Patent wars or copycat threats? Protecting innovation & market power with patent litigation

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Abstract

Patents and intellectual property protection (IPP) activities vary systematically across firms and are a source of market power. I develop a tractable Schumpeterian growth model with firm dynamics where product entry barriers are shaped by firms' IPP investments and strategic litigation activity. Due to IPP investments both protecting the quasi-monopoly rents of their incumbent products and helping defend against infringement suits when entering rival markets, market power and IPP interact and feedback into firm innovation incentives and dynamically generate so-called 'superstar firms.' Although these high market power firms are more innovative, their net effect on innovation may be negative due to the higher entry barriers for new, non-superstar innovators. Furthermore, persistent asymmetries in IPP investment efficiencies, like retaining a high quality in-house IP legal team integrated with the R&D activity of the firm, can result in predatory behavior of stronger IPP incumbents 'litigating to kill' weaker rivals innovations. Thus, whether IP protection activities promote or stifle innovation is a quantitative question and depends on the distribution of innovative and IPP investment capacity. Leveraging a new firm-matched dataset linking firm patent portfolios, legal counsel and IP litigation data to US firm balance sheets, I empirically assess and quantify the net effects of court IP enforcement on firm competition and aggregate growth. I find that strengthening court IP enforcement boosts growth with a near one to one trade-off ins firm concentration, while improving the expertise of court adjudication by eliminating the ability of asymmetric IPP investments to influence outcomes both reduces average firm concentration by 2.4% and raises aggregate growth 3.1 percentage points.

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1 Introduction

Creative destruction, the revolutionary process by which new innovative products supplant the products of stagnating incumbents, is a primary determinant of long-run economic growth. Such disruptive innovations are costly to discover, but, without intellectual property protections, often easy for others to copy. Thus, patents which give the owner a monopoly, that is, the legal right to block any product which directly infringes on their intellectual property, may be essential for growth.

Enforcement of intellectual property ownership however is neither automatic or clearcut, with the lines distinguishing two ideas blurry and often requiring subject matter expertise to fully apprehend. Adjudication is done by judges and juries, who have little domain specific knowledge on the ideas in question. Consequently, the outcomes of patent litigation suits are uncertain, with more sophisticated legal strategies and better substantiated arguments able to sway court opinions away from what technical experts would agree upon.[1] As a result, any persistent asymmetries in IPP capital can distort entry barriers across incumbent firms and entrants. In this paper I seek to quantify these distortions and answer the question: what are the effects of changing court IP enforcement on competition and aggregate innovation?

To answer this question, I build and estimate a (Schumpeterian) endogenous growth model with IPP investments and strategic IP litigation. I empirically assess the theory and estimate the model on a new firm-matched dataset of (dynamic) patent portfolios, IP lawyers, federal IP litigation suits and firm balance-sheets. I find that strengthening court IP enforcement boosts growth with a near one to one tradeoff in firm concentration, while improving the expertise of court adjudication by eliminating the ability of asymmetric IPP investments to influence outcomes both reduces average firm concentration by 2.4% and raises aggregate growth 3.1 percentage points.

My theory builds off the canonical, firm-based Schumpeterian growth model of Klette & Kortum (2004), where firms grow by innovating upon and stealing rivals' product monopolies. In contrast to the standard model where every innovation translates to the obsolescence of the incumbent firm's product, here I allow incumbents' to litigate the entry of the new product for infringement to either (a) obtain a stochastic court ruling which may preclude the entry of the new innovation or (b) bargain royalties with the innovator to receive compensation for their IP embedded in the new innovation. I assume that court rulings are a function of the relative IPP investments of the incumbent and new innovator, so that asymmetries in IPP investments across the two influence the probability that the incumbent will win a product ban of the innovator.

Reflecting the common occurrence of litigation suits and countersuits in inter-firm patent disputes, IPP investments serve two purposes for the firm. First, they complement firm's R&D by protecting quasi-monopoly rents of past innovations from rival innovators and copycats. Second, they help differentiate and defend their new

[1] Amidst a large surge of patent litigation in the 19th century, an opinion article by a prominent electrical engineer in a leading technical periodical stated, "There are not many leading counsel who add to their professional ability the power of mastering thoroughly the scientific facts upon which they are "coached" for the occasion – there are very few who have the technical knowledge necessary to deal effectively with unexpected issues which may arise – in a complicated patent case..It has now been alleged (both in the Times and in many technical journals) that in such a suit the more wealthy of the two parties (the one may be a company with a capital of a million, the other a struggling mechanic or instrument maker) is generally found to have retained by means of a general retainer all the leading counsel who have distinguished themselves in electrical patent law, and that they, with the aid of the most experienced scientific experts available, succeed in misleading the Judges by causing them to misunderstand patent specifications and consequently give unjust judgments." The Electrician July 23, 1886, George Forbes

innovations from infringement claims brought against them by incumbent litigation. The fact that IPP investments both help defend existing products and the successful entry into new product spaces introduces a feedback effect between firm's profits (i.e. market power) in existing product spaces to their innovative capacity. This interplay dynamically results in high market power, high IPP-intensive and high growth firms. Although these high market power firms are more innovative, their net effect on innovation may be negative due to the higher entry barriers for new, pre-revenue innovators. Furthermore, persistent asymmetries in IPP investment efficiencies, like retaining a high quality in-house IP legal team integrated with the R&D activity of the firm, can result in predatory behavior of stronger IPP incumbents 'litigating to kill' weaker rivals' innovations (akin to the killer acquisitions examined in Cunningham et al. (2020)). Finally, besides adding an IPP investment choice, to capture the value of IP enforcement and offer a tradeoff between monopoly distortions and innovation incentives, I introduce two types of costly R&D, frontier and copying, with the former more expensive than the latter.

Due to the tradeoffs between (1) monopoly distortion and innovation incentives and (2) superstars higher innovative capacity vs predatory, entry inhibition behaviour, the aggregate effects of court IP enforcement on competition, aggregate growth and consumer surplus depends on the distribution of monopolies, incumbent firm types and potential entrant masses. As such, micro-measures of firm IPP investments and their realized effects on firm dynamics are crucial to credibly inform the model's predictions on policy reforms. I construct such a dataset linking (using fuzzy string matching) Compustat firm balance sheet data with (1) patent litigation data from the Federal Judicial Center, supplemented with the (2019) USPTO patent litigation docket dataset, (2) patent ownership data for Compustat firms from USPTO (patent) Assignment Dataset, as well as data on registered licensing agreements. The constructed patent ownership dataset provides a contribution relative to existing alternatives (including the NBER patent dataset of 2006 which only tracks ownership at the time of patent grant) by updating the parent subsidiary relationships using Capital IQ corporate tree and M&A data, thereby giving more accurate representations of patent ownership. With these enhancements, my coverage of patent ownership increases to $> 40\%$ of US granted utility patents, compared to the NBER patent datasets 32% coverage.[2] Finally, to identify differences in quality IP lawyer capital, I use executive compensation data on chief legal officers (using a span of control argument) and data on the lawyer / legal department filing patents on behalf of each firm over time (where firm's use of in-house lawyers for patent filings suggest paying of a fixed cost to retain the employee, thereby reducing the marginal cost of patenting activity). Altogether this gives me an expansive firm-linked patent and IP litigation datasets.

I measure and empirically assess the ability of IPP investments to influence firm market power and innovation dynamics for large incumbent firms found in Compustat. Using a variance decomposition of patent grant values based on identified lawyer characteristics who initially filed (and presumably constructed) the patent, I find that the lawyer input contributes about 5% to patent grant value variation and that having well-integrated in-house lawyers constructing the IP of the firm is associated with 48% higher patent grant values. I then give evidence consistent with asymmetric IPP investments and high quality in-house IP legal teams able to influence court IP rulings in their favour using matched IPP for both plaintiffs and defendants. Third, I leverage the identity of judges in federal litigation cases to instrument winning a patent litigation with judge 'biases' and document it's effect on firm markups and R&D intensity. Finally,

[2]This increase in coverage is not due to differences in fuzzy string matching algorithms, as my algorithms are found to be more conservative in identifying patents on the NBER patent dataset pre-2001. Hence coverage is increasing due to expanding the set of subsidiaries owned by large public firms.

studying the differential dynamics of plaintiffs / defendants following litigation events against un-litigated counterparts, I find that patent litigation defendants' markups increase relative to non-litigated defendants in the year following a patent litigation suit.

Finally, I combine the model and the documented empirical associations of IPP investments, firm innovation and market power in a structural estimation of a cross-section of firms tracked primarily between 2003 and 2007. Identification of the model is based variation in innovation incentives, survival probabilities and profitability across different intensities of IPP investments, as well as firm dynamics around litigation events and variations in court judgements. The estimated model suggests a high threat of copying, with high entry barriers for frontier expanding innovations and substantial sensitivity of court IP enforcement to IPP investment asymmetries. It also suggests that substantial asymmetries exist between incumbent firms with in-house IP departments and those who hire IP lawyers externally and on an ad hoc basis. These asymmetries have substantial bite as I find that roughly as many frontier expanding innovations are blocked by these high market power, in-house IP firms as allowed entry.

Equipped with this estimated model, I then examine the quantitative impact of various, in principle, actionable policy interventions. In particular, I examine (1) an elimination of court susceptibility to influence by asymmetric IPP investments (through hiring independent, technical experts as adjudicators of infringement), (2) a 10% unconditional increase in court IP enforcement, (3) a ban on in-house IP legal departments, (4) a total elimination of patent protections, and (5) perfect patent enforcement (i.e. blocking of copycats, no blocking of frontier innovators).

The organization of the rest of the paper is as follows. In the remainder of this section, I discuss how this paper contributes to the existing literature. In section 2, I provide a simplified, illustrative version of the model to provide intuition on the mechanisms and guide the empirical analysis. In section 3, I describe the data and empirically assess the observable links between firm IPP investments, patent litigation and firm dynamics. In Section 4, I present the full quantitative model used in the estimation. In Section 5, I present some theoretical implications of the model and in Section 6, I present the identification, structural estimation and quantitative impact of various counterfactual policy reforms. In Section 7, I conclude.

1.1 Related literature

This paper contributes and complements a number of distinct different strands of literature. First, this paper seeks to contribute to the academic and policy debate over recent aggregate trends in intangibles, market power, productivity, flagging investment and the labour share documented by, amongst others, De Loecker et al. (2020), Bloom et al. (2020), Crouzet and Eberly (2018), and Akcigit and Ates (2021). On the one-side, Autor et al. (2020) and De Ridder (2019) argue in support of a superstar firm hypothesis that rising productivity differences between the top performers and the rest in an industry have led to an efficient re-allocation of sales. On the other side, Gutiérrez and Philippon (2017), Gutiérrez and Philippon (2019), Grullon et al. (2018), Cunningham et al. (2020) and Kamepalli et al. (2020) examine and document anti-competitive / anti-trust behaviour with very different implications for policy. My paper tries to be agnostic and speak to both sides of the debate with IPP investments and litigation endogenously creating superstars and anti-competitive behaviour. It also helps rationalize (1) the puzzling trends in the elasticities of investment to Tobin's Q examined by Crouzet and Eberly (2018), (2) the documented expansion of scope by Hoberg and Phillips (2021) (through increased copycat activity of well-IP protected superstar firms), and

(3) the contrasting trends of increasing aggregated concentration and declining local concentration documented by Rossi-Hansberg et al. (2021).

Second, this paper contributes to a recent, growing literature empirically examining the role of patent wars and litigation on firm competition and innovation. On the theory side, is work by Bessen and Meurer (2006), Farrell and Shapiro (2007), Choi and Gerlach (2017) and Lemus and Temnyalov (2017) which examines on the micro-level how inadvertent infringement and litigation dispute activity between pairs of rivals affects innovation incentives. Choi and Gerlach (2017) study how the size of firm patent portfolios can affect firm R&D incentives, while and Lemus and Temnyalov (2017) study how non-practicing entities have a comparative advantage in extracting royalties from firms due to the negligible risk of countersuits jeopardizing the sale of their own products.[3] My paper builds and complements these works by considering the dynamic and general equilibrium implications of patent litigation disputes.

The empirical end of the economic impacts of patents is expansive (see for instance surveys by Boldrin and Levine (2013) and Sampat (2018)), however, empirical studies on patent litigation has been relatively limited until recently due to the lack of accessible data. A few recent and notable exceptions include Galasso and Schankerman (2018), Mezzanotti (2021) who use assignment of judges in patent appeals and a supreme court ruling to estimate the effect of litigation on incumbent firm R&D activity. Akin to Galasso and Schankerman (2018), I utilize judge assignment in federal IP litigation suits to instrument firms' winning an infringement suit. However, the focus of my paper is not on average treatment effects on incumbent firms, but to infer the effects of IP enforcement on both incumbent and latent entrant firm R&D incentives. Relatedly, Lee et al. (2019a) empirically document differences in firm dynamics between firms involved in a patent litigation suit as a plaintiff and defendant using a hand-collected sample of S&P 500 inter-firm litigation suits. My paper complements theirs by examining a wider pool of litigants and studying the conditional dynamics involving asymmetric patent portfolios and legal departments. Finally, Cohen et al. (2019) offer evidence that patent trolls more frequently target firms with smaller legal departments, which supports my theory of in-house IP firms as being stronger opponents in litigation suits. To my knowledge, the only structural and general equilibrium examinations of patent litigation is work by Abrams et al. (2019) who examine the equilibrium role of non-practicing entities (patent trolls) in facilitating or inhibiting innovation.

Finally, this paper builds on and contributes to the applied growth, creative destruction literature following the seminal works of Aghion and Howitt (1992) and Grossman and Helpman (1991) and later quantitative assessments such as Klette & Kortum (2004), Lentz & Mortensen (2008) and Acemoglu et al. (2018). Recent applications of this class of models have been used to examine the interplay of market power and growth dynamics. For instance, Akcigit and Ates (2021) decompose recent innovation dynamics and in their framework find that declining knowledge diffusion across firms can best rationalize observed patterns consistent with the rise of IP litigation examined in my paper. A second closely related paper is De Ridder (2019) who examines the implications of a different aspect of intangible capital, IT, on firm dynamics, market power and growth. In his model, IT / intangibles offer a cost shifting from marginal to fixed costs, giving those with high fixed costs an ability to undercut less IT-intensive innovators, thereby increasing entry barriers. Besides measuring different important aspects of intangible capital and having substantially different implications for policy, our mechanisms differ in that IPP investments are dynamic forward looking choices with feedback between market power and innovation as opposed to the static intangi-

[3] This thoery, combined with the majority of patents asserted by NPEs originating from operating firms, suggests that the NPEs can be interpreted as outsourced patent litigation activity by the operating firms.

bles fixed cost choice. In another example, Olmstead-Rumsey (2019) decomposes the role of heterogeneous innovations across incumbents.

Numerous other works in the area examine patent ownership, market power and innovation including work by Akcigit and Kerr (2018), Abrams et al. (2019), and Argente et al. (2020). Akcigit and Kerr (2018) document substantial heterogeneity in the types and quality of innovation by small, young firms relative to large incumbents and study the distributional impacts of size-biased heterogeneous innovation quality, but doesn't explicitly have a role for IP protection in the model. Abrams et al. (2019) study the middleman role of non-practicing entities (NPE) providing IP protection services to small inventors who don't have the scale / IP protection capabilities to enforce their patent rights on larger firms. Here the terms of settlement / royalties in patent litigation between NPEs (so-called 'patent trolls') and incumbent firms are exogenously biased in the favour of larger firms, as opposed to generated endogenously through. This exogeneity restricts the type of counterfactuals which can be examined by patent reforms since dispersion in IP protection investments will not be invariant to such policies. Argente et al. (2020) study the distinction between firm patent filings and product creation in the consumer goods market and finds that over half of new product creation comes from non-patenting firms, and increases to 65% on a quality adjusted basis.

Finally, my paper also contributes to the endogenous growth literature by examining dynamic efficiency tradeoffs of firm investments into expanding market power and obtain a cross-sectional distribution of firm markups generated by firm dynamics. Peters (2020) studies the impacts of firm investment in expanding market power which increases the monopoly distortion through higher markups over marginal costs, whereas in my setting, investments in IP protection inhibit the entry of new products (thereby stifling knowledge spillovers), but reduces the business stealing externality arising from creative destruction through surplus splitting royalties. In addition to examining the effects of investment on different externalities, my paper provides a technical contribution of obtaining the firm-size, markup distribution using advances in applied math on Quasi-Birth Death Processes (for details, see Bean and Latouche (2010), Baumann and Sandmann (2010), Kharoufeh (2011) and Cordeiro et al. (2019)). This technical contribution allows for the study of non-trivial joint dependence between market power and firm-size distributions, potentially yielding a fatter firm-size distribution and better capturing the higher intensity of innovative activities, and declining innovation.

2 Shaping entry barriers with court IP enforcement

In this section, I provide a simple framework for how court IP enforcement can distort innovation incentives across incumbents and potential entrants and interact with firm market power. This framework is a boiled down version of the full model to give intuition.

Time is continuous and runs forever. There is a unit measure of horizontally differentiated products operated by distinct incumbent firms and a fixed pool of potential entrants of mass K_E. Each incumbent has a quality leading product and earns flow profits π. Incumbents and potential entrants can invest $c_\iota(\iota)$ in R&D for rate of discovery of ι. Upon discovery of a quality improvement on an incumbent product, the incumbent of that product litigates the innovator for infringement and wins a court product ban on the new innovator with probability $1 - a(\cdot)$. If the innovator wins, they replace the incumbent as the quality leader in that product and earn π in this additional market.

For simplicity in this section, assume that this new product is sold / spun-off into a separate operating firm so that each incumbent is only operating a single product line. Suppose the probability that the court will allow entry of the new innovator is given by $a(\ell - \hat{\ell}) = a_0 + a_1(\ell - \hat{\ell}) + \varepsilon$ where ℓ is the IPP investments of the innovator and $\hat{\ell}$ is the IPP investments of the incumbent, where $a_0, a_1 > 0$ and $\mathbb{E}[\epsilon] = 0.$[4], Thus, if the innovator invests relatively more in IPP they are able to increase upwards the probability of their innovation fending off an infringement suit and winning the product market. Let $c_\ell(\ell)$ denote the convex costs of IPP investment. Finally, let δ denote the rate at which some rival firm innovates on a given incumbent, which in equilibrium will equal the aggregate rate of innovation across incumbents and entrants, so if $\eta = \iota_E K_E$, then $\delta = \iota \cdot 1 + \eta$.

Observe that with this court ruling form, a_0 is the unconditional probability of court deeming patent infringed (given symmetric firms) and giving the plaintiff the right to preclude entry of the innovator. Consequently, $-a_0$ can be interpreted as a strengthening of the validity and court enforcement of incumbent IP.

The investment problem facing an incumbent firm f at a given instant is:

$$r\Pi = \max_{\tilde{\iota},\tilde{\ell}} \pi - c_\iota(\tilde{\iota}) - c_\ell(\tilde{\ell}) + \tilde{\iota}\mathbb{E}[a(\tilde{\ell} - \hat{\ell})]\Pi' - \delta\mathbb{E}[a(\hat{\ell} - \tilde{\ell})]\Pi$$

where Π is the present-value of the incumbent firm's product and Π' is the value of a new product they acquire.

