falls, and listing propensity falls

   - Proof: $|P| = |P \cup S| - |S|$, and $|S|$ fixed in short-run, hence by above result follows.
   - Since $|S|$ fixed and $|P|$ falls in short-run, listing propensity falls

3. Average public CEO pay ambiguous

   - Proof: Similar arguments as ideas' harder to find

4. Average PE premium rises

   - Proof: The mass of privately financed firms doesn't change (and set of private financiers), and hence, the sum of PE premiums is sufficient to characterize the difference. Consider the initial matching, $m(\nu) = \{\theta : \pi'(\theta) = \min\{\Upsilon(\mu), \pi\}\}, \pi'(\nu)$. After the reform, their payoffs matched to the same firm is $\pi'_{SR}(\nu) - \nu = \min\{\Upsilon(\mu), \pi + \zeta\} - \nu > \min\{\Upsilon(\mu), \pi\} - \nu$. Since these matchings are feasible, the optimal matching generated in equilibrium (which given the submodularity is efficient) it must be that this total sum exceeds this. QED

[58] Suppose not, if $\underline{\mu}^{SR} < \underline{\mu}$, then the implied private financier set strictly contains the original, hence $|S^{SR}| > |S|$, contradiction. By definition of $\underline{\pi}^{SR}$, $\underline{\pi}^{SR} = \Upsilon(\underline{\mu}^{SR}) - \zeta$, so $\underline{\pi} - \underline{\pi}^{SR} = \Upsilon(\underline{\mu}) - (\Upsilon(\underline{\mu}^{SR}) - \zeta) = \zeta + (\Upsilon(\underline{\mu}) - \Upsilon(\underline{\mu}^{SR})) > 0$, where the last inequality follows from $\underline{\mu} > \underline{\mu}^{SR}$.

Long-run

In the long-run, PE funds expand so that (41) holds with equality. Thus, using analogous arguments as for previous comparative statics, $B^{LR} > B$, $\bar{v}^{LR} > \bar{v}$, $\underline{\mu} < \underline{\mu}^{LR} < \underline{\mu}^{SR}$ and $\underline{\pi} > \underline{\pi}^{LR} > \underline{\pi}^{SR}$. Using similar arguments as above, we have the following results:

1. Public listings fall relative to short-run, but listing propensity change ambiguous
2. Output rises relative to short-run, but still below initial
3. Public CEO pay and average PE premium ambiguous

Proof of Theorem 4 - (v) Productive disclosure

This is equivalent to a reduction in firm intangibility / information premia, use intangibility increase arguments but flip predictions for a decrease.

Appendix B Appendix - Data Details

We download the Capital IQ accounting and financial data about US firms from 1993 to 2016. We do so applying three filters: firms must be US incorporated, they must have and they must not be financial firms (SIC codes from 6000 to 6999), utilities (SIC codes 4900 to 4999), and quasi-governamental (SIC codes from 9000). We drop firms with missing central index key (CIK) for merging reasons and with missing SIC code. To find each Compustat identifier, GVKEY, associated with each Capital IQ firm in our sample we use the link tables provided on the Wharton Research Data Services (WRDS) website. To obtain the listing status of each firm in our sample we use the Compustat Snapshot data since this has historical rather than the most recent values of the listing status of each firm. Furthermore, we assume that if a firm is not present in Compustat Snapshot, it is not publicly listed. To compute the age of the companies in our sample, we obtain information about their foundation years from Capital IQ. We augment this data with the one provided by Jay Ritter on his website. We consider a firm's foundation year as the one provided by Jay Ritter if available, otherwise we consider the one provided by Capital IQ. If none of the two data sources has this information, we consider the first year in which a firm appears in our sample as the end of the first year of life of such a corporation. Following Jay Ritter's convention, we cap the age of a firm at 80. We obtain information about LBOs and IPOs from Capital IQ. For the first type of corporate event, we categorize a given firm-year observation as undergoing an LBO if either Capital IQ or Compustat Snapshot reports this type of episode and the listing status supports this information (otherwise, we consider the LBO as attempted but not completed). For the second type of corporate event, we categorize given firm-year observation as undergoing an IPO if either Capital IQ or Jay Ritter's data (see Loughran and Ritter (2004) and Field and Karpoff (2002) for more information) reports this type of episode.