The optimal incumbent firm investment decisions are then given by

$$\Gamma(\ell)\Pi' = c'_\iota(\iota) \tag{1}$$

$$[\iota\Gamma'(\ell) - \delta\Lambda'(\ell)]\Pi = c'_\ell(\ell) \tag{2}$$

where $\Gamma(\ell) = \mathbb{E}[a(\ell - \hat{\ell})]$ and $\Lambda(\ell) = \mathbb{E}[a(\hat{\ell} - \ell)]$.

The investment problem of a potential entrant is the same as an incumbent with the exception that they don't have any incumbent profits to protect π, thus, their optimal decisions are given by

$$\Gamma_E(\ell_E)\Pi' = c'_\iota(\iota_E) \tag{3}$$

$$[\iota_E\Gamma'_E(\ell_E) + 0]\Pi = c'_\ell(\ell_E) \tag{4}$$

where $\Gamma_E(\ell_E) = \mathbb{E}[a(\ell_E - \hat{\ell})]$ is the expected probability of winning a litigation suit as a defendant upon innovating on a random rival with IPP investments $\hat{\ell}$.

Given the court ruling sensitivity is linear in ℓ, we have immediately that $\Gamma'(\ell) = \Gamma'_E(\ell) = -\Lambda'(\ell) = a_1$. In addition, as all quality leading products yield the same profits, $\Pi' = \Pi$, which then re-arranging the firm's value function yields

$$\Pi = \frac{\pi - c_\iota(\iota) - c_\ell(\ell)}{r + \delta\Lambda(\ell) - \iota\Gamma(\ell)}$$

provided $r + \delta\Lambda(\ell) - \iota\Gamma(\ell) > 0$.

Further, as costs are homogenous, and using the assumed functional form of the ruling function, we get that the expected successful entry of a new innovation for existing incumbents is $\Gamma(\ell) = a_0$ (since the innovator and incumbent both invest the

[4]Note, this definition of a() holds for any $\ell, \hat{\ell}, \varepsilon$ such that $a() \in [0, 1]$

same amount in IPP ℓ). On the flip-side, as an incumbent being innovated (realization of a δ shock), the counterparty may be either another incumbent or entrant, with probability $\frac{\eta}{\delta}$ that it is the latter. Thus, the probability of successfully litigating / blocking a new innovator through court is $\Lambda(\ell) = a_0 + \frac{\eta}{\delta} a_1(\ell_E - \ell)$.

In contrast, for entrants, the successful entry probability is $\Gamma_E(\ell_E) = \mathbb{E}[a(\ell_E - \hat{\ell})] = a_0 + a_1(\ell_E - \ell)$ which may be above or below the incumbents probability depending on their relative levels of IPP investment. Because potential entrants earn the same product value Π upon successful innovation, but currently lack any existing products of their own their level of IPP investments, it turns out that their IPP investments ℓ_E will be less than an incumbent who has value in IPP both in defending their own and litigating rivals innovation. Thus, $\Gamma_E(\ell_E) < \Gamma(\ell)$. Putting all of this together, yields the following lemma.

Lemma 2.1. *If $a_0, a_1 > 0$, $a_0 - a_1(\ell - \ell_E) \in (0,1)$, $r + (\delta - \iota)a_0 + a_1(\ell - \ell_E) > 0$, the optimal investment policies of an incumbent firm and potential entrant are as follows.*

Incumbent optimal investment policies:

$$a_0 \cdot \Pi = c_\iota'(\iota) \tag{5}$$

$$a_1[\iota + \delta] \cdot \Pi = c_\ell'(\ell) \tag{6}$$

Potential entrant optimal investment policies:

$$[a_0 - a_1(\ell - \ell_E)] \cdot \Pi = c_\iota'(\iota_E) \tag{7}$$

$$a_1 \cdot \iota_E \cdot \Pi = c_\ell'(\ell_E). \tag{8}$$

where

$$\Pi = \frac{\pi - c_\iota(\iota) - c_\ell(\ell)}{r + \delta\Lambda(\ell) - \iota\Gamma(\ell)}.$$

From these optimal investment policies, we can see that innovation incentives for incumbents and pre-revenue startups are distorted relative to standard innovation models. Further, intuitively since startups earn the same product value Π upon successful innovation, but currently lack any existing products of their own their level of IPP investments, their IPP investments ℓ_E will be less than an incumbent who has value in IPP both in defending their own and litigating rivals innovation. In other words, court IP enforcement which is sensitive to asymmetric IP investments implies entry barriers are endogenously higher for entrants than incumbents. This implies that patent rights and court IP enforcement will have asymmetric impacts on the innovation incentives of incumbents and startups. In particular, an increase in court IP enforcement (a decrease in a_0) will unambiguously increase entrant innovation but may increase or decrease incumbent R&D investment depending on the size of the court enforcement asymmetry in protecting their incumbent products from entrants relative to their discount rate.

Theorem 2.2. *Denote $x = a_1(\ell - \ell_E)$. If $a_0 \in (0,1), a_1 > 0$, $a_0 - a_1(\ell - \ell_E) \in (0,1)$, $r + \delta a_0 - \eta x > 0$ and c_ι, c_ℓ strictly increasing and convex, then*

1. *Incumbent investment will be strictly greater than entrants, $\ell_E < \ell$ and $\iota_E < \iota$*

2. *Entrant R&D is strictly decreasing in the strength of IP enforcement, $-a_0$, while incumbent R&D increases if $r > \eta x$ and decreases otherwise*

$$\frac{d\iota_E}{da_0} = \frac{r(\Pi x + c_\iota''(\iota)r^*) + \Pi\eta x(a_0 - x)}{c_\iota''(\iota_E)r^*(a_0 + r^* c_\iota''(\iota)\Pi^{-1})} > 0,^5 \tag{9}$$

$$\frac{d\iota}{da_0} = \frac{r - \eta x}{a_0 + r^* c_\iota''(\iota)\Pi^{-1}}, \tag{10}$$

where $r^* = r + (\delta - \iota)a_0 - \eta a_1(\ell - \ell_E)$, $x = a_1(\ell - \ell_E)$.

Proof. Proof given in Appendix B.1. □

These results suggests that the determination process of IP enforcement through the legal system can generate significant barriers to entry and favour some firms over others. Consequently, whether strengthening or weakening IP protections is better for growth depends on whether there is more latent innovative capacity amongst startups being blocked by the IP system or more IP incentives for incumbents. In light of recent trends of declining entry rates and some suggestion that more disruptive innovations come from new entrants, based on this simplified environment it may seem that eliminating court IP enforcement (e.g. $a_0 = 1$ and $a_1 = 0$) is optimal for growth.

A crucial component of IP protection missing is the threat of copied innovations diluting the quasi-monopoly rents requisite to incur the sunk costs of R&D. Concretely, taking λ^* to denote the aggregate rate of successfully copied innovation, the value of an innovation Π becomes

$$\Pi = \frac{\pi - c_\iota(\iota) - c_\ell(\ell)}{r + \delta\Lambda(\ell) - \iota\Gamma(\ell) + \lambda^*}.$$

Hence, if copycat threats are sufficiently high, so $\lambda^* \to \infty$, court IP enforcement is essential to for innovation based growth to occur. Thus, optimal policy for court IP enforcement depends on the level of copycat threats λ and the degree of distortions. In the remainder of the paper, I will use new firm-matched data on IPP investments and litigation events and a quantitative model of IPP enforcement to determine the net effects of IP enforcement on competition and aggregate growth.

3 Measuring IPP investments and their links to firm dynamics

3.1 Data construction

The primary data for this paper combines patent litigation, and patent ownership transfers data with balance sheet, and executive compensation data for public firms. The patent litigation data is obtained from the civil suits database of Federal Judicial Center (FJC) Integrated Database (obtained from WRDS) supplemented with the (2019) USPTO patent litigation docket dataset described in Marco et al. (2017) and Schwartz et al. (2019). Patent transaction data is obtained from the USPTO (Re-)Assignment Dataset which tracks the ownership of patents across time since the time of initial filing, and covers the universe of patents filed in the US (described by Marco, Graham, Myers, D'Agostino and Apple (2015)). The resulting dataset firm-matched dataset differs from the initial NBER patent dataset and a number of extensions like those of Bessen

[5]Note that $a(\ell_E - \ell) = (a_0 - x)$ and as a() is a probability a() is restricted to be between 0 and 1.

(2010), and Kogan et al. (2017), in that it updates parent-subsidiary relationships that have evolved since the initial NBER patent dataset construction using 2001 corporate relationship structures.

An initial impediment to utilizing these rich micro-datasets is a lack of firm level identifiers for these administrative datasets. Furthermore, in the midst of a surge of M&As and expanding size of public firms, accurate characterization of firms patent portfolio and litigation exposures should take into account evolving parent-subsidiary relationships. To tackle these data challenges, I follow a similar broad approach to Hall et al. (2005) by using fuzzy string matching between the patent assignment database and utilizing firm ownership / merger information to track transfers of patents.[6] To assist in linking patents to their ultimate owners, I use Capital IQ corporate tree data to fuzzy string match not just the ultimate parent to any patent assignment transfers involving them, but also those of any subsidiary. Unfortunately, the corporate tree data required manual downloading, so I at this stage only downloaded 262 corporate trees yielding 78,229 former or existing subsidiaries.[7] The 262 ultimate parent companies of the top 100 S&P 500 firms in terms of market cap (July 21, 2020), the top 50 patent holders as of 2017 (https://www.ificlaims.com/ultimate-owners-2018.htm) and other S&P 500 firms in more innovative GIC-subsectors.[8] As these corporate trees only reflect the current situation, I use capital IQ M&A data to determine the data which these firms became subsidiaries of the parent.

The patent assignment dataset provides well over 11.5 million patent assignment transactions, recorded from 1970-present and with respect to over 7.5 million patents. Of which 92% are identified as either ownership assignment transactions or mergers, with the remaining 8.1% being name changes. Between the tracking of parent-subsidiaries and refinements to the string matching developed since the original NBER dataset was constructed, my firm-matched patent coverage increases over 30% relative to the original.

In the fully linked dataset, across the full-sample, I linked 7,606 unique Compustat to holding at least one patent over 75,289 firm-year observations.The max matched patent stock to a single firm-year in sample is IBM in 2018 with 120,626 patents, while according to IBM's own blog, they have obtained since 1920, over 150,000 U.S. patents.

The number of unique Compustat parent plaintiffs linked to patent litigation cases which terminated between 1979 - 2019 is 1,939, 3,125 were defendants, 1,403 firms were both defendants and plaintiffs, and total coverage is 3,661 unique Compustat firms across 14,735 firm $\times$ year observations. When restricted to the sample period from 1996 - 2016 yields 2,542 unique Compustat linked parent defendants, 1,515 unique Compustat linked parent plaintiffs, and 1,403 were both, giving 3283 unique Compustat firms with a completed patent litigation suit in-sample across 10,979 firm $\times$ year observations. Finally, restricting to the set of firms which existed in the 2003 Compustat cross-section

[6]Due to the many variations in the recorded name of the firm in the assignment dataset, I do another fuzzy string matching exercise to disambiguate firms which differ only by a typo / formatting issue or mislabel of inc, etc. I do two procedures, 1) the same fuzzy string matching routine as with the other datasets, 2) sorting alphabetically, looking at pairwise string distance metrics and grouping firm names together which are in close proximity. The procedures I follow can be seen as refinements of the method done by Mann (2018)

[7]Note the coverage of Capital IQ only starts in the 1990s, so other datasources are needed to determine corporate parents in earlier time-periods.

[8]The GIC subsectors I included are: Commodity chemicals / materials, diversified chemicals, ag chemicals, speciality chemicals, application software, comm eq, data processing, electric components, electric eq, electric manufacturing, IT infrastructure, IT consulting, Semi conductors, systems software, tech hardware, aerospace / defense, ag machinery, electric eq components, industrial conglomerates, bio tech, health eq, health supplies, life science tools, pharma, drug retail, auto parts, household appliances, advertising, cable and satellite, ITS, interactive home entertainment,Wireless Telecommunication Services.

of firms and to the sample up to 2016, reduces to 1,885 unique firms across 7,103 firm $\times$ years.

Finally, in addition to the firm-patent / patent litigation matching, in an attempt to identify the legal representatives for firm's patent transactions using the name / address of the legal representative listed on the transaction document. I match the name of the legal representative organization to the firm as well as to a list of public law firms given by Capital IQ as well as legal representatives for other patent transactions / firms to identify if the legal representative is in-house or an external representative.

3.2 Measuring the value of lawyers and litigation on patent values

The value of a patent depends not only on the quality of the patented idea but in the precise legal language and crafting of the patent document to maximize the breadth in cases in which it will be held valid against a counterparty in an infringement case. In this section, I empirically assess the contribution and variation of lawyer input to patent value using identified characteristics of the legal entity the identified legal representative is associated with and legal representative fixed effects.

To measure the contribution of lawyers to the strength or quality of patent protections, I merge my patent and legal representative data to Kogan et al. (2017) patent grant value data, and assess the contribution of the identity and characteristics of the lawyer input to the initially filed patent in explaining the variation of the inferred patent grant values from stock market responses.[9] The lawyer characteristics I use besides named legal representative and firm fixed effects are (i) an in-house lawyer indicator, (2) the 2020 revenue of the public law firm the lawyer works for and (3) the revenue per employee of the public law firm of the legal representative, where for the latter two I set the value to zero for non-public law firms.

Table 1 presents the results. Controlling for the named legal representative and firm (not ultimate parent) fixed effects as well as year and USPTO patent technology class, I find that patents constructed and filed by in-house lawyers are associated with between 43-58% higher patent values upon announcement of patent grants. This semi-elasticity is comparable to a roughly 16% increase in forward citations. In contrast, patents filed by an externally hired lawyer of a large public law firm are lower than a private external law firm consistent with the view that private boutique law firms which specialize and are retained by a small set of clients provide higher quality work than the large public counterparts. This negative effect of large public law firms is counter-balanced by higher quality public law firms exemplified by high revenue per employee. However, these two effects lose statistical significance when restricted to the sub-sample of patents which were filed by the ultimate public parents.

As patents are ultimately valued for their perceived enforceability in court, I link the patents with patent litigation events associated with these transactions using the (2019) USPTO patent litigation docket supplement which includes patent numbers matched to litigation cases. I find that patents which are ultimately named by a public firm defendant is associated with a 5.6% higher patent grant value, but this effect is drowned out after controlling for being used as a plaintiff or is involved in license agreements. Court awarded royalties have a significant semi-elasticity with patent values, where a one thousand dollar increase in royalties ever awarded to the patent is linked to a 11.5% increase in patent grant value. Relatedly, but in addition, patents

[9] I restrict attention to only the first legal representative of a patent filed in order to capture the lawyer input on the construction of the patent.

which are named by a plaintiff of having been infringed in a patent suit are tied to 7.8% higher patent grant values.

Finally, I assess the total contribution of the lawyer input to patent grant values by doing a variance decomposition of the firm fixed effects and the pooled lawyer input including legal rep fixed effects, the in-house indicator and the public law firm covariates. I find that 13.8% of the total variation in patent grant values is attributed to the lawyer input and 85.4% to the firm fixed effects. As with Abowd et al. (1999), this lawyer contribution is likely biased downwards due to low mobility of legal representatives across firms (although many law firms do have relationships with numerous corporations in the cross-section and over-time). Supporting this, I find the correlation between the in-house firms and firm fixed effects is 36.1% suggesting substantial positive assortativeness between in-house legal departments and high quality firms.

Altogether these results suggest that integration of lawyers with the R&D team and the implied specialization that results provides substantial value. "More and more, large international corporations are building small law firms within their own borders...Things like intellectual property, maintaining the IP portfolio, doing a lot of the patent prosecution, handling a lot of the licensing, is I think more efficiently handled by people who are experts internally, who not only have the skills to do the licensing, but also have the knowledge of what is at the core of the corporations needs in this space. What kinds of rights they need in order to do what they want to do. That's hard to communicate to outside counsel, and is much easier for in-house counsel," Bruce Sewell, Columbia law school interview, 2019.[10]

3.3 Rival IPP investments and their influence on court outcomes

An important component of the theoretical link between court IP enforcement and distortions in firm innovation incentives and entry barriers is that asymmetric IPP investments are able to influence court IP enforcement outcomes. To empirically validate this link, I use a proxy for ℓ the sum of the patents assigned (either by being internally filed or acquired) to the plaintiff and defendant over the year of the litigation and the previous three years and regress this log count against a plaintiff judgement. I restrict to patent litigation cases which have parents matched to patent portfolios on both sides of the litigation. Further, due to limitations on data availability for some series in later exercises on firm dynamics, I restrict to the sample period between 1996 to 2016 leaving me with 18,866 patent litigation parent company case pairs.

The results are given in Table 2. Regressing the plaintiff and defendant count separately, I find in column (1) that both contribute significantly and largely counterbalance each-other, with a 10% percent increase in the plaintiff's accumulated patents increasing the average probability of the plaintiff winning by about 6.1% while for a defendant a 10% increase in their accumulated patents over the past 3 years reduces the probability of the plaintiff winning by 4.6%.Consistent with the theory (where a_1 is the sensitivity to the difference in IPP investments), taking the difference between these log counts of the plaintiff, in column (2) I find that the effect of the difference is the mid-point between the estimated coefficients in column (1) for the two counterparties individually.[11] Furthermore, comparing the standard errors, we see that it is even more precisely estimated than those in column (1), suggesting the difference is the relevant

[10]Interview available at `https://youtu.be/-wuf3KI76Ds?t=1658`.

[11]Note that in the model $a()$ was the probability of the defendant winning the litigation suit, while here we are considering $1 - a()$.

factor in plaintiff winning likelihood. Moving to columns (3) - (5) with various other controls the net difference remains statistically and economically significant.

Using the average share of assigned patents represented by an in-house lawyer in the past three years to the plaintiff and defendant respectively, I find that the in-house share of the plaintiff is statistically significant with a 10 p.p. increase in in-house patents increasing the probability of the plaintiff winning by .4%. Defendant's in-house patents are not found to be relevant. In addition, asymmetry in the plaintiff and defendant in their status as a public firm is not found to be significant suggesting either that asymmetries between the size of firms is not relevant or that the selection effects weed out all but the stronger private firms in these litigations. I also find that the concentration of the plaintiff's patent technology within classes is associated with a higher winning probability.

Now observe that the last two columns of this table used judge fixed effects. This data is not available in the FJC dataset. Instead, judge initials presiding over each case were obtained from the USPTO patent docket supplement from the tailing strings in the raw case numbers following court conventions. Besides being relevant controls, these judge fixed effects will be useful in subsequent empirical exercises as an instrument on favourable litigation outcomes for the plaintiff. From the last specification, I extract the judge fixed effects 'bias'. In total, there are 345 individual judge fixed effect estimates. A summary of their 'biases' are given in Table 3. Overall there is substantial dispersion in judge 'biases', with a standard deviation of 0.21.

As the final exercise for this subsection, in Table 4 I examine how these effects have changed over time as well as examine alternative measures of IPP investments coming from the lawyer input in patent construction. Column (1) contains the results pooled across all bilaterally matched litigation suits in sample since 1978. Ignoring for the moment any concerns about selection in cases which ultimately make it to trial, the constant maps to $J_0 = 1 - a_0$ in the model. I find that pooled across the sample $\hat{J}_0 = 1 - \hat{a}_0 = 5.9\%$ while pre-2000s the unconditional level of enforcement was 3.3 p.p. higher, and following the America Invents Act (AIA) of 2011, the unconditional level of enforcement fell by 1.4 p.p. These three effects together suggest that unconditional court patent enforcement has declined over time. It is important to note however that patents and patent litigation have 'exploded' since court reforms in the early 80s and reached peak levels around 2011 before falling somewhat.

Next I consider simultaneously estimates of $J_1 = a_1$ from (a) my primary measure of IPP investments, the sum of assigned patents to the plaintiff / defendant over the past three years, and (b) an alternative measure based on the sum of lawyer fitted values from column (1) specification of Table 1. Both have numerically similar coefficient estimates for the full sample and are statistically significant. However, the former is in logs while the latter is in levels suggesting economically distinct effects.[12] For generality, I also consider the addition of a quadratic term J_2 to the court ruling function capture in the two quadratic regressor terms. In the full sample, these non-linear effects are found to be insignificant.

Moving to column (2) of Table 4 I consider changes in court IP enforcement sensitivity to IPP investment asymmetries of the plaintiff / defendant prior to 2000s. Although only significant at the 10% level, I find that the point estimate effectively undoes the entire pooled effect for the patent measure.This implies that court IP

[12]Observe that given the larger support and variation in patents to fitted values of the legal input, and the fact that the fitted legal values do not statistically hold across all four samples, my primary measure of IPP investments appears stronger and more robust. In addition, and relatedly, the fitted legal inputs are only available for patents granted to public firms and so is more restrictive than my patent count measures which consist of patents obtained by public parent companies and any of their private subsidiaries.

enforcement susceptibility to influence, a_1 was insignificant during the boom growth period of the 90s but has increased in the period since where productivity growth has declined.

Finally, in columns (3) and (4) of Table 4 I study sub-samples of the second half of the sample, from 1996- 2007 and 2008 - 2016 respectively. The former is broadly consistent with the full sample, although the legal fitted value becomes insignificant. The latter is qualitatively distinct with the quadratic term on the legal fitted value becoming statistically significant. This corresponds in the model to $a_2 > 0$ which can be shown to be associated with a strategic complementarity or arms race dynamic.

3.4 Effects of IP litigation on market power and innovation

In this last subsection, I link the patent and litigation data to Compustat firm balance-sheets in order to assess the impacts of IP litigation on firm market power and innovation dynamics. I restrict attention to non-financial, and not heavily regulated or government sponsored industries of utilities, tobacco and defense as classified by Fama-French 48 industries.

To provide a plausibly causal interpretation of estimated treatment effects on public incumbent firms, I follow a strategy similar to Galasso and Schankerman (2018) and use estimated judge biases from specification (5) of Table 2 as an instrument for a plaintiff favourable ruling. To be interpretable, I standardize the judge fixed effects, so that one unit corresponds to a standard deviation increase in the fixed effect. I first consider the instrumented effect of a favourable plaintiff ruling on firm's R&D expenditure (XRD) scaled by physical capital (PPNT). Commensurate with classic Q theory and in-line with my model where Π is the scale free, average Tobin's Q (equity value / PPNT) in the absence of court IP enforcement or other frictions should capture firm investment incentives.

The results are given in columns (1) - (3) of Table 5. I find that favourable court rulings to the plaintiff spur higher R&D intensity for the plaintiff, and this is robust to both year and firm fixed effects.[13] Lagged Tobin's Q has a positive and significant effect on R&D intensity, however with the inclusion of firm fixed effects its elasticity on R&D is the same magnitude as a one-standard deviation increase in judge bias from last years litigation. In columns (2) and (3) I include the lagged / logged difference in patents assigned over the past three years and indicators of whether the firm is currently a defendant or plaintiff. While past asymmetric IP investments have a statistically significant effect in column (2) this disappears with firm fixed effects. Neither of the litigation indicators in the contemporaneous year have any effect on R&D.

Second, in columns (4) - (6) I conduct the same exercise but consider the influence of winning a patent litigation suit on firm markups. As noted by De Loecker et al. (2020), markups are up to an industry specific constant given by revenues over marginal costs, and so following Crouzet and Eberly (2018), I define markups in Compustat as simply sales over cost of goods (SALE/COG) and examine only percent changes so that the industry specific constant / level is immaterial. I find that a one standard deviation higher probability of winning a plaintiff litigation suit increases markups by 8.5% with year fixed effects. However, I find that with the addition of additional controls and firm fixed effects the effect on markups is washed out.

[13] Note that the sample size here is quite small driven by only those litigations covered by the USPTO docket supplement and including matched public plaintiffs / defendants with non-missing balance sheet data for the 4 years around the litigation are included.

In Table 6 I consider the same two dependent variables but try to estimate the differential dynamics of plaintiffs / defendants following litigation events against un-litigated counterparts. In column (1) with just year fixed effects, we see that firms participating in patent litigation events are associated with higher markups (greater market power) than those who aren't. Controlling for industry in column (2), selection of firms involved in IP litigation on either side is tied to higher markups. In contrast, with the addition of firm fixed effects in column (3) we see that being a defendant last year in IP litigation has a positive and significant association with markups. This implies that defendants in these IP litigation suits are not simply cheap copycat infringers who undercut the original innovators, but are offering new products which have some degree of product differentiation.

Moving to the differential effect on R&D intensity in columns (4) - (6) we see that R&D intensity falls on average for those who have been subject to IP litigation suits or, after controlling for industry, are found on both sides of litigation in a likely patent litigation war. However, none of these dynamics are robust to firm fixed effects.

Putting all these results together, we have evidence of the following three stylized facts:

1. IPP investments and integrated, high quality lawyers have substantial influence on IP litigation outcomes

2. IP litigation outcomes impact subsequent firm innovation intensities

3. patent litigation defendants subsequently inherit substantial market power.

These stylized facts are key ingredients which inform the subsequent quantitative model building and estimation.

4 Model

I present the model in steps beginning with the investment-savings decision of an individual firm as well as the patent litigation stages. I then describe entry and the birth-death process of firms generating a firm size, productivity and cash distribution, and the determination of the aggregate innovation rate in general equilibrium.

4.1 Preferences and Technology

The economy consists of a unit continuum of differentiated goods. Consumers have symmetric Cobb-Douglas preferences across the goods so that their expenditure on each good is the same. Household's can borrow or lend at interest rate r_t and maximize their path of consumption given their present-value flow of labour income and profits from firms,

$$U_t = \max_{C_s} \int_t^{\infty} \log C_s e^{-\rho(s-t)} ds \tag{11}$$

s/t $P_t C_t = E_t$, $\int_s C_s ds = \int_s w_s L ds$. Optimal consumption expenditure must then solve the differential equation $\frac{\dot{E}}{E} = r_t - \rho$. Following GH(1991), we choose numeraire so that $E_t = 1 \forall t$, which implies $r_t = \rho$.

I set the numeraire so that household expenditure is constant at one ($E_t = P_t C_t = 1$.) Since time is continuous there is thus a unit flow of expenditure on each good. [14] The consumption good is an aggregate of a unit measure of intermediate goods with productivity/quality $A_t(j)$ as follows:

$$\log C_t = \int_0^1 \log(A_t(i)x_t(i))di \tag{12}$$

Demand for individual products (varieties) $x(i)$ is Cobb-Douglas, and treat different vintages of a given variety as perfect substitutes. Consequently, demand for a given variety i, vintage j is

$$x_{tj}(i) = \begin{cases} \frac{E_t}{p_{tj}(i)} & p_{tj}(i) = \min p_{tj'}(i) \\ 0 & o/w. \end{cases} \tag{13}$$

The productivity / quality of the intermediate goods follow a Poisson jump process, where the quality A is given by the number of disruptive innovations (jumps), N_{it} on a quality increment $q > 1$,

$$A_{it} = q^{N_{it}+1} \tag{14}$$

4.2 Intermediate Good Prices

Intermediate good producers price compete against other vintages / variants of their product. A given product markets (for variety i), can be in two different states: monopoly (m) or competitive (c) states. In a monopoly state, there is a single intermediate good quality leader with quality $A_{it} = q^{N_{it}}$, while in the competitive state there are at least two firms with the same quality.

Whereas in the competitive state, price competition leads to price equalling marginal cost, $p(i) = w$, in the case of a monopoly, the quality leader solves the following static profit maximization problem:

$$\pi(i) = \max_p px(p, \hat{p}) - wx(p, \hat{p}) \tag{15}$$

provided that their quality adjusted price, $\frac{p}{A(i)}$ is the cheapest available for product i. As there exists at least one firm which has the pre-existing quality of the product $q^{N_{it}-1}$, the optimal price for a single quality leader is then the limit price, $p(i) = wq$ which makes this rival indifferent to competing in the market and generates monopoly profits for the quality leader of

$$\pi(i) = 1 - \frac{1}{q}. \tag{16}$$

[14] See **?**, **?** and **?** for more details.

Let ν denote the fraction of product markets which have a single quality leader and $1-\nu$ the fraction which have competing quality leaders. The total equilibrium expected quantity of intermediate goods supplied is

$$\log X_t = \int_0^1 \log x(i)di = \nu \log x_m + (1-\nu)\mathbb{E}_t[\log x_c(i)] = \nu \log(\frac{1}{wq}) + (1-\nu)\mathbb{E}[\log(\frac{1}{w})] \tag{17}$$

4.3 Aggregate growth rate

As the level of aggregate consumption is $\log C_t = \int_0^1 \log(A_t(i)x_t(i))di$ we have that the growth rate of aggregate consumption, δ_C, is given by

$$\frac{d\mathbb{E}[\log C_t]}{dt} = \mathbb{E}\left[\frac{d\log A_t}{dt} + \frac{d\log X_t}{dt}\right] = \log q \cdot \delta^*, \tag{18}$$

where δ^* is the endogenous product creation rate.

4.4 A firm and it's growth technologies

A firm is defined by the portfolio of n intermediate goods that they are the original innovator of (and hence own the intellectual property rights and the capital underlying the quality advantage), their firm type τ, the net royalties received, ρ and for each product i, the competitive status of the product m_i. Firms can invest in new disruptive innovations to add new product lines to its portfolio intensity ι or try to "copycat" (backward engineer) another producers product and steal $\phi\pi$ of the incumbent's profit stream at intensity γ. Finally firms can invest in IP protection, ℓ to help protect the sales of their existing products and new product innovations. All three of these investment activities come from separate convex labour costs, $wc_\iota(\iota)$, $wc_\gamma(\gamma)$, and $wc_\ell(\ell;\tau)$. Note that the convex labour cost in IP protection ℓ is in general type dependent, where the marginal cost is uniformly lower for the high type firm.

The firm standalone problem is given by

$$rV(n,\{m_i\},\rho,\tau) = \max_{\iota_i,\gamma_i,\ell_i} \sum_i m_i\pi + r\rho - w\sum_i [c_\iota(\iota_i) + c_\gamma(\gamma_i) - c_\ell(\ell_i;\tau)] \tag{19}$$

$$+ \mathbb{E}[\Delta V(n,\{m_i,\ell_i,\iota_i,\gamma_i\},\rho,\tau)] \tag{20}$$

where, letting $\hat{x} = (\hat{n},\{\hat{M},\hat{\ell}\},\hat{\rho},\hat{\tau})$ denote the rival firms type, we have

$$\Delta V(n,\{m_i,\ell_i,\iota_i,\gamma_i\},\rho,\tau) = \mathbb{E}[\iota_i D_i(n,\{m_i,\ell_i\},\rho,\tau,\hat{x})] + \sum_i \gamma_i \mathbb{E}[C_i(n,\{m_i,\ell_i\},\rho,\tau,\hat{x})]$$
$$+ \sum_i \delta\mathbb{E}[P_i(n,\{m_i,\ell_i\},\rho,\tau,\hat{x})] + \sum_i \lambda m_i \mathbb{E}[Q_i(n,\{m_i,\ell_i\},\rho,\tau,\hat{x})].$$

From here-on denote $[m_i]_n = \{m_i\}_{i=1,..,n}$.

Without any litigation, the ex-ante value of being a defendant with a disruptive innovation is

$$D(n,\{m_i,\ell_i\},\rho,\hat{x}) = \mathbb{E}[V(n,\{m_i,M_{n+1}=1\},\rho,\tau) - V(n,[m_i]_n,\rho,\tau)] \tag{21}$$

while for a copycat innovation facing a plaintiff with monopoly market type $\hat{m}$ is:

$$C_i(n, [m_i]_n, \{\ell_i\}, \rho, \hat{x}) = \mathbb{E}[V(n, [m_i]_n, \rho + \phi\pi\hat{m}) - V_\tau(n, [m_i]_n, \rho, \tau)]. \tag{22}$$

On the flip-side, the ex-ante value of being a plaintiff from a disruptive innovation is:

$$P_i(n, [m_i]_n, [\ell_i]_n, \rho, \hat{x}) = V(n-1, [m_i]_n \setminus m_i, \rho, \tau) - V(n, [m_i]_n, \rho, \tau) \tag{23}$$

that is, a plaintiff loses the product line entirely after a disruptive innovation, while in the case of a copycat, the size of their firm doesn't change (they retain the production / R&D team), but the net profits are diluted through the change in royalties.[15] Note, the implicit assumption about R&D teams is that only the first discoverer retains the R&D team, copycat innovators do not grow in firm size nor affect the firm size of others.

Finally, the ex-ante value of being a plaintiff from a copycat innovation is:

$$Q(n, [m_i]_n, \{\ell_i\}, \rho, \hat{x}) = V(n, [m_i]_n \setminus m_i, \rho, \tau) - V(n, [m_i]_n, \rho, \tau). \tag{24}$$

4.5 Patent litigation trials

Upon discovery of an innovation (either disruptive or coypcat), a litigation suit is automatically initiated between the incumbent of that product and the new innovator. Each firm, after realizing the counterparty and whether the product is currently with a single quality leader, makes a discrete choice to attempt to settle the suit or refuse to negotiate and take to court (note a single firm electing to prefer court will yield court trial as the outcome).[16]

Patent infringement cases are adjudicated by the court who based on the legal case presented either (i) dismiss the case if the court deems the case spurious, (ii) awards licensing / royalty fees, or (iii) bans the infringing product from the market. The outcome of a court battle depends on the perceived distance, d, by the judge / jury between the litigant's intellectual property and the target of the litigation. This perceived distance is a noisy signal of the the true overlap between the new innovation and the existing patented technology $d_0 \sim F(d_0)$,

$$d = d_0 f\left(\ell^D - \ell^P + \sigma\varepsilon\right)$$

where $\varepsilon \sim G_{d_0}(\varepsilon)$. Firms can strategically bias the court's perceived distance in their favour (away from the true product similarity, d_0) by expending more legal counsel man-hours than their opponent.[17] Denote $J(\ell^P, \ell^D)$ as the ex-ante probability of a judgment in favour of the plaintiff given plaintiff and defendant expenditures ℓ^P and ℓ^D respectively, where $J()$ is of the form:

$$J(\ell, \hat{\ell}) = \min\{1, \max\{J_0 f(\ell - \hat{\ell}), 0\}\}. \tag{25}$$

Similarly, denote $\bar{J} = 1 - J$. A judgment in favour of the plaintiff results in an injunction, that is a total barring of the defendant's new product from entering the

[15] Accounting for the copycat entry this way allows us to avoid keeping track of the history of copycat entry into a product market.

[16] Of course, firms may also choose to withdraw, but we will assume costs are such that this outside option is never taken.

[17] Legal hours expended relating to the case is private information to the firm, and hence the court cannot back-out the true similarity from the expenditures.

market (as well as repayment of any profits taken in the interim). On the other hand, a judgment against the plaintiff results in the status quo of the new entrant entering and competing with the incumbent.

Disruptive innovation:

The value of the plaintiff from a trial with facing a disruptive innovation is given by

$$\begin{aligned}\tilde{P}_i(trial, n, \{m_i, \ell_i\}, \rho, \hat{x}) =& \bar{J}(\ell_i, \hat{\ell}_j)\left[V(n-1, [m_i]_n \setminus m_i, \rho, \tau) - V(n, [m_i]_n, \rho, \tau)\right] \\ &+ J(\ell_i, \hat{\ell})\left[V(n, [m_i]_n, \rho - \xi) - V(n, [m_i]_n, \rho, \tau)\right]\end{aligned}$$

which (since trial costs $\xi = 0$) simplifies to

$$\tilde{P}_i(trial, n, \{m_i, \ell_i\}, \rho, \hat{x}) = \bar{J}(\ell_i, \hat{\ell}_j)\left[V(n-1, [m_i]_n \setminus m_i, \rho, \tau) - V(n, [m_i]_n, \rho, \tau)\right]. \tag{26}$$

On the flip-side, the defendant with a disruptive innovation in a trial obtains (regardless of whether a monopoly existed in the product market prior to their innovation):

$$D_i(trial, n, [m_i]_n, \{\ell_i\}, \rho, \hat{x}) = \bar{J}(\hat{\ell}, \ell_i)\left[V(n+1, \{[m_i]_n, 1\}, \rho, \tau) - V(n, [m_i]_n, \rho, \tau)\right]. \tag{27}$$

Copycat innovation:

Trials in the presence of copycat innovations have the same form as above, where the only difference is that the plaintiff doesn't lose their R&D team (i.e. n remains fixed) and the innovator only gains short-term profits $\phi\pi$.

$$Q_i(trial, n, \{m_i, \ell_i\}, \rho, \hat{x}) = \bar{J}(\ell_i, \hat{\ell})\left[V_\tau(n, [m_i]_n, \rho, \tau) - V(n, \{[m_i]_n \setminus m_i, m_i = 0\}, \rho, \tau)\right] \tag{28}$$

$$C_i(trial, n, \{m_i, \ell_i\}, \rho, \hat{x}) = \bar{J}(\hat{\ell}, \ell_i)\left[V(n, [m_i]_n, \rho + \phi\pi) - V(n, [m_i]_n, \rho, \tau)\right]. \tag{29}$$

Patent litigation settlements

Rather than have a stochastic settlement in court and bear the fixed trial costs ξ, firms can elect to settle with each other outside court. The value of settlement for a plaintiff given a disruptive innovation is

$$\tilde{P}_i(settle, n, \{m_i, \ell_i\}, \rho, \hat{x}) = V(n-1, [m_i]_n \setminus m_i, \rho + b) - V_\tau(n, [m_i]_n, \rho, \tau) \tag{30}$$

while the value of settlement for a defendant given a disruptive innovation is

$$\tilde{D}_i(settle, n, \{m_i, \ell_i\}, \rho, \hat{x}) = V_\tau(n+1, \{[m_i]_n, 1\}, \rho - b) - V_\tau(n, [m_i]_n, \rho, \tau). \tag{31}$$

Let x denote the state of the plaintiff and $\hat{x}$ the defendant. The bargained settlement royalties between plaintiff and defendant with team i and j respectively solves:

$$b_{ij}(x,\hat{x}) = \arg\max_{\tilde{b}} \left(\mathbb{S}_P(\tilde{b},x,\hat{x})\right)^{0.5}\left(\mathbb{S}_D(\tilde{b},x,\hat{x})\right)^{0.5} \tag{32}$$

where in the case of a disruptive innovation the settlement surplus is

$$\mathbb{S}_P(i,j,\tilde{b},x,\hat{x}) = P_i(settle,x,\hat{x}) - P_i(trial,x,\hat{x}) \tag{33}$$

and

$$\mathbb{S}_D(i,j,\tilde{b},x,\hat{x}) = D_i(settle,x,\hat{x}) - D_i(trial,x,\hat{x}) \tag{34}$$

and symmetrically replacing P with Q and D with C yields the individual surpluses with copycat innovation.