We analyze the Capital IQ text containing information about the IPO to be sure that was actually completed and not just initiated. Even in this case, we check that the listing status confirms the success of the operation. To compute the stock of intangible capital we follow Peters and Taylor (2017). We adapt their methodology to our case by assuming that firms start to accumulate intangible capital since their inception and that the IPO does not affect the way in which the intangible expenses contribute to a firm stock of intangible capital. We use the depreciation rate of the knowledge capital (the one that stems from R&D expenditures) estimated by Li & Hall (2020) and we set those which are not covered by them to 15% as standard in the literature. Similarly we set the depreciation of the organization capital (the one that stems from SG&A expenditures) to 20% following Falato et al. (2020).

We download the Capital IQ CEO data about US firms from 2001 to 2016. We restrict out attention to the set of firms for which we have also accounting and financial data. Given that Capital IQ has header information about which executive was the CEO of a given firm at a given point in time, we use Execucomp as well as Capital IQ data about corporate to help identify the CEO from the set of executives who are linked to a given firm in a given year following Gao et al. (2017). Where ambiguity remains in the identify of the CEO for a given firm-year (occurred in 6.5% percent of observation) we take the highest paid executive in terms of total compensation and we exclude from our consideration all those executives whose salary is either missing or non positive.

We interpolate EBITDA, PPEGT and CAPEX using non-missing neighbour values. Given the structure of Capital IQ data, we consider 0 EBITDA and CAPEX as being missing and non-positive PPEGT as being missing. We drop observations with non positive assets and total revenues. The tangible assets of a firm are given by the different between its total assets and the intangible assets on its balance sheet, that is the sum of goodwill and the accounting item other intangibles on the balance sheet. The intangible assets of a firm are given by the sum of knowledge capital, organizational capital, goodwill and other intangibles on the balance sheet. Our proxy of τ is computed as the ratio of tangible assets and the sum of tangible and intangible assets. The total cash-flows measure, y_t is given by the ratio of EBITDA and the denominator of τ. In the reduced form analysis, the proxy of σ_z^2 which we use is the 3 year firm level variance of the total cash flows.

The total CEO compensation and individual parts (meaning, the salary, the fixed bonuses, other incentives, stocks and options) are built following the Execucomp manual using the Capital IQ data. The compensation shares so to have consistency between the cross-sectional and time series analysis. This means that the fixed share of CEO compensation is computed as the sum of salary and fixed bonuses divided by total compensation, the incentive shares as the sum of long-term incentive plan and non equity incentive payments and stocks and options divided by total compensation, and the equity share as the sum of stocks and options divided by total compensation.

We drop from our analysis observations of firms that underwent an IPO or an LBO or which were listed on a minor stock exchange in a given year, or with a ROA (computed as the ratio of EBITDA over total book assets) less than -100% (since these firms would never be able to go public conditional on being private and they might be forced to delist in the other case).

We yearly winsorize at 1% all the ratio variables while all of the are in 2016 US dollars.

To build the data for the reduced form analysis using historical, we clean the data in a similar way as before and we build the variables in a consistent way with the cross-sectional analysis and across the different periods of time (since, as explained above, up to 1991 we use Frydman and Saks (2010) data while after 1992 we use Execucomp data).

Finally, to build the data for the structural estimation analysis, we use the panel built for the cross-sectional reduced form analysis, and we consider a firm as being public if it has appeared more often as such in our sample and vice versa if has appeared less (dropping firm with an equal number of appearances).