4.6 Startup problem

Besides incumbent firms already endowed with past disruptive innovations / R&D teams, there is a fixed mass μ of potential entrants, who conduct both types of R&D, invest in IP protection for their in-progress R&D. They are initially endowed with the high marginal cost IP technology but can pay a fixed cost κ to get a lottery that gives them the low marginal cost IP upon successful entry as a quality leader in a product market. That is, the entrant does not know if they have the low or high IP cost type until after finishing the litigation with the incumbent firms. Besides the choice to pay the fixed cost or not and not being initially endowed with any products, the problem is the same as the incumbents above:

$$rV_0 = \max_{\iota,\gamma,\ell,k\in\{0,1\}} \quad -wc_\iota(\iota) - wc_\gamma(\gamma) - wc_\tau(\ell) - k\kappa + \mathbb{E}[\Delta V_0(\ell,\iota,\gamma,k,\hat{x})] \tag{35}$$

where

$$\mathbb{E}[\Delta V_0(\ell,\iota,\gamma,k,\hat{x})] = \iota\mathbb{E}[D_0(\ell,k,\hat{x})] + \gamma\mathbb{E}[C_0(\ell,\hat{x})]$$

and $\mathbb{E}[D_0(\ell,k,\hat{x})]$ is the same as for the incumbent but taking $n=0, m=0$ as baseline.

4.7 Firm Value Decomposition

Will conjecture that firm value takes the following form:

$$V(n,[m_i]_n,\rho,\tau) = \rho + \sum_i \Sigma_{\tau m_i} \tag{36}$$

That is, firm value is comprised of the present value of being the quality leading innovator in n products, with retained monopoly in $\sum_i m_i$ products, and a non production based term ρ which simply depends on the history of past innovations involving the firm.

With this guess, the value of a trial is as follows:

Disruptive innovation trials:

The value of the plaintiff from a trial facing a disruptive innovation is given by

$$P_i(trial, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\bar{J}(\ell_i, \hat{\ell})\Sigma_{\tau m_i} \tag{37}$$

On the flip-side, the defendant with a disruptive innovation in a trial obtains (regardless of whether a monopoly existed in the product market prior to their innovation):

$$D_i(trial, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \bar{J}(\hat{\ell}, \ell_i)\Sigma_{\tau 1}. \tag{38}$$

where here m_i is the innovator's original product line monopoly status which conditional on ℓ_i is immaterial for the defendant's payoffs.

Copycat innovation trials:

Trials in the presence of copycat innovations have the same form as above, where the only difference is that the plaintiff doesn't lose their R&D team (i.e. n remains fixed) and the innovator only gains short-term profits $\phi\pi$.

$$Q_i(trial, m_i = 1, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\bar{J}(\ell_i, \hat{\ell})[\Sigma_{\tau 1} - \Sigma_{\tau 0}] \tag{39}$$

$$C_i(trial, m_i, \ell_i, \tau, \hat{m} = 1, \hat{\ell}, \hat{\tau}) = \bar{J}(\hat{\ell}, \ell_i)\phi\pi \tag{40}$$

where there are no profits to be made for the copycat if they should copy a non-monopoly product.

Disruptive innovation settlement:

Similarly, with the guess, the value of settlement for a plaintiff given a disruptive innovation on a monopoly becomes

$$P_i(settle, b, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Sigma_{\tau m_i} + b \tag{41}$$

while the value of settlement for a defendant given a disruptive innovation is

$$\tilde{D}_i(settle, b, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \Sigma_{\tau 1} - b \tag{42}$$

Copycat innovation settlement:

In the case of copycat innovation, the settlement values are

$$Q_i(settle, b, m_i = 1, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Sigma_{\tau 1} + b$$

and

$$C_i(settle, b, m_i, \ell_i, \tau, \hat{m} = 1, \hat{\ell}, \hat{\tau}, b) = \phi\pi - b.$$

4.8 Litigation equilibrium outcomes

We can now solve for the equilibrium outcomes of litigation between a given defendant and plaintiff and the two innovation types, disruptive and copycat. Since a trial is the outside option for both parties, standard results of Nash bargaining implies bargaining is weakly preferred provided the total trade (settlement) surplus is positive.[18]

From the analysis in the previous section, we see that only the monopoly status of the product market i which was innovated on for the plaintiff and the monopoly status of the product line which did the innovating matters (all the other products are independent). From here-on, we will drop the i subscript and instead use simply m for the status of the ith product market of the firm in question and $\hat{m}$ for the rival firm.

Given the guess of the value functions, the total settlement surplus in the case of a copycat is:

$$\mathbb{S} = -(\Sigma_{\tau 1} - \Sigma_{\tau 0}) + \bar{J}(\ell_i, \hat{\ell})(\Sigma_{\tau 1} - \Sigma_{\tau 0}) + \phi\pi - \bar{J}(\hat{\ell}, \ell_i)\phi\pi = -J(\ell_i, \hat{\ell})\left[(\Sigma_{\tau 1} - \Sigma_{\tau 0}) - \phi\pi\right] < 0$$

where the last inequality holds for any ϕ not too large. This gives us our first result.

Lemma 4.1. *Provided $\Sigma_{\tau 1} - \Sigma_{\tau 0} > \phi\pi$ trial is the equilibrium outcome of copycat innovation on a monopoly market, (and the outcome is irrelevant in the case of $m = 0$). The interim values of the copycat plaintiff and defendant are respectively:*

$$C(\ell, \hat{m}, \hat{\ell}) = \pi\phi\hat{m}\bar{J}(\hat{\ell}, \ell) \tag{43}$$

and

$$Q(\ell, m, \tau, \hat{\ell}) = -\bar{J}(\ell, \hat{\ell})(\Sigma_{\tau 1} - \Sigma_{\tau 0}). \tag{44}$$

Moving to the case of a disruptive innovation, the settlement surplus for the plaintiff is

$$\mathbb{S}_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Sigma_{\tau m} + \tilde{b} - \left(- \bar{J}(\ell_i, \hat{\ell})\Sigma_{\tau m}\right)$$

which simplifies to

$$\mathbb{S}_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -J(\ell_i, \hat{\ell})\Sigma_{\tau m} + \tilde{b}. \tag{45}$$

Similarly for the defendant,

$$\mathbb{S}_D(\tilde{b}, \ell_i, m, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = J(\hat{\ell}, \ell_i)\Sigma_{\tau 1} - \tilde{b}. \tag{46}$$

Thus, the total surplus from settlement for a disruptive innovation is

$$\mathbb{S}(m, x, \hat{x}) = \mathbb{S}_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) + \mathbb{S}_D(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}),$$

that is,

$$\mathbb{S}(m, x, \hat{x}) = J(\ell, \hat{\ell})[\Sigma_{\hat{\tau}1} - \Sigma_{\tau m}]. \tag{47}$$

[18]With positive trial costs ξ then any weak preference here would be strict preference for settlement. To keep the model as simple as possible, I have abstracted from these costs here since qualitatively nothing changes for ξ not too large to induce some firms to entirely forfeit a litigation proceeding.

Given two types τ, there are 6 cases with disruptive innovation. In the first two cases, $\tau = \hat{\tau}$ and plaintiff has a monopoly ($m = 1$) then the surplus is zero (but is strictly positive if trial costs are positive) hence settlement is the equilibrium outcome. In the case where the plaintiff has a monopoly, but is of a lower type than $\hat{\tau}$, assuming monotonicity in τ so that $\Sigma_{\tau m} \geq \Sigma_{\tau' m}$ for $\tau > \tau'$, settlement surplus is strictly positive, hence settlement again. In the fourth case, the plaintiff is of a higher type than the defendant, in which case the total surplus is negative, and hence trial is the equilibrium outcome. Finally, the last two cases involve asymmetric τ and plaintiff has no monopoly. Conjecturing that the value of a monopoly product for the low type is higher than the value of not owning a monopoly for the high type, we obtain settlement is optimal in this case as well.

Lemma 4.2. *Assume that $\Sigma_{\tau 1} > \Sigma_{\tau 0}$ and $\Sigma_{\tau' 1} \geq \Sigma_{\tau 0}$ for $\tau' < \tau$. With disruptive innovation, the unique equilibrium litigation outcomes for two firms where plaintiff is of type τ with monopoly product status m and defendant is of type $\hat{\tau}$ is to settle in all meetings except when the plaintiff firm is of the low marginal cost type $\tau = 1$, the defendant is of the high marginal cost type $\hat{\tau} = 0$ and the plaintiff's product under threat is a monopoly $m = 1$.*[19] *The equilibrium interim payoffs of the defendant is given by*

$$D(\ell, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \begin{cases} \bar{J}(\hat{\ell}, \ell)\Sigma_{\tau 1} & \hat{\tau} = 1 > \tau = 0, \hat{m} = 1 \\ \Sigma_{\tau 1} - b(\hat{m}, \hat{\ell}, \hat{\tau}, \ell, \tau) & else \end{cases} \tag{48}$$

while the interim value of a plaintiff with ℓ, τ, m facing a defendant in state $\hat{\ell}, \hat{\tau}$ is given by

$$P(\ell, \tau, m, \hat{\ell}, \hat{\tau}) = \begin{cases} -\bar{J}(\ell, \hat{\ell})\Sigma_{\tau m} & \tau = 1 > \hat{\tau} = 0, m = 1 \\ -\Sigma_{\tau m} + b(m, \ell, \tau, \hat{\ell}, \hat{\tau}) & else \end{cases} \tag{49}$$

where the bargained royalties in the case of settlement are

$$b(m, \ell, \tau, \hat{\ell}, \hat{\tau}) = \frac{1}{2} J(\ell, \hat{\ell}) \left[\Sigma_{\tau m} + \Sigma_{\hat{\tau} 1} \right]. \tag{50}$$

Thus far, we have only considered the case of two incumbent firms meeting in litigation. If we assume the entrant knows their IP type upon innovation, but not prior, then the forward looking settlement / trial decisions are identical to the incumbent case, but since the entrant did not know their type ex-ante, their IP protection investment will lie between the non-monopoly high marginal cost and the non-monopoly product low marginal cost IP protection levels. Hence the size of transfers / the probability of a favourable judgment will differ for the case with an entrant defendant.

Lemma 4.3. *With an entrant's innovation, provided the entrant knows their incumbent type τ upon innovation, then under the same assumptions as above, the same equilibrium outcomes of trial or settlement occur conditional on the realized type of the entrant, however, the royalties differ in value, and are given by*

$$b_E(m, \ell, \tau, \ell_E, \tau_E) = \frac{1}{2} J(\ell, \ell_E) \left[\Sigma_{\tau m} + \Sigma_{\tau_E 1} \right]. \tag{51}$$

When it comes to structurally estimating the model, it will in fact be convenient to assume that entrant's only discover their incumbent type τ after the innovation / litigation with the incumbent, so their value itself is uncertain.

[19] On the other hand, if $M_{01} + R_{01} < R_{10}$ then trial occurs also (for the same $\tau = 1, \hat{\tau} = 0$) when the plaintiff's product under threat is a non-monopoly product.

4.9 Ex-ante value of litigation

Finally, to determine the optimal innovation and IP protection levels, we must take expectations over the type of counterparty the firm will face in each. Commensurate with our guess of the value-function, this implies that all policy variables for a given product line i only depend on τ, m_i.

Denote K_τ as the mass of incumbents of type τ, $\nu_{\tau m}$ as the probability of that firm type product being a monopoly status m, $K_E = \mu$ as the mass of potential entrants. Then the probability of a plaintiff meeting a given type $\hat{m}, \hat{\tau}$ disruptive innovator is

$$\theta^P_{\hat{\tau}\hat{m}} = \frac{\iota_{\hat{\tau}\hat{m}} \nu_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\sum_{\tau'} \sum_{m'} \iota_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \iota_E K_E} \tag{52}$$

and meeting an entrant of type τ is:

$$\theta^P_{\hat{\tau}E} = \frac{\iota_E \phi_{\hat{\tau}} K_E}{\sum_{\tau'} \sum_{m'} \iota_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \iota_E K_E} \tag{53}$$

while the probability of a defendant meeting a given type plaintiff $\hat{m}, \hat{\tau}$ is simply

$$\theta^D_{\hat{\tau}\hat{m}} = \frac{\delta \nu_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\sum_{\tau'} \sum_{m'} \delta \nu_{\tau' m'} K_{\tau'}} = \nu_{\hat{\tau}\hat{m}} K_{\hat{\tau}}. \tag{54}$$

where the last equality follows from there being a unit measure of products, $\sum_{\tau', m'} \nu_{\tau'}(m') K_{\tau'} = 1$.

Using the above, (and applying symmetric reasoning for the defendant) yields the following lemma.

Lemma 4.4. *Assuming $\Sigma_{\tau 1} > \Sigma_{\tau 0}$ and $\Sigma_{01} \geq \Sigma_{10}$, the equilibrium expected net payoff given a disruptive innovation is as follows.*

Disruptive plaintiff

$$E[P|\ell, m, \tau] = -\Sigma_{\tau m} \Psi^P_{\tau m}(\ell) + T^P_{\tau m}(\ell) \tag{55}$$

where the effective probability of losing the product market for a plaintiff is

$$\Psi^P_{\tau m}(\ell) = \sum_{\tau'} \sum_{m' \in \{0,1,E\}} \theta^P_{\tau' m'} \left\{ B_{settle}(\tau, \tau', m) \left(1 - \frac{J(\ell, \ell_{\tau', m'})}{2}\right) + B_{trial}(\tau, \tau', m) \bar{J}(\ell, \ell_{\tau', m'}) \right\} \tag{56}$$

while the expected gain of royalties (obtained in settlement) from the sharing of the rival innovators surplus is

$$T^P_{\tau m}(\ell) = \sum_{\tau'} \sum_{m'} \theta^P_{\tau' m'} B_{settle}(\tau, \tau', m) \frac{J(\ell, \ell_{\tau' m'})}{2} \Sigma_{\tau' 1} \tag{57}$$

Disruptive defendant

$$E[D|\ell, m, \tau] = \Sigma_{\tau 1} \Psi^D_{\tau m}(\ell) - T^D_{\tau m}(\ell) \tag{58}$$

$$\Psi^D_{\tau m}(\ell) = \sum_{\tau'} \sum_{m'} \theta^D_{\tau' m'} \left\{ B_{settle}(\tau', \tau, m') \left(1 - \frac{J(\ell_{\tau' m'}, \ell)}{2}\right) + B_{trial}(\tau', \tau, m') \bar{J}(\ell_{\tau' m'}, \ell) \right\} \tag{59}$$

$$T^D_{\tau m}(\ell) = \sum_{\tau'}\sum_{m'} \theta^D_{\tau' m'} B_{settle}(\tau', \tau, m') \frac{J(\ell_{\tau' m'}, \ell)}{2} \Sigma_{\tau' m'} \tag{60}$$

Lemma 4.5. *If $\Sigma_{\tau 1} - \Sigma_{\tau 0} > \phi\pi$ for all τ. The ex-ante payoffs of copycat innovation are*

Copycat plaintiff

$$E[Q|\ell, m, \tau] = -(\Sigma_{\tau 1} - \Sigma_{\tau 0})\Psi^Q_{\tau m}(\ell) \tag{61}$$

$$\Psi^Q_{\tau m}(\ell) = \sum_{\tau'}\sum_{m'} \theta^Q_{\tau' m'} \bar{J}(\ell, \ell_{\tau', m'}) \tag{62}$$

Copycat defendant

$$E[C|\ell, m, \tau] = \phi\pi\Psi^C_{\tau m}(\ell) \tag{63}$$

where

$$\Psi^C_{\tau m}(\ell) = \sum_{\tau'}\sum_{m'} \theta^C_{\tau' m'} m' \bar{J}(\ell_{\tau' m'}, \ell) \tag{64}$$

$$\theta^Q_{\hat{\tau}\hat{m}} = \frac{\gamma_{\hat{\tau}\hat{m}} \nu_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\sum_{\tau'}\sum_{m'} \gamma_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \gamma_E K_E} \tag{65}$$

and

$$\theta^Q_{\hat{\tau} E} = \frac{\gamma_E \phi_{\hat{\tau}} K_E}{\sum_{\tau'}\sum_{m'} \gamma_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \gamma_E K_E} \tag{66}$$

while the probability of a defendant meeting a given type plaintiff $\hat{m}, \hat{\tau}$ is

$$\theta^C_{\hat{\tau}\hat{m}} = \frac{\lambda \nu_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\lambda \sum_{\tau'}\sum_{m'} \nu_{\tau' m'} K_{\tau'}}. \tag{67}$$

4.10 Optimal investment policies

Having solved for the expected value of innovation for the firm $\mathbb{E}[\Delta V]$, I can now determine the optimal R&D and legal protection investment policies, ι, γ, ℓ for both types of incumbents as well as startups.

Incumbent optimal policies

$$\mathbb{E}[D|\ell_{\tau m}, m, \tau] = wc'_{\iota}(\iota_{\tau,m}) \tag{68}$$

$$\mathbb{E}[C|\ell_{\tau m}, m, \tau] = wc'_{\gamma}(\gamma_{\tau,m}) \tag{69}$$

$$\iota_{\tau,m} \frac{\partial \mathbb{E}[D|\ell, m, \tau]}{\partial \ell} + \gamma_{\tau,m} \frac{\partial \mathbb{E}[C|\ell, m, \tau]}{\partial \ell} + \delta \frac{\partial \mathbb{E}[P|\ell, m, \tau]}{\partial \ell} + \lambda \frac{\partial \mathbb{E}[Q|\ell, m, \tau]}{\partial \ell} = wc'_{\tau}(\ell) \tag{70}$$

By inspection, we see that the optimal policies of ℓ, ι, γ depend as conjectured only on m, τ.

Theorem 4.6. *Assume $c_\iota(\iota) = \frac{1}{a_\iota}\iota^{\alpha_\iota+1}$ and $c_\gamma(\gamma) = \frac{1}{a_\gamma}\gamma^{\alpha_\gamma+1}$, $\alpha_\gamma < \alpha_\iota$, $a_\gamma > a_\iota$, and $c_\tau(\ell) = \frac{1}{a_\ell(\tau)}\ell^{\alpha_\ell(\tau)+1}$. Also let $\xi \to 0$, then optimal firm policies are given by*

$$\iota_{\tau m} = c_\iota(\iota)^{-1'}\left(w^{-1}\mathbb{E}[D|\ell_{\tau m}, m, \tau]\right) \tag{71}$$

$$\gamma_{\tau m} = c_\gamma(\gamma)^{-1'}\left(w^{-1}\mathbb{E}[C|\ell_{\tau m}, m, \tau]\right) \tag{72}$$

$$\ell_{\tau m} = c_\ell(\ell, \tau)^{-1'}\left(\frac{w^{-1}\partial\mathbb{E}[\Delta V]}{\partial \ell}\right) \tag{73}$$

where

$$\frac{\partial\mathbb{E}[\Delta V|x]}{\partial \ell} = \iota_\tau\left(\frac{\partial\mathbb{E}[D|x]}{\partial \ell^D}\right) + \delta\frac{\partial\mathbb{E}[P|x)]}{\partial \ell^P} + \gamma_\tau\frac{\partial\mathbb{E}[C|x]}{\partial \ell^D} + \lambda\frac{\partial\mathbb{E}[Q|x]}{\partial \ell^P}.$$

4.11 Solving for value functions

Given the above, I am now equipped to solve for $\Sigma_{\tau m}$ (and thus the value function V). Plugging in the expected values of IP litigation events into the HJB yields:

$$\begin{aligned} rV = \sum_i m_i\pi + r\rho - w\sum_i[c_\iota(\iota_{\tau m_i}) + c_\gamma(\gamma_{\tau m_i}) + c_\ell(\ell_{\tau m_i};\tau)] \\ + \sum_i\left\{\iota_{\tau m_i}(\Psi^D_{\tau m_i}\Sigma_{\tau 1} - T^D_{\tau m_i}) - \delta(\Psi^P_{\tau m_i}\Sigma_{\tau m_i} - T^P_{\tau m_i})\right\} \\ + \sum_i \gamma_{\tau m_i}\Psi^C_{\tau m_i}\phi\pi - \lambda m_i\Psi^Q_{\tau 1}(\Sigma_{\tau 1} - \Sigma_{\tau 0}) \end{aligned}$$

which plugging in the guess of V on the LHS of FirmProblem and matching coefficients of $\hat{m}, \hat{n}$:

$$\begin{aligned} r\Sigma_{\tau 1} = \pi - w[c_\iota(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_\ell(\ell_{\tau 1};\tau)] \\ +\iota_{\tau 1}(\Psi^D_{\tau 1}\Sigma_{\tau 1} - T^D_{\tau 1}) - \delta(\Psi^P_{\tau 1}\Sigma_{\tau 1} - T^P_{\tau 1}) - \lambda\Psi^Q_{\tau 1}\Sigma_{\tau 1} + \gamma_{\tau 1}\Psi^C_{\tau 1}\phi\pi + \lambda\Psi^Q_{\tau 1}\Sigma_{\tau 0} \end{aligned}$$

and

$$r\Sigma_{\tau 0} = -w[c_\iota(\iota_{\tau 0}) + c_\gamma(\gamma_{\tau 0}) + c_\ell(\ell_{\tau 0};\tau)] + \iota_{\tau 0}(\Psi^D_{\tau 0}\Sigma_{\tau 1} - T^D_{\tau 0}) - \delta(\Psi^P_{\tau 0}\Sigma_{\tau 0} - T^P_{\tau 0}) + \gamma_{\tau 0}\Psi^C_{\tau 0}\phi\pi$$

Solving for $\Sigma_{\tau 1}$ and $\Sigma_{\tau 0}$ yields

$$\Sigma_{\tau 1} = \frac{\pi - w[c_\iota(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_\ell(\ell_{\tau 1};\tau)] + \gamma_{\tau 1}\Psi^C_{\tau 1}\phi\pi - \iota_{\tau 1}T^D_{\tau 1} + \delta T^P_{\tau 1}}{r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1}} + \frac{\lambda\Psi^Q_{\tau 0}\Sigma_{\tau 0}}{r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1}} \tag{74}$$

$$\Sigma_{\tau 0} = \frac{-w[c_\iota(\iota_{\tau 0}) + c_\gamma(\gamma_{\tau 0}) + c_\ell(\ell_{\tau 0};\tau)] - \iota_{\tau 0}T^D_{\tau 0} + \delta T^P_{\tau 0} + \gamma_{\tau 0}\Psi^C_{\tau 0}\phi\pi}{r + \delta\Psi^P_{\tau 0}} + \frac{\iota_{\tau 0}\Psi^D_{\tau 0}\Sigma_{\tau 1}}{r + \delta\Psi^P_{\tau 0}} \tag{75}$$

The joint solution of the two equations is fully described in the next theorem.

Theorem 4.7. *The value function of a firm is given by*

$$V_\tau(m,n,\rho,\theta) = \rho + m\cdot\Sigma_{\tau 1} + (n-m)\cdot\Sigma_{\tau 0}$$

where

$$\begin{pmatrix}\Sigma_{\tau 1}\\ \Sigma_{\tau 0}\end{pmatrix} = A^{-1}\begin{pmatrix}\pi - sgna_{\tau 1} + \gamma_{\tau 1}\Psi^C_{\tau 1}\phi\pi + \delta T^P_{\tau 1} - \iota_{\tau 1}T^D_{\tau 1}\\ -sgna_{\tau 0} + \gamma_{\tau 0}\Psi^C_{\tau 0}\phi\pi + \delta T^P_{\tau 0} - \iota_{\tau 0}T^D_{\tau 0}\end{pmatrix} \tag{76}$$

$$A = \begin{pmatrix} r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1} & -\lambda\Psi^Q_{\tau 1}\\ -\iota_{\tau 0}\Psi^D_{\tau 0} & r + \delta\Psi^P_{\tau 0}\end{pmatrix}$$

and $sgna_{\tau m} = w[c_\iota(\iota_{\tau m}) + c_\gamma(\gamma_{\tau m}) + c_\ell(\ell_{\tau m};\tau)]$.

Startups optimal policies

The startups FOCs are exactly the same as for incumbents on a product line without a monopoly, except that they don't yet know which type τ they will be when they become an incumbent (this status gets drawn upon the discovery, but prior to the litigation).

The FOCs are:

$$c'_\iota(\iota_E) = \sum_\tau \phi_\tau\{\Sigma_{\tau 1}\Psi^D_{\tau 1}(\ell_E) - T^D_\tau(\ell_E)\} \tag{77}$$

$$c'_\gamma(\gamma_E) = \Psi^C_E(\ell^E)\phi\pi \tag{78}$$

$$c'_\ell(\ell_E;\underline{\tau}) = \iota_E\sum_\tau \phi_\tau\left\{\Sigma_{\tau 1}\frac{\partial}{\partial\ell}\Psi^D_{\tau 1}(\ell_E) - \frac{\partial}{\partial\ell}T^D_\tau(\ell_E)\right\} \tag{79}$$

4.12 Equilibrium firm, markup distribution

Observing that all policies and equilibrium objects do not depend on the firm-size distribution conditional on the mass of firms of each type, K_τ and the share of those type with monopolies ν_τ, in this subsection we characterize the steady state distribution of firms across firm and monopoly status types.

Let $\Gamma_{\tau\tau'}(m,m') = 1 - \mathbb{1}_{trial}(\tau,\tau',m)J(\ell_{\tau'm'},\ell_{\tau 1})$ denote the probability of a disruptive innovation not being blocked conditional on the incumbent is of type (τ,m) and the defendant is of type τ',m' (where in the case of an entrant, $m'=E$).

First, as the only way to have a quality leading product team without a monopoly is for a copycat to have destroyed it, and the only way for these product teams to be destroyed is a new quality innovation on the product, the net flows into $(\tau,0)$ is given by

$$0 = \lambda\nu_\tau K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^Q_{\tau'm'}\bar{J}(\ell_{\tau 1},\ell_{\tau'm'})\right) - \delta(1-\nu_\tau)K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^P_{\tau'm'}\Gamma_{\tau\tau'}(0,m')\right). \tag{80}$$

The net flows into $(\tau,1)$ are a bit more complicated since entrants discovering new innovations and becoming type τ will also contribute to the flows in, and flows out either stem from destruction of copycat or other type quality innovation.

$$
\begin{aligned}
0 =& -\lambda\nu_\tau K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^Q_{\tau'm'}\nu_{\tau'}(m')\bar{J}(\ell_{\tau 1},\ell_{\tau'm'})\right) - \delta\nu_\tau K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^P_{\tau'm'}\Gamma_{\tau\tau'}(1,m')\right)\\
&+(1-\nu_\tau)\iota_{\tau 0}K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^D_{\tau'm'}\Gamma_{\tau'\tau}(m',0)\right)+\nu_\tau\iota_{\tau 1}K_\tau\left(\sum_{\tau'}\sum_{m'}\theta^D_{\tau'm'}\Gamma_{\tau'\tau}(m',1)\right)\\
&+\phi_\tau K_E\iota_E\left(\sum_{\tau'm'}\theta^D_{\tau'm'}\Gamma_{\tau'\tau}(m',E)\right).
\end{aligned}
$$

Define $\delta^*_{\tau m} = \delta\left(\sum_{\tau'}\sum_{m'}\theta^P_{\tau'm'}\Gamma_{\tau\tau'}(m,m')\right)$ as the effective destruction rate of type τ, monopoly status m, and define ι^*_τ and η^*_τ as the effective (non-blocked entry rates given the innovator is of type τ)

Using the definition of δ^*_τ, λ^*_τ, and similarly defining ι^*_τ and η^*_τ as the effective (non-blocked entry rates given the innovator is of type τ), the above can be re-expressed more compactly as $0 = -\lambda^*_\tau\nu_\tau K_\tau - \nu_\tau\delta^*_{\tau 1}K_\tau + \iota^*_\tau K_\tau + \phi_\tau\eta^*_\tau$ or solving for K_τ :

$$
K_\tau = \frac{\phi_\tau\eta^*_\tau}{\nu_\tau\delta^*_{\tau 1} + \lambda^*_\tau\nu_\tau - \iota^*_\tau}. \tag{81}
$$

Similarly, from (80) we have that the steady state share of monopolies for type τ is:

$$
\nu_\tau = \frac{1}{\frac{\lambda^*_\tau}{\delta^*_{\tau 0}} + 1}. \tag{82}
$$

That is, the steady state monopoly share in type τ is determined by the relative rate of destruction of monopolies by copycats relative to the disruptive innovation of non-monopoly products. The monopoly share is 1 if $\lambda^* = 0$ and is decreasing in the relative rate of copycat destruction of τ with a monopoly to creative destruction for type τ without a monopoly.

4.13 Aggregate product creation / destruction rates

Firm type specific product destruction rates:

$$
\delta^*_{\tau m} = \delta\sum_{\tau'}\sum_{m'=0,1,E}\Gamma_{\tau\tau'}(m,m')\theta^P_{\tau'm'} \tag{83}
$$

or, $\delta^*_{\tau m} = \sum_{\tau'}\sum_{m'=0,1}\nu_{\tau'}(m')\iota^*_{\tau'm'}K_{\tau'} + \sum_{\tau'}\Gamma_{\tau\tau'}(m,E)\phi_{\tau'}\eta$.

The aggregate product destruction rate is

$$
\delta^* = \sum_\tau\left\{\delta^*_{\tau 1}\nu_\tau + (1-\nu_\tau)\delta^*_{\tau 0}\right\}K_\tau \tag{84}
$$

while the aggregate innovation rate, δ is

$$
\delta = \sum_\tau[\nu_\tau\iota_{\tau 1} + (1-\nu_\tau)\iota_{\tau 0}]K_\tau + \iota_E K_E. \tag{85}
$$

Similarly, the aggregate copycat innovation rate λ is

$$
\lambda = \sum_\tau[\nu_\tau\gamma_{\tau 1} + (1-\nu_\tau)\gamma_{\tau 0}]K_\tau + \gamma_E K_E. \tag{86}
$$

while the effective monopoly destruction rate λ^* is

$$\lambda^* = \sum_{\tau} \nu_\tau \lambda^*_\tau K_\tau \tag{87}$$

where

$$\lambda^*_\tau = \lambda \sum_{\tau'} \sum_{m' \in \{0,1,E\}} \theta^Q_{\tau' m'} \bar{J}(\ell_{\tau 1}, \ell_{\tau' m'}) \tag{88}$$

4.14 Labour market clearing

Aggregate labour demand by type τ with monopoly status $m = 1$ is given by

$$L_{\tau 1} = x_m + c_\iota(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_\tau(\ell_{\tau 1}) \tag{89}$$

while for the non-monopoly labour demand

$$L_{\tau 0} = x_0 + c_\iota(\iota_{\tau 0}) + c_\gamma(\gamma_{\tau 0}) + c_\tau(\ell_{\tau 0}) \tag{90}$$

and finally potential entrants labour demand is

$$L_E = c_\iota(\iota_E) + c_\gamma(\gamma_E) + c_\tau(\ell_E) \tag{91}$$

where $x_m = \frac{1}{wq} = \frac{1-\pi}{w}$ and $x_0 = \frac{1}{w}$ is the labour demand of the monopoly or non-monopoly product. There is a fixed pool of labour supply L, so that

$$L = \sum_{\tau} \Big\{ \nu_\tau L_{\tau 1} + (1 - \nu_\tau) L_{\tau 0} \Big\} K_\tau + L_E K_E \tag{92}$$

4.15 Equilibrium definition

A stationary equilibrium is a set of allocations $\{y, x_m, x_0, L_{\tau m}, \iota_{\tau m}, \gamma_{\tau m}, \ell_{\tau m}, \iota_E, \gamma_E, \ell_E, \delta^*_{\tau m}, \lambda^*_\tau, \lambda, \delta, C\}_{it}$, litigation decision rules $B_{trial}(\tau, m, \tau', m')$, matching monopoly probabilities ν_τ and prices $\{w_{it}, r_t, p_{it}, b_{it}(\tau, m, \tau', m')\}$ such that

1. all aggregate variables grow at a constant rate
2. consumers choose C_t, y_{it} to maximize lifetime utility (11)
3. firms choose $\{p_{it}, \iota_{\tau m}, \gamma_{\tau m}, \ell_{\tau m}\}$ to maximize firm value (15) and (20)
4. potential entrants optimally solve Entrant's problem with $\iota_E, \gamma_E, \ell_E$
5. litigation rules $B_{trial}(\tau, m, \tau', m')$ are a Nash equilibrium outcome in the litigation game
6. bargained royalties solve (32)
7. markets clear and beliefs are consistent, i.e.
   (a) w solves labour market clearing condition
   (b) firm and aggregate product / monopoly destruction and litigation rates, $\lambda^*_\tau, \delta^*_\tau, \lambda, \delta$ etc. are given by (83) - (88)
   (c) anticipated matching rates $\theta^C_{\tau m}, \theta^Q_{\tau m}, \theta^D_{\tau m}, \theta^P_{\tau m}$ are given by (67), (65), (54), (52) and $\theta^P_{\tau E}, \theta^Q_{\tau E}$ by (53), (66)
   (d) $K_{\underline{\tau}}, K_{\bar{\tau}}, \nu_\tau$, given by (81) , (82).

4.16 Equilibrium existence

Observe that this model nests Lentz & Mortensen (2005), by eliminating the patent litigation and accumulation of IP protection. By continuity of the equilibrium relationships described above, L large but finite an equilibrium also is guaranteed for small perturbations into this setting. Further if $\bar{\tau} = \underline{\tau}$ it is assured to be unique. For general parameters, an analogous argument to Lentz & Mortensen (2005) may be given provided the convex investment costs are sufficiently more convex than the sensitivity of judgment to IP protection expenditures (to avoid multiplicity).

5 Dynamics of Protected Market Power and Growth

The model developed in this paper implies that heterogeneity in litigation exposure and marginal costs of IP protection has important implications for the sources of market power and aggregate growth. The model links the share of monopoly products to the churn of knowledge diffusion (ratio of uninhibited copycat innovated products to disruptive innovated products). This monopoly share ν_τ in turn is tied to aggregate growth, with higher monopoly shares inducing greater innovation incentives due to the spillover effects of firms accumulating more IP protection resources in more valuable products.

To see this more precisely, observe, for a firm of size n with m monopolies, their innovation intensity is $\frac{m}{n}\iota_{\tau 1} + \frac{(n-m)}{n}\iota_{\tau 0}$. Using the solved for expressions of firm value given in Theorem 4.7, and the optimal innovation policies in Theorem 4.6, we have the following result:

Theorem 5.1. *Innovation intensities and IP protection are complements and are both strictly increasing in the share of monopolies, and strictly decreasing in the marginal costs of IP protection. That is, firm innovation intensity $I(\cdot)$*

$$I(\frac{m}{n}, \tau) = \frac{m}{n}\iota_{\tau 1} + \frac{(n-m)}{n}\iota_{\tau 0}$$

is strictly increasing in $\frac{m}{n}$ and in τ as is firm IP protection $\ell^f_\tau = \frac{m}{n}\ell_{\tau 1} + \frac{(n-m)}{n}\ell_{\tau 0}$.

Consequently, since $\nu_\tau = E[\frac{m}{n}|\tau]$ the higher this share the higher aggregate growth will be ceteris paribus. Notice from the steady state condition for ν_τ that ν_τ is declining in the knowledge diffusion churn, implying the more pervasive is market power in this sense the higher is aggregate growth. Through this churn, we see that with homogeneous type firms (which results in no equilibrium stifling of disruptive innovation) for $J = 0$ increasing patent protection and the likelihood of an injunction marginally will strictly increase growth.

From Theorem 4.7, we can decompose the value of the firm into three components:

$$\Sigma_{\tau 1} = \underbrace{\frac{\pi - sgna_{\tau 1}}{r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1}}}_{PV(\pi)} + \underbrace{\frac{\delta T^P_{\tau 1} - \iota_{\tau 1} T^D_{\tau 1}}{r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1}}}_{\textit{Factorless royalty income } R_{\tau 1}} + \underbrace{\frac{\gamma\Psi^C_{\tau 1}\phi\pi + \lambda\Psi^Q_{\tau 1}\Sigma_{\tau 0}}{r + \delta\Psi^P_{\tau 1} + \lambda\Psi^Q_{\tau 1} - \iota_{\tau 1}\Psi^D_{\tau 1}}}_{O_{\tau 1}}$$

The first component is the direct profits from production net the costs of production and costs associated with R&D and IP protection investments (SG&A). The second component is the expected factorless income component arising from the net expected royalties earned upon a rival innovating vs the costs they anticipate having to pay when they innovate. Finally, the third term consists of the option value associated with the copycat technology, where they gain the copycat profits if they innovate on a rival and transition to the non-monopoly value if a copycat innovation occurs upon them.

Using the values of $T^P, T^D, \iota_{\tau m}, \delta$ that for the high marginal cost monopolist, the expected factorless income component will always be positive since $\delta > \iota_{\tau 1}$ and $T^P_{\tau 1} > T^D_{\tau 1}$ when the probability of settlement tends to one.[20]

Theorem 5.2. *Intangible firm value from royalties*

For $Pr(B(\cdot,\cdot)) \to 1$ (i.e. the probability of fighting in trial tends to zero), (1) expected factorless royalty income $R_{\tau 1}$ is strictly positive for all monopoly producers and (2) the factorless income for $\tau = I$ is higher than that of the high marginal cost firm $R_{I1} > R_{O1}$.

5.1 Firm markup dynamics

The model suggests that markups measured by De Loecker et al. (2020) can be decomposed into a classic production-based markup over marginal costs and a royalties component which arises from cross-firm licensing of past innovations for use by product innovations. To be more precise, following the approach of De Loecker et al., from the cost minimization dual of the firm maximization problem with Cobb-Douglas production elasticity of a given factor ϑ, markups μ are given by

$$\frac{price}{marginal\ cost} = \mu = \frac{output\ elasticity\ to\ labour\ \vartheta}{factor\ revenue\ share\ \frac{wx}{py}}$$

where wx is the input factor costs and py is the firm revenue.

In my baseline model, and assuming copycat innovation is zero for simplicity, the only input is labour and output is produced one to one and given the limit pricing in a monopoly market we have that the production revenues per product are $py = px = 1$, $\vartheta = 1$ and $wx = \frac{1}{q}$, so the production based markup per-product is simply q.[21]

However, from firm balance sheet data, observed firms revenues include licensing transactions (net royalties ρ) giving us

$$\mu^f = \frac{npy + \rho}{nwx} = q + \frac{q\rho}{n}.$$

Similarly, from our firm value function decomposition (without copycats) we have the stock price of a firm is

$$V(n, \rho) = n\Sigma + \rho.$$

Using the restriction of no copycat innovation, we can solve for individual firm dynamics

$$\dot{p}_{\tau,n}(t; n_0) = (n-1)p_{n-1}(t)\iota^*_\tau + (n+1)p_{\tau,n+1}(t)\delta^*_\tau - (\iota^*_\tau + \delta^*_\tau)np_{\tau,n}(t).$$

[20] Recall that upon a disruptive innovation, the innovator always obtains a monopoly and hence the transfer the plaintiff receives is always the share of $\Sigma_{\tau' 1}$, while as a defendant they will transfer back some of the plaintiff's value, which may not be a monopoly.

[21] In this subsection, to facilitate analytical characterizations of firm dynamics, I will assume the copycat technology is sufficiently costly that the mass of non-monopolies is 0 with probability 1.

Firms with no products exit with $\dot{p}_{\tau,1}(t;n_0) = \delta^*_\tau p_{\tau,1}(t;n_0)$.

Considering the lifecycle of firm, who is born with one product, $p_{\tau,n}(t) = p_{\tau n}(t;1)$. For convenience I will drop the dependence on τ and distinction between ι^* and ι since there is no ambiguity here. Following the arguments of Klette & Kortum (2004), this differential system has a unique solution given by

$$p_0(t) = \frac{\delta}{\iota}\psi(t), \psi(t) = \frac{\iota(1-e^{-(\delta-\iota)t})}{\delta - \iota e^{-(\delta-\iota)t})}$$
$$p_1(t) = [1-p_0(t)][1-\psi(t)]$$

and $p_n(t) = p_{n-1}(t)\psi(t)$ for any $n > 1$. By induction this implies the distribution of firm size conditional on surviving to age t is geometric $\frac{p_n(t)}{1-p_0(t)} = [1-\psi(t)]\psi(t)^{n-1}$.

With this, we can return to the determination of the dynamics of the markups / value of the firm in our setting. Define $G^V_t = \frac{V_t - V_0}{V_0}$ as the growth rate of the stock (value) of a firm starting with n_0 product lines and no royalties, we then have the conditional growth rate on a firm starting with n_0

$$E[G^V_t|n_0] = e^{-(\delta-\iota)t} - 1 + \frac{E[\rho_t|n_0]}{n_0\Sigma}.$$

The evolution of net royalties ρ is given by the settlements $b()$ transacted upon a successful disruptive innovation. In general, whether (and what size) a settlement b is depends on the counterparties and their respective monopoly status. For illustrative purposes, consider a firm which upon innovating pays b^- and upon being innovated upon, receives b^+. Let A_t denote the Poisson process (rate $n_0\iota$) counting the number of product lines they innovate on up to t and D_t the Poisson process (rate $n_0\delta$) counting the number of product lines innovated upon them. Then starting at $\rho_0 = 0$,

$$\rho_t = \begin{cases} -A_t b^- + D_t b^+ & D_s - A_s < n_0 \forall s \le t \\ -A_{\hat{t}} b^- + D_{\hat{t}} b^+ & \hat{t} = \min\{s : D_s - A_s \ge n_0\} \end{cases}. \tag{93}$$

Unfortunately, we lack an exact analytic characterization for $E[\rho_t|n_0]$. However, we are able to characterize a closely related object which abstracts from the downward censored truncation. That is, consider $\hat{\rho}_t$ as given by $-A_t b^- + D_t b^+$ if $N_t > 0$ and $\hat{\rho}_t = 0$ if $N_t = 0$ with A_t, D_t independent of N_t. Then

$$E[\hat{\rho}_t|n_0] = E[-A_t b^- + D_t b^+|n_0, N_t > 0]Pr(N_t > 0|n_0) = n_0 t[-\iota b^- + \delta b^+](1 - p_0(t)^{n_0})$$

where $1 - p_0(t)^{n_0} = 1 - [\frac{\delta}{\iota}\psi(t)]^{n_0}$.

First observe that for n_0 large, the survival truncation consideration is negligible. Second, note for $n_0 = 1$, $t[1 - \frac{\delta}{\iota}\psi(t)] = t\frac{(\delta-\iota)e^{-(\delta-\iota)t}}{(\mu-\lambda)e^{-(\mu-\lambda)t}} \underset{t\to\infty}{\to} 0$, hence it follows that the expected royalties $\hat{\rho}_t$ is finite. Finally note that for small horizons t and large n_0, the differences between $\hat{\rho}_t - \rho_t$ vanish. Returning to the firm value growth expression, using this $\hat{\rho}_t$ approximation we have

$$E[G^V_t|n_0] = e^{-(\delta-\iota)t} - 1 + \frac{1}{\Sigma}t[-\iota b^- + \delta b^+](1 - p_0(t)^{n_0})$$

thus expected firm stock value growth is no longer independent from firm size n_0. Further since $\delta > \iota$ then if $b^+ > b^-$ we have expected firm stock value growth increasing in firm size n_0.

5.2 Firm size $\times$ monopoly distribution and the network of product rivals

The stationary firm type, size, monopoly and copycat product distribution $M_\tau(n, m, n_{c,I}, n_{c,O})$ can be decomposed into the firm type $\times$ size $\times$ monopoly distribution $M_\tau(n, m)$ and the conditional distribution $M^c_\tau(n_{c,I}, n_{c,O}|n, m)$. This is because the quasi-birth-death process of the firm is independent of any copied products $n_{c,I}, n_{c,O}$. The firm type $\times$ size $\times$ monopoly distribution $M_\tau(n, m)$ is pinned down by solving for the quasi-birth-death (QBD) process stationary firm-size monopoly distribution which can be computed given the equilibrium objects $\delta^*_{\tau m_i}, \iota^*_{\tau m_i}, \gamma^*_{\tau m_i}, \lambda^*_\tau, K_\tau, \nu_\tau$ and the entry type prob. parameters ϕ_τ.

Of course, given these same objects and the distribution $M_\tau(n, m)$ we can then obtain the distribution of $n_{c,I}, n_{c,O}$ which follows a simple birth-death process conditional on the τ, n, m. The aggregate destruction rate δ^* and λ^* are also pinned down by the same objects. This conditional distribution of copied products $M^c_\tau(n_{c,I}, n_{c,O}|n, m)$ implies a stochastic network of firm product rivals, defined by those whose frontier product has been copied or is copying. This network of product overlap creates a notion of firm specific product rivals, and metrics of local product concentration HHI.

Now since τ is fixed for the lifetime of the firm, I can solve separately for the distribution $M_\tau(n, m)$ which in general is a level dependent quasi-birth death process with tridagonal phase.[22] That is, the infinitisimal generator matrix of the system (characterizing the balance equations for steady state) for a given τ is (dropping the τ subscript) given by

$$Q^\tau = \begin{pmatrix} B^\tau_0 & A^\tau_0 & 0 & .... & & & \\ C^\tau_1 & B^\tau_1 & A^\tau_1 & 0 & .... & & \\ 0 & C^\tau_2 & B^\tau_2 & A^\tau_2 & 0 & ... & \\ & & & ... & & & \end{pmatrix}$$

where A_n is a matrix of dimension $n \times n+1$ governing jumps from state n to $n+1$ with columns corresponding to movement from monopoly state m to m', B_n is a matrix of dimension $n \times n$ governing no switches in the level n (but possibly switches in the phase m), and C_n is the $n \times (n-1)$ matrix governing jumps from n to $n-1$.

First, since a jump to $n+1$ occurs with a gain of a new monopoly $m+1$ every time is given by

$$(A^\tau_n)_{ij} = \begin{cases} m\iota^*_{\tau 1} + (n-m)\iota^*_{\tau 0}, & i = m, j = m+1, m = 0, .., n \\ 0 \end{cases}.$$

Second, to fall backwards in size to $n-1$ only occurs with a successful frontier shock on either a monopoly or non-monopoly line,

$$(C^\tau_n)_{ij} = \begin{cases} m\delta^*_{\tau 1} & i = m, j = m-1, m = 1, .., n \\ (n-m)\delta^*_{\tau 0}, & i = m, j = m, m = 0, .., n \\ 0 \end{cases}.$$

Third, to stay at the same size n, either (i) no shock occurs, or (ii) a copycat innovation shock succeeds on a monopoly product,

[22] The quasi-birth death process is a generalization of a birth-death process allowing for a second dimension, for details see for instance Kharoufeh (2011).

$$(B_n^\tau)_{ij} = \begin{cases} m\lambda_\tau^* & i=m, j=m-1, m=1,..,n \\ -m(\delta_{\tau 0}^* + \lambda_\tau^*) + (n-m)\delta_{\tau 0}^*, & i=m, j=m, m=0,..,n \\ 0 \end{cases} .$$

As the last piece to characterize the QBD process, the initial forward (entry) transition is

$$(C_0^\tau)_{ij} = \begin{cases} \iota_E^* \phi_\tau K_E & ,i=0, j=1 \\ 0 \end{cases}$$

and the initial local transition is the opposing

$$(A_0^\tau)_{ij} = \begin{cases} -\iota_E^* \phi_\tau K_E & ,i=0, j=0 \\ 0 \end{cases} .$$

Finally, the type conditional copycat distributions are more standard, separable birth-death processes which are destroyed when the copied producer loses their frontier product at rate $n_{c,\tau'}\delta_{\tau' 0}^*$ and born at rate $m\gamma_{\tau 1}^* \theta^C(\tau') + (n-m)\gamma_{\tau 0}^* \theta^C(\tau')$. See **?** for details of its computation.

5.3 Existence and computation of the stationary firm-size $\times$ monopoly distribution

The existence of a stationary distribution for this quasi-birth death process is akin to that of the standard linear birth death model (i.e. that the death rates exceed the birth rates), given by Neut's 'mean drift condition: $KFe < KBe$ where e is a column vector of ones and K is the stationary distribution of the irreducible Markov chain such that $KQ = 0, Ke = 1$. Unfortunately, unlike the standard linear birth death model, obtaining a closed form solution for the distribution is more challenging and seems beyond the scope of this paper.[23] Thus, in order to compute the stationary firm-size $\times$ monopoly distribution by type τ, I resort to numerical methods. To do so, I use the matrix continued fractions and the numerical algorithm recently developed by Baumann and Sandmann (2010).

6 Estimation

If the same patents and other IP protection investments both support firms own innovation and help preclude competition from rivals, then differences in firm market power will be associated with different levels of IP protection and hence innovation.

The model has sharp predictions about a one to one relationship between IP protection and market power ($\frac{m}{n}$) due to the public good nature of IP investments. However, the model's notion of market power does not map one to one to HHI or markups since firm's own incentives and effectiveness to copycat activity also increase with higher IP investments, ℓ. Consequently, identification of the investment

[23] For a discussion on conditions for existence, and a basic algorithm for computation see `https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1068.1327&rep=rep1&type=pdf`. See also a review `https://epubs.siam.org/doi/pdf/10.1137/19M1289625`. One possible exception is if somehow the $\iota(\frac{m}{n})$, δ, λ can be mapped to the processes in `https://reader.elsevier.com/reader/sd/pii/S0022247X21001086?token=59396F55593C1916F59C4C1AC3CD2CBAC8F68E155FF71BDEA0010C0E8F1F7FA809FE92A4595955CE18259192D817D1F6`.

cost functions and $a_0, a_{0,c}$ will come from investment, profits and expense moments conditional on τ and quantiles of the IP protection investments scaled by physical capital.

6.1 US firm IPP and litigation data

The theory predicts that firms with greater market power will have more IP protection and so will grow faster than firms with less profits. However, because royalty payments increase with innovations and incentives to copy also increase with market power ($\frac{m}{n}$), markups / HHI do not necessarily immediately increase with more growth.

The model is estimated on an unbalanced panel of 3,105 Compustat firms and a second event panel of IP litigation cases matched to counterparties. As in Lentz and Mortensen (2008), I will restrict to firms which appear in the initial cross-section and follow their dynamics over time. I also restrict to firms who have positive R&D or at least one patent in 2003. In the estimation, the initial observed cross-section will be interpreted as the steady state, whereas the years following will deviate from steady state due to the selection of survival across firm types / market power. Gradual exit of the initial cross-section will be leveraged for identification and since entry into the Compustat dataset suffers from a selection of older and larger public firms (which has suffered by an additional selection bias of declining listing propensities) and is not necessary for identification I omit it from the panel (outside of any litigation which may occur between an entrant and incumbent). However, by including the surviving cross-section (of 2007), any dynamic processes changing the composition of survivors will be captured in the estimation.

For the patent litigation cases data, I restrict to cases with a single parent plaintiff and defendant, cases filed in 1996 and completed by 2016. This leaves me with a sample of 16,876 unique litigation cases, of which, restricting to matched Compustat firms on both sides consists of 3,126.[24] I use the data range 1996 to 2016 for the patent litigation data, rather than restricting to the same period of the data, due to the relative sparsity of litigation cases.[25]

6.2 Targeted moments

Descriptives of the cross-section(s) I target for the estimation are given in Table 7. The first three columns pertain to the 2003 cross-section of US R&D / IP intensive firms sorted into terciles based on their IPP intensity. The second three columns pertain to the surviving subset of the 2003 cross-section in 2007. I measure IPP intensity as the number of patents assigned to the Compustat parent (directly or to one of its subsidiaries) in the past 5 years. All variables besides indicator variables (fraction inhouse / survivors) are de-trended for industry / year and re-centered at the 2003 unconditional mean. Due to the de-trending, average IP protection intensity (patents to physical capital) for the lowest tercile in both 2003 and 2007 is below zero reflecting a substantial number of firms without any patents assigned in the past five years. Since patent count is just one measure of IPP and imperfectly tracks to the full IPP of the firm, I will not use it for the estimation beyond its use to rank the firms IP protection investment levels.

I identify in-house firms as those with average share of patents filed done by an inhouse lawyer above 8% or if the firm includes a CLO amongst their top 5 most paid

[24]Note that this is about twice the number of cases as covered by Lee et al. (2019a) who cover the interval between 2000 - 2006 using hand-collected data from PACER.

[25]I start in 1996 due to a limitation on the Hoberg-Phillips product similarity data and for the sake of consistency with the diff in diff regressions I run around litigation cases with the product rival HHI.

executives.[26] Note, that as my measures of inhouse are imperfect proxies, like the IPP investments, I will not use the level of them to identify anything, but instead will use the correlation and differential dynamics of those firms which are identified as inhouse from those which are not. HHI is taken to be the Hoberg-Phillip (2016) product rivals with min score $\geq 1/12$ and firms required to be in the sample, and set to 1 for firms which have product rival scores, but not above the cutoff level or in the existing firm set. As was discussed earlier, to reflect total firm value, Tobins Q is measured as equity value + long-term debt -current assets, $(CSHO * PRCC_F + DLLT + DLC - ACT)/PPENT$.[27]

In addition to the moments in this table, I use the full sample estimates of the court regressions, the 1996 to 2016 sample estimates of the judge bias IV's and diff and diff's presented in the earlier section as additional moments to target for the estimation.

6.3 Model estimator

An observation of the panel is given by

$$\psi_{it}^{P} = \{Profits_{it}, R\&D_{it}, SG\&A_{it}, PP\&E_{it}, Wage_{it}, Sales_{it}, Stock\ value_{it}, ...$$

$patent\ stock_{it}$, in-house$_{it}$, {product similarity$_{ijt}$}, # plaintiff litigations$_{it}$, # defendant litigations$_{it}$} and an observation of the litigation event panel is

$$\psi_{ijt}^{L} = \{judge_{ijt}, \text{plaintiff covariates}_i, \text{defendant covariates}_j\}.$$

Let $\psi_i = \{\psi_{it}^P, \psi_{it}^L\}_{t=0,..,T}$ and $\psi = \{\psi_i\}_{i=1,..N}$. The model is estimated by indirect inference. Define $\Psi(\psi)$ as the vector of auxiliary parameters.

I produce a simulated panel $\tilde{\psi}^S(\Theta)$ based on the parameters Θ. The model simulation draws the initial firm distribution from the steady state, and then simulates dynamics by drawing the destruction shocks to each incumbent along with the identity of the rival who innovated upon them.

The simulated auxiliary parameters are then given by

$$\tilde{\Psi}^S(\Theta) = \frac{1}{S}\sum_{s=1}^{S} \Psi(\tilde{\psi}(\Theta)^s)$$

where S is the number of simulated repetitions.

The estimator is the choice of model parameters which minimizes the weighted distance between the data and auxiliary parameters, with $g(\Theta) = (\tilde{\Psi}^S(\Theta) - \Psi(\psi))$

$$\widehat{\Theta} = \arg\min_{\Theta} g(\Theta)' W g(\Theta).$$

Since all targeted moments are normalized, an identity weight-matrix is used, with two modifications. First, due to computational instabilities with small sample bias concerns from short panel simulations and the equal weighting, the moment criterion in the estimation drops computed moments from the criterion which are three standard deviations removed from the median gap and second,

[26] Note while the latter is based on execucomp data and hence restricted to roughly the S&P 1500 firms, the in-house lawyer segment is identified from patent transactions and so faces no such sample selection.

[27] Belo(2013) citetBelo2013 also use net capital PPENT for the replacement cost of capital, including in Tobins Q, due to a better mapping to the model. Notice that for a firm in the model, $\frac{PPENT}{sales} = \frac{\kappa[m+(n-m)s_0+n_c s_c]}{[m+(n-m)s_0+n_c s_c]}$.

6.4 Model parameters and endogenous objects

The model is characterized by the following set of parameters:

$$\Theta = \{r, q, \phi_H, J_0, J_{0,c}, J_1, J_2, \{(c_{0j}, c_{1j})\}_{j\in\{\iota,\gamma\}}, \{c^{\tau}_{0\ell}, c^{\tau}_{1\ell}\}, L, K_E, \kappa, s_0, s_c, \sigma_{judge}, \sigma_{idio,trial}, \{\sigma_{\tau,\ell}\}\}$$

where r is the interest rate, q is the quality increment, $\phi_L = 1 - \phi_H$ is the entry share of low IP capital, J_0, J_1, J_2 are the IP litigation judgment parameters for frontier innovation (where $J() = 1 - a()$), and $J_{0,c}, J_1, J_2$ are the parameters governing court IP enforcement for copycat infringement cases, c_{0j}, c_{1j} are the scalar and exponential R&D / copying investment cost terms, $\{c^{\tau}_{0\ell}, c^{\tau}_{1\ell}\}$ are the IPP investment cost terms for each type τ, κ if the unit cost of capital per unit of output, L is total labour supply, s_c is the copycat share of market after entry (= share of flow profits at time of entry), s_0 is the incumbent share of the market after a copycat entry, $1 - s_c - s_0$ is assumed to be the share of a competitive fringe and K_E is total fixed mass of potential entrants. ξ is a fixed cost for litigation trials.[28]

The endogenous objects from the model are:

$$X = \{w, g, \delta, \lambda, \iota^*_E, \{\iota_{\tau m}, \gamma_{\tau m}, \ell_{\tau m}, \lambda^*_{\tau m}, \delta^*_{\tau m}\}_{\tau\in\{L,H\},m\in\{0,1,E\}},$$
$$B^{disruptive}_{trial}(\tau m, \tau' m'), \{\nu_\tau, K_\tau\}_\tau, M_\tau(n, m, n_{c,I}, n_{c,O})\}.$$

Given a parameter vector Θ, simulation of the model generates time paths for ψ_i including a litigation panel. The firm type distribution ϕ_τ is taken to be a two-point discrete distribution. The cost function is parameterized as

$$c(\iota, \gamma, \ell) = c(c_\iota(\iota) + c_\gamma(\gamma) + c_{\tau,\ell}(\ell))^{\vartheta}$$

where $c_z(z) = \frac{1}{c_{0,z}} z^{c_{1,z}+1}$,

For the estimation, I assume that each unit of capital requires 1 unit of labour with price w and one unit of capital with price κ. While in the model, the distribution of sales of a product across firms is immaterial after copycat entry (since profits are zero), when mapping to the data, we must impose some structure. I assume that after copycat entry, s_0 is the share of sales retained by the original frontier innovator, s_c is the share retained by the first copycat and s_{fringe} is the share of sales retained on aggregate by a competitive fringe of producers who crowd in after the first copycat success. Thus, for a firm, sales sales is given by

$$sales_{f,t} = sales_f(n, m, R, \tau, n_c) = m \cdot 1 + (n - m)s_0 + n_c s_c + R\{R > 0\}$$

, $COGs_{f,t} = (w + \kappa) \cdot (m + (n - m)s_0 + n_c s_c)$, and PP&E is given by $\kappa(m + (n - m)s_0 + n_c s_c)$.

Assuming lump-sum transient royalties (for simplicity),[29], a firm's product rival sales concentration is given by:

$$HHI_{f,t} = \left(\frac{sales_i}{\sum_j sales_j + sales_i}\right)^2 + \sum_{i\in S_{f,t}} \left(\frac{sales_j}{\sum_j sales_j + sales_f}\right)^2$$

[28]This last parameter is useful to help pin down selection in litigation cases.

[29]Relaxing this is an interesting and likely empirically relevant exercise, but computationally costly as it requires an additional simulation per set of parameters to identify the

where $\mathbb{S}_{f,t}$ is the set of firms which firm f has a product overlap with and each firm's sales at a point in time is given by

$$Sales_{f,t} = sales_f(n, m, R, \tau, n_c) = m \cdot 1 + (n-m)s_0 + n_c s_c + R\{R > 0\}$$

given by $s_0 + s_c = 1 - s_{fringe}$. That is, s_0, s_c are parameters governing the share of sales a firm with a frontier product retains after being copied, while s_c is the share that the first copycat retains on a copied product (and residual share s_{fringe} is given by an outside fringe of copiers who have no other products and earn no profits but enter into the market after the first successful copycat). The product rivals are identified based off a minimum positive threshold pairwise firm product similarity score, set to 1/12, with product overlap being the sum of product for which either firm has copied the other.

Firm's market value is decomposed as $V_{f,t} = V(n, m, R, \tau, n_c) = R + m\Pi_{\tau 1} + (n - m)\Pi_{\tau 0}$. The firm's stock return, $G^V_{t+\Delta} = \frac{V_{t+\Delta} - V_t}{V_t}$ can be locally approximated by taking m and n to evolve as independent Poisson processes with rates $\delta_{\tau 1} + \lambda_\tau - \iota_{\tau 1}$

$$\mathbb{E}[G^V_t | n_0, m_0, \tau] = \frac{\left[\frac{m_0}{n_0}(\Pi_{\tau 1} - \Pi_{\tau 0})\left(e^{-(\delta_{\tau 1} + \lambda_\tau - \iota_{\tau 1})t} - 1\right) + \frac{n_0 - m_0}{n_0}\Pi_{\tau 0}\left(e^{-(\delta_{\tau 0} - \iota_{\tau 0})t} - 1\right)\right] + \frac{1}{n_0}\mathbb{E}[R_t | n_0, m_0]}{\frac{m_0}{n_0}(\Pi_{\tau 1} - \Pi_{\tau 0}) + \frac{(n_0 - m_0)}{n_0}\Pi_{\tau 0}}.$$

[30]

Profits $= m \cdot \pi + (n'_c - n_c)\pi_c + R\{R > 0\}$, note here that with the timing assumption in the simulation that royalties are paid lump-sum upon the innovation, and so only one direction of royalties can occur in an instant. Similarly, R&D $= w \cdot n\bar{c}_\iota(\iota_\tau(\frac{m}{n})) + \bar{c}_\gamma(\gamma_\tau(\frac{m}{n}))$ and SG&A = $w \cdot n\bar{c}_{\tau,\ell}(\ell_\tau(\frac{m}{n})) + R\{R < 0\}$. Finally, Tobins Q = $\frac{V}{PP\&E}$, where $V = R + m\Pi_{\tau 1} + (n-m)\Pi_{\tau 0} + \pi_c\{n'_c - n_c\}$.

Given the endogenous model objects, the steady state firm-size monopoly distribution is computed using properties of a QBD stationary distribution. For the forward simulation, I divide each annual period into Δ_T sub-periods, and for each incumbent firm draw (i) whether a destruction shock occurs (δ or λ) given the total destruction rate $n\delta + m\lambda$ (ii) determine the type of destruction shock and the monopoly state of the product it is on (iii) I then draw the identity of the rival firm based on the matching probabilities Θ^P, Θ^Q, (iv) draw the judge and (v) determine the outcome of the trial if it occurs by a random draw. Firm statistics are then obtained by averaging the per-sub-period statistics across the year (i.e. over Δ_T).

6.5 Identification

The interest rate is set at $r = 0.01$ (in line with the closing US one year t-bill rate of 1.24%). Since the model abstract from heterogeneous inputs to production and connects investment and production inputs through a market clearing wage, the unit cost of labour and capital will be identified simply as the sales-weighted average labour expense to employees (XLR/EMP), $w = 70.61$. While the wage w is an equilibrium object, as with LM (2008) identifying the wage upfront saves from having to do a costly second fixed point algorithm, and the parameter L (mass of labour) maps one to one with w given all the other parameters / equilibrium objects. Finally, due to current empirical challenges in separately identifying ϕ, I will fix it at $\phi = 1$ so that a copycat steals the full profits of the rival initially upon entry before competition drives away the profits.

[30] Notice that Gibrat's law (firm level growth rates being independent of firm size) is weakly violated in this model, even abstracting from the expected royalties term for any $t > 0$, except when $n_0 = m_0$ or $m_0 = 0$. However, the growth is increasing in firm size.

The stationary firm type, monopoly and copycat product distribution $M_\tau(n, m, n_{c,I}, n_{c,O})$ can be decomposed into the firm type $\times$ monopoly distribution $M_\tau(n, m)$ and the conditional distribution $M^c_\tau(n_{c,I}, n_{c,O}|n, m)$. This is because the quasi-birth-death process of the firm is independent of any copied products $n_{c,I}, n_{c,O}$. The firm type $\times$ monopoly distribution $M_\tau(n, m)$ is pinned down by solving for the QBD process stationary firm-size monopoly distribution which can be computed given the equilibrium objects $\delta^*_{\tau m_i}, \iota^*_{\tau m_i}, \gamma^*_{\tau m_i}, \lambda^*_\tau, \iota^*_E K_E$ and the entry type prob. parameters ϕ_τ. Of course, given these same objects and the distribution $M_\tau(n, m)$ we can then obtain the distribution of $n_{c,I}, n_{c,O}$ which follows a simple birth-death process conditional on the τ, n, m. Equivalently, given the other parameters, the mass of incumbent firm types M_τ maps one to one with the initial entry probability type ϕ_τ.[31] These quasi-birth-death process parameters are directly reflected in the survival patterns of firms across the terciles and the resulting distributional effects on HHI and average (scaled) investment costs.

The average level of Tobin's Q is primarily driven by the size of quality driven monopoly markups q and, as it is a ratio against physical capital, the level of κ. Variation across terciles is then driven by different proportion of product monopolies and IPP cost types τ. Given cost functions, pooled average R&D over sales and average investment costs scaled by physical capital jointly pin down κ (since the two denominators differ only by κ barring any royalty payments $R\{R > 0\}$) which then allows average Tobins Q to isolate q.

Since sales is driven by both copycat products n_c and the monopoly / copied original frontier products of the firm, while R&D efforts are only linked to their own frontier products n, variation of the R&D sales ratio across two of the terciles can pin down s_0 and s_c given the R&D cost function and the firm distribution. Similarly, local product rival sales HHI is determined by the dispersion in sales shares across firms which have copied products of each other. Moving across terciles puts increasing average frontier monopoly product shares and varies the fraction of τ firms and so the loadings on s_0, q and s_c will vary systematically across the different terciles.

From the model, increasing Tobin's Q across the IPP terciles arises solely due to $J_1 > 0$. Consequently, the magnitude of the increase in Q across terciles can pin down J_1. This variation combined with correlation between Tobin's Q and defendant litigation events speaks to the unconditional court enforcement parameters $J_0, J_{0,c}$. Hence variation in this correlation of Q and defendant litigation events across terciles will be driven by different proportions of defendants as copycats vs frontier innovators, thereby disentangling J_0 and $J_{0,c}$. The correlation of plaintiff status and returns, and plaintiff and in-house provide further variation that helps disentangle the J_0 and $J_{0,c}$ parameters.

It then remains to identify and estimate the investment cost parameters (8 unknowns). The model has sharp predictions about a one to one relationship between IP protection and market power ($\frac{m}{n}$) due to the public good nature of IP investments. That is a product line without a monopoly has different incentives to protect / defend than a product line with a monopoly.[32] Consequently, as there are three investment decisions, two monopoly states for a product and two types τ of incumbent, we have 12 equations and 8 unknowns. The variation across the within IPP tercile average investment costs scaled and R&D over sales can disentangle the four R&D cost function parameters.

[31] This conditional distribution of copied products $M^c_\tau(n_{c,I}, n_{c,O}|n, m)$ implies a stochastic network of firm product rivals, defined by those whose frontier product has been copied or is copying. This network of product overlap creates a notion of firm specific product rivals, and metrics of local product concentration HHI.

[32] However, note that the model's notion of market power does not map one to one to HHI or markups since firm's own incentives and effectiveness to copycat activity also increase with higher IP investments, ℓ.

Finally, the correlation of in-house status and IP investments and correlation of SG&A (cost of IP investments) and R&D are then sufficient to pin down the IPP investment cost parameters.

6.6 Estimation results and model fit

The estimated parameters are given in Table 8. Noting that $\gamma, \iota, \ell < 1$, we see that the copycat cost function is strictly cheaper / flatter than the frontier R&D, and that the asymmetries between the low and high IP cost firms parameters are of the same order of magnitude, but the level effect is about 5 times lower. The mass of potential entrants is found to be large, and the share of in-house on entry relatively small (but I find becomes more populated due to selection on survival in sample). The court IP enforcement function has a large slope of 0.3 and the unconditional probability of a plaintiff favourable outcome is markedly higher for a frontier innovation than for a copycat. Notice that these parameters as estimated do not just reflect the ex-post probability of litigation conditional on the two seeking a judgement, but rather also captures the blocked entry through aggressive litigation and predatory practices prior to the suit being filed. Hence, this likely reflects incumbents have more incentive to heavily litigate more disruptive innovations than say a third generic competitor.

The results of the estimation are given in Table 9. The model matches the variation of Tobin's Q across IPP terciles in the initial cross-section quite well. Average local rival sales concentration (HHI) matches quite well the mid and upper terciles, but substantially underestimates the lower tercile. This is likely driven by firms / industries like Walmart where IP protection was not a crucial determinant of value for these firms in the 2000s. The variation in the dispersion of HHI across terciles slightly undershoots across the 2003 cross-section, but matches the substantial drop in dispersion at the top tercile relative to the other two. The increasing average scaled investment costs across terciles is replicated with the moments matching reasonably in the center of the IPP distribution but undershooting in the two tails. In contrast R&D over sales, while also sharing an upward sloping pattern, is best matched in the lowest tercile of the distribution. The non-monotonic correlation of IP to inhouse status is successfully captured by the model, but is substantially more volatile than found in the 2003 data. Although the across group variation in Tobin's Q is well-matched, the model substantially overshoots the within group correlation of R&D to Tobin's Q, particularly for the bottom tercile which includes substantially less R&D intensive firms than the others. Similar to overall investment costs, correlation between R&D and SG&A is captured well in the center of the IPP distribution and qualitatively matches the inverted u pattern, but over-exaggerates, especially in the top end of the IPP distribution where the correlation becomes significantly negative. The correlations of SG&A and returns, defendant and Q and plaintiff / inhouse are all captured well in the center of the distribution but are off on the tails.

Moving to the 2007 cross-section, we see first that while there is more survival than found in the 2007 data, the model successfully captures the u-shaped pattern of survival across the 2003 terciles. The model undershoots in level but captures a relatively flat average / std of HHI in the 2007 cross-section. It also captures the flat investment costs for the bottom two terciles and relatively persistently elevated investment costs of the 2003 high tercile group. The model also does quite well in capturing the correlation of plaintiff litigation events to returns and inhouse status for the middle group in the '07 cross-section. Note that besides the survival and HHI moments, all others were weighted at 50% relative to the 2003 moments, reflecting the greater information content on the 2003 distribution in these other moments.

While not essential for the identification, as an additional validity check of model fit, I replicate the base specifications of the court ruling, judge IV and difference in differences on the simulated data. The results for the court rulings, Judge IV, and difference in differences are given in Table 10 - Table 12. The markup judge IV is replicated quite well as is the coefficient for lagged Tobin's Q. The model also successfully matches the implied court ruling constant $\hat{J}_0 = .05$ despite the structurally estimated frontier and copycat ruling functions exceeding 0.05. The model however substantially overshoots the sensitivity of court rulings, failing to capture selection effects of firms willing to pay the substantial fixed costs of trial which I have currently abstracted from. Similarly, although the diff in diff coefficients match for the plaintiff, they fail to do so for the defendant, coming from the high proportion of copycat innovation in the estimated model, and lack of selection effect for defendants willing to fight a litigation suit.

6.7 Counterfactual policy interventions and court IP enforcement reforms

In this sub-section I consider the aggregate effects of various actionable policy interventions and reforms of court IP enforcement through the lens of my estimated model. I consider four distinct but potentially actionable policy reforms. First, the removal of asymmetric IPP investments ability to influence court rulings (i.e.$J_1 = 0$) which could be plausibly implemented through the appointment of independent technical experts as adjudicators of infringement suits. Second, I consider a uniform 10% increase in court IP enforcement. Third, I consider a ban on in-house legal departments, imposing an arms-length relationship between legal arbiters and corporations (e.g. setting the τ IPP investment cost functions to be the same and equal to the higher cost one). Fourth, I consider Boldrin and Levine (2013)'s stark proposal to eliminate patents completely (setting $J_0 = J_1 = 0$). For all these counterfactuals, I hold fixed the equilibrium wage fixed, leaving only the GE entry rate to adjust.[33]

The results are summarized in Table 13. I find that improving the expertise of court adjudication by eliminating the ability of asymmetric IPP investments to influence outcomes both reduces average firm concentration by 2.4% and raises aggregate growth 3.1 percentage points. The key driver of this is the large reduction in blocked frontier innovation rate which drops from 0.042 to 0.001, with a mild increase in the copycat innovation blocking rate coming from less in-house incumbents excessively encroaching on rivals products. With the lower blocking, we have less dispersion in firm profits.

Moving to the second counterfactual, I find that an unconditional strengthening of court IP enforcement boosts growth with a near one to one trade-off in firm concentration (with a commensurate reduction in consumer surplus). This is driven by a proportionate increase in copycat innovation being blocked than frontier innovation as seen by the only 0.002 point increase in frontier blocked innovation rate but a near doubling of the copycat blocking rate. This increase in unconditional IP enforcement substantially spurs entrant innovation who have higher expected values conditional on entry as a non-inhouse incumbent, with a 8 p.p. increase in entrant innovation share.

Third, I find that banning in-house legal departments has a quite similar quantitative effect as reforming the judgement, suggesting heterogeneity in IPP capabilities and activities amongst incumbents is the fulcrum to the apparent stifling of growth. That is forcing an arms length relationship between firms and lawyers leads to the same

[33] I do this so as to isolate any GE effects in the labour market coming from the substitution between lawyers and engineers / manufacturers owing to the segmentation which seems likely to be prevalent between these occupations.

elimination of the predatory IP litigation practices as the expert adjudicators. The key difference between the two is that with symmetric IP protection costs, firms engage in wasteful expenditure in IPP to try to sway opinion in their favour while in equilibrium facing no better odds than if all firms had set IPP investments to zero.

Finally, in the last column I consider eliminating patents completely. This counterfactual has an implausibly large impact on growth and improvement of welfare. This result is likely driven by a lack of heterogeneity in the quality of incumbent producers, whereby patent rights and litigation may offer substantial benefits in terms of reallocating products and ameliorating business stealing externalities, which was found by Lentz & Mortensen (2015) to be the largest distortion in their estimated variant of the workhorse model. As such, this suggests that incorporating firm heterogeneity in innovation calibre q may be important to fully capture the global tradeoffs in the introduction / elimination of patent rights.

7 Conclusion

I study how intellectual property protection investments distort innovation incentives and contribute to firm concentration through asymmetric court IP enforcement. I measure the influence these investments have on firm dynamics using a new firm matched dataset of IPP investments and litigation. Further, using a structural model of IPP investments and litigation with Schumpeterian growth, I illustrate and quantify the entry distortions which arise with IP litigation. I find that the quantitative distortions of court IP decision making are substantial and that there may be actionable reforms which can promote firm entry and aggregate growth while also improving consumer surplus. Interesting next steps to improve the model fit and explicitly account for selection margins include (1) incorporating fixed costs in trials, (2) adding the opportunity for M&A to side-step litigation and (3) adding persistent firm heterogeneity into the quality of firm innovations.

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A Tables

Table 1: Decomposition of patent grant values with lawyer input

Dependent Variable:	Log(Patent Grant Announcement Value)				
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Inhouse lawyer	0.4793***	0.5882***	0.4793***	0.4793***	0.4345***
	(0.1192)	(0.0926)	(0.1192)	(0.1192)	(0.0894)
Log(1+ # forward citations)	0.0319***	0.0304***	0.0318***	0.0318***	0.0267***
	(0.0061)	(0.0068)	(0.0061)	(0.0061)	(0.0043)
Public law firm revenue	-0.0091***	-0.0566	-0.0091***	-0.0091***	0.0012
	(0.0026)	(0.2062)	(0.0026)	(0.0026)	(0.0030)
Public law firm revenue/employee	7.842***	27.60	7.837***	7.833***	2.978*
	(1.619)	(85.43)	(1.619)	(1.619)	(1.536)
Held by public defendant			0.0559**	0.0129	-0.0089
			(0.0285)	(0.0322)	(0.0311)
Royalties awarded total			0.1145***	0.1100***	0.1059**
			(0.0338)	(0.0337)	(0.0419)
# of litigations vs private defendants			0.0002	-0.0011	-0.0012
			(0.0041)	(0.0041)	(0.0039)
# license agreements				-0.1119*	-0.1009*
				(0.0649)	(0.0607)
# private firm licenses given				0.1018	0.0921
				(0.0986)	(0.0931)
Used by public plaintiff				0.0780**	0.0706**
				(0.0320)	(0.0311)
Fixed-effects					
Firm	Yes	Yes	Yes	Yes	Yes
Legal representative	Yes	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes	
Patent tech class	Yes	Yes	Yes	Yes	
Class X Year					Yes
Fit statistics					
Observations	556,024	419,080	556,024	556,024	556,038
R^2	0.89227	0.89409	0.89227	0.89228	0.90089
Within R^2	0.00419	0.00612	0.00422	0.00425	0.00342

One-way (consolidated_company_pdpco)) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Column 1: Baseline, Column 2: Baseline, only patents filed by NBER parent, Column 3: Baseline + litigation, Column 4: Baseline + litigation + licensing, Column 5: Column 4 + class x year FEs. Kogan et al. (2017) patent grant values matched to patent filing transaction and legal representative that represented the firm. Patent litigation data from federal judicial center patent infringement suits, licensing and patent data from USPTO patent re-assignment dataset. Patent grant dates from 1887 - 2010. Firms exclude utilities and finance.

Table 2: IP investments effects on court outcomes

Dependent Variable:	Plaintiff Judgement				
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
(Intercept)	0.0498***	0.0527***			
	(0.0025)	(0.0016)			
Log (1 + # Patents Plaintiff)	0.0061***				
	(0.0008)				
Log (1+ # Patents Defendant)	-0.0046***				
	(0.0007)				
$\text{Log}(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}})$		0.0052***	0.0035**	0.0030*	0.0034**
		(0.0005)	(0.0015)	(0.0015)	(0.0015)
Avg. Share Patents Inhouse - Plaintiff			0.0422**	0.0459**	0.0450**
			(0.0169)	(0.0202)	(0.0206)
Avg. Share Patents Inhouse - Defendant			0.0086	0.0050	0.0063
			(0.0101)	(0.0126)	(0.0130)
Plaintiff Public vs Defendant Private			0.0018	0.0018	0.0003
			(0.0109)	(0.0111)	(0.0112)
Plaintiff Private vs Defendant Public			-0.0058	-0.0025	-0.0023
			(0.0080)	(0.0077)	(0.0074)
Plaintiff Tech Concentration			0.0198**	0.0215**	0.0211*
			(0.0095)	(0.0107)	(0.0110)
Defendant Tech Concentration			-0.0073	-0.0016	-0.0011
			(0.0085)	(0.0101)	(0.0098)
Fixed-effects					
Year			Yes	Yes	Yes
Judge				Yes	Yes
District					Yes
Fit statistics					
Standard-Errors	Standard	Standard	Year	Judge	Judge
Observations	18,866	18,866	7,681	7,291	7,291
R^2	0.00512	0.00500	0.01066	0.13293	0.14297
Within R^2			0.00552	0.00605	0.00620

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Sample US federal litigation cases, plaintiff / defendant IP portfolio matched, 1996 - 2016. Column 1: Separate Patents, Column 2: Patent Differences (Baseline), Column 3: Baseline + base controls, Column 4: Column 3 + tech controls + judge fixed effects, Column 5: Column 4 + district fixed effects. Firm matched patent stock data. Firm patent holding characteristics = sum over 3 years prior to filing. Year fixed effects for termination date

Table 3: Estimated judge fixed effects column (5) of 2

	Judge fixed effects
Min.	-1.20
1st Qu.	-0.07
Median	-0.03
Mean	0.00
3rd Qu.	0.05
Max.	0.98
Std. Dev	0.21

Table 4: Estimated court ruling function across full-sample

Dependent Variable:	Plaintiff wins litigation			
Model:	(1)	(2)	(3)	(4)
Variables				
Constant	0.0593***	0.0607***	0.0604***	0.0552***
	(0.0027)	(0.0022)	(0.0033)	(0.0032)
$\text{Log}(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}})$	0.0032***	0.0036***	0.0029***	0.0040***
	(0.0006)	(0.0006)	(0.0010)	(0.0008)
Fitted legal input differences	0.0032***	0.0036***	-0.0002	0.0059***
	(0.0006)	(0.0006)	(0.0009)	(0.0008)
$[Log(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}})]^2$	2.17×10^{-5}		0.0002	-4.63×10^{-5}
	(0.0001)		(0.0002)	(0.0002)
(Fitted legal input differences)^2	0.0001		-9.77×10^{-5}	0.0003**
	(0.0001)		(0.0002)	(0.0001)
Post 2011 AIA	-0.0143***	-0.0140***		
	(0.0038)	(0.0038)		
Pre-2000	0.0329***	0.0327***		
	(0.0043)	(0.0042)		
$\text{Log}(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}})$ pre-2000		-0.0032*		
		(0.0018)		
Fitted legal input differences pre-2000		-0.0023		
		(0.0015)		
Fit statistics				
Observations	22,509	22,509	9,279	9,167
R^2	0.00854	0.00878	0.00107	0.01238
Adjusted R^2	0.00827	0.00852	0.00064	0.01195

Normal standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Full litigation sample of matched parent firm portfolios on both sides: 1978 - 2016. # patents = number of patents assigned to plaintiff / defendant over the year of the litigation and 3 prior years. Fitted legal input differences are the sum of the pooled legal input effects for plaintiff / defendant over the past 3 years extracted from the regression in column (1) of Table 1. Column (1) and (2) is on the full sample. Column (3) is the sub-sample from 1996 - 2007 and Column (4) is 2008 to 2016.

Table 5: Impact of judge bias on firm innovation and market power

Dependent Variables:	Log(R&D/PPENT)			Log(SALES/COGS)		
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Avg. last year judge fixed effects as plaintiff	0.0784**	0.0769**	0.0276**	0.0852***	0.0825***	0.0061
	(0.0301)	(0.0319)	(0.0121)	(0.0213)	(0.0216)	(0.0107)
Log(Tobins Q)	0.6574***	0.6370***	0.0798***	0.3950***	0.3867***	0.1243***
	(0.0275)	(0.0272)	(0.0261)	(0.0186)	(0.0181)	(0.0389)
Lag: Log($\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}}$)		-0.0604***	-0.0016		-0.0109	0.0027
		(0.0116)	(0.0056)		(0.0070)	(0.0055)
Defendant litigation in year		-0.0630	0.0285		0.0091	0.0110
		(0.0714)	(0.0287)		(0.0451)	(0.0310)
Plaintiff litigation in year		0.0743	0.0160		0.2557***	0.0024
		(0.0515)	(0.0262)		(0.0435)	(0.0369)
Fixed-effects						
Year	Yes	Yes	Yes	Yes	Yes	Yes
Firm			Yes			Yes
Fit statistics						
Standard-Errors	Year	Year	Firm	Year	Year	Firm
Observations	1,672	1,672	1,672	1,824	1,824	1,824
R^2	0.36592	0.37965	0.97018	0.26498	0.27933	0.93235
Within R^2	0.34461	0.35880	0.02975	0.24467	0.25942	0.04890

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Column 1: Base, Column 2: Year, Column 3: Firm FE, Column 4: Base, Column 5: Year, Column 6: Firm FE. Firm matched patent stock data. Firm patent stocks = averages over 3 years prior to filing. Year fixed effects corresponding to litigation termination date filing year.

Table 6: Differences in Differences of firm dynamics around litigation events

Dependent Variables:	Log(SALES/COGS)			Log(R&D/PPENT)		
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Variables						
Plaitiff litigation last year	0.2037***	0.1165*	0.0208	0.0710	-0.0333	-0.0208
	(0.0316)	(0.0605)	(0.0139)	(0.0587)	(0.0578)	(0.0177)
Defendant litigation last year	0.0634**	0.0548	0.0220**	0.0398	-0.0085	0.0123
	(0.0269)	(0.0410)	(0.0097)	(0.0611)	(0.0872)	(0.0138)
Both sides litigation last year	0.0596	-0.0087	-0.0196	-0.0753	-0.1915***	0.0112
	(0.0370)	(0.0421)	(0.0164)	(0.0670)	(0.0686)	(0.0226)
Ever Defendant	0.0192	0.1241**		-0.5639***	-0.3096*	
	(0.0187)	(0.0486)		(0.0292)	(0.1781)	
Ever Plaintiff	0.1212***	0.1128**		-0.0222*	-0.0311	
	(0.0075)	(0.0487)		(0.0125)	(0.0683)	
Fixed-effects						
Year	Yes	Yes	Yes	Yes	Yes	Yes
Fama-French 48 industry		Yes			Yes	
Firm			Yes			Yes
Fit statistics						
Standard-Errors	Year	Fama-French 48 industry	Firm	Year	Fama-French 48 industry	Firm
Observations	31,087	30,761	31,087	24,556	24,324	24,556
R^2	0.01636	0.33579	0.83776	0.01029	0.48421	0.92075
Within R^2	0.01306	0.00916	0.00032	0.00777	0.00704	0.00014

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Column 1: Base, Column 2: Year, Column 3: Firm FE, Column 4: Base, Column 5: Year, Column 6: Firm FE. Firm matched litigation data. Year fixed effects corresponding to litigation termination date filing year.

Table 7: Cross-section summary statistics

	2003			2007		
	L	M	H	L	M	H
Npatents intensity quantile	1.000	2.000	3.000	1.000	2.000	3.000
num survivors	1037.000	1031.000	1037.000	729.000	667.000	783.000
avg IP ppnt	-0.172	0.040	0.251	-0.159	0.009	0.366
avg TobinsQ	21.737	57.387	67.816	13.771	46.771	104.136
avg net income scaled	-0.028	0.022	0.067	0.050	0.101	0.083
avg HHI	0.864	0.872	0.869	0.874	0.874	0.869
std HHI	0.181	0.183	0.174	0.163	0.170	0.165
fraction inhouse	0.125	0.102	0.273	0.198	0.180	0.328
avg invest costs	0.085	0.108	0.263	0.160	0.151	0.136
std invest costs	1.904	0.362	1.371	1.490	0.577	4.487
avg rnd sales	0.355	1.434	2.362	0.204	4.120	6.050
avg num plaintiff cases	0.084	0.048	0.273	0.024	0.065	0.440
avg num defendant cases	0.074	0.103	0.348	0.004	0.119	0.605
cor inhouse markups	-0.027	-0.044	-0.006	-0.013	-0.015	0.011
cor inhouse IP	0.020	-0.128	0.115	0.107	-0.007	-0.024
corr rnd Q	0.471	0.796	0.893	0.432	0.265	0.930

Summary statistics of R&D-intensive, non-financial / utility firms from 2003 sorted by 2003 IP intensity terciles. IP intensity is defined as number of patents assigned to parent over the past 3 years (ie since 2000 for 2003 sample). All variables (besides identifier indicators) are industry x year detrended with the 2003 average added back in. Columns 1 - 3 summarize the 2003 cross-section taken to be steady state and the last columns pertain to the surviving set of these same firms in 2007.

Table 8: Table of parameter estimates / values
ϕ, w and r are fixed outside of the estimation

w	q	s_0	s_c	ϕ	a_ι	α_ι	a_γ	α_γ	a_{ℓ_I}
70.61	9.512	0.045	0.076	1.000	0.006	1.759	0.268	18.691	0.005
r	a_{ℓ_O}	α_{ℓ_I}	α_{ℓ_O}	J_0	J_1	$J_{0,copy}$	K_E	κ	ϕ_I
0.01	0.001	11.827	11.446	0.762	0.300	0.097	2.470	9.802	0.230

Table 9: Target and Simulated Moments for 2003 Firm Cross-Section

	L - D	L - S	M - D	M - S	H - D	H - S	L - D (07)	L - S (07)	M - D (07)	M - S (07)	H - D (07)	H - S (07)
num survivors	1.000	1.000	1.000	1.000	1.000	1.000	0.703	0.956	0.647	0.833	0.755	0.999
avg TobinsQ	21.737	19.837	57.387	64.292	67.816	71.742	13.771	50.511	46.771	51.794	104.136	54.240
avg HHI	0.864	0.658	0.872	0.852	0.869	0.888	0.874	0.786	0.874	0.794	0.869	0.800
std HHI	0.181	0.222	0.183	0.220	0.174	0.191	0.163	0.233	0.170	0.225	0.165	0.228
avg invest costs scaled	0.085	0.037	0.108	0.122	0.263	0.136	0.160	0.095	0.151	0.098	0.136	0.102
avg rnd / sales	0.355	0.299	1.434	0.954	2.362	1.125	0.204	0.779	4.120	0.774	6.050	0.844
avg net income scaled	-0.028	-0.007	0.022	-0.112	0.067	-0.124	0.050	-0.076	0.101	-0.082	0.083	-0.085
cor inhouse IP	0.020	0.075	-0.128	-0.838	0.115	0.654	0.107	0.132	-0.007	0.060	-0.024	0.047
corr rnd Q	0.471	0.981	0.796	0.919	0.893	0.988	0.432	0.997	0.265	0.998	0.930	0.994
corr rnd sgna	0.370	0.940	0.791	0.693	0.509	-0.014	0.246	0.930	-0.420	0.969	0.809	0.936
corr sgna returns	0.107	-0.019	0.135	0.231	0.122	-0.026	-0.138	-0.004	-0.118	0.011	-0.117	-0.030
corr inhouse returns	-0.110	-0.007	-0.086	0.268	-0.123	-0.003	0.027	-0.033	-0.018	-0.074	0.052	-0.074
corr plaintiff term returns	-0.049	0.003	-0.027	-0.184	-0.090	-0.037	-0.010	0.131	0.027	0.075	0.062	0.089
corr defendant term Q	-0.031	0.083	-0.039	-0.053	-0.022	-0.236	-0.042	0.010	-0.020	0.011	-0.011	0.029
cor plaintiff inhouse	0.132	0.039	0.147	0.149	0.194	0.013	0.075	0.005	0.132	0.129	0.119	0.038

NB: 2003 firm cross-section(s) target and simulated moments for 2003 and 2007. Firms are split in terciles of ℓ after the first sample year. Summary stats for these terciles are then computed on both the initial cross-section of 2003 and for the subset of surviving firms from this sample again in 2007.

Table 10: Replicated baseline Judge IV regression on simulated (S) data

	Judge Bias - D	Judge Bias - S	Q - D	Q - S
R&D	0.0784	-0.087085	0.6574	0.74619
Markups	0.0852	0.034371	0.395	0.18083

Table 11: Simulated vs data regression coefficients of litigation event Diff-in-Differences

	Data (D)	Sim (S)
plaintiff	0.0208	0.0022
defendant	0.0220	-0.0329

Table 12: Simulated vs data regression coefficients of court ruling function

	Data (D)	Sim (S)
$\hat{J}_0$	0.0527	0.0102
$\hat{J}_1$	0.0052	1.9700

Table 13: Counterfactual policy interventions

	Baseline	Expert Judges	Strong Patents	Ban In-House	Eliminate Patents
E[Q]	52.143	51.366	49.196	50.453	22.208
E[HHI]	0.805	0.786	0.824	0.787	0.758
E[gross profits]	0.098	0.094	0.092	0.094	0.029
SD[gross profits]	0.277	0.272	0.276	0.275	0.144
Entrant innovation share	0.704	0.716	0.780	0.715	0.715
Entrant copying share	0.712	0.712	0.711	0.712	0.712
Frontier block rate	0.042	0.001	0.044	0.001	0.000
Copycat innov block rate	1.131	1.267	2.535	1.105	0.000
Copycat innov. prob.	0.987	0.986	0.987	0.986	0.982
Cons. growth	0.037	0.068	0.063	0.067	0.092
Cons. level	-4.323	-4.381	-4.378	-4.378	-4.298
Welfare	-66.322	237.977	190.000	229.845	485.188

B Appendix - proofs

B.1 Proof of Theorem 2.2

Proof. Theorem 2.1(i):

Observe that if $a_1 = 0$ then the incumbent and entrant FOCs are identical and given the strict concavity of the objective function, these conditions are necessary and sufficient and uniquely pin down their investment policies. With $a_1 > 0$, $\Gamma'(\ell) = \Lambda'(\ell) = \Gamma'_E(\ell) = \Lambda'_E(\ell_E) = a_1$,

Noting symmetry amongst incumbent firms, and given that only incumbent firms can be innovated on, $\Gamma(\ell) = \mathbb{E}[a_0 + a_1(\ell - \hat{\ell})] = a_0$ and $\Gamma_E(\ell_E) = \mathbb{E}[a(\ell_E - \hat{\ell})] = a_0 + a_1(\ell_E - \ell)$. Slightly differently, as $\frac{\eta}{\delta}$ is the probability of an entrant innovating on an incumbent product, the probability of an incumbent losing an infringement suit as a plaintiff depends on whether the innovator is an incumbent or entrant. That is, again using symmetry amongst incumbent firms, $\Lambda(\ell) = a_0 + \frac{\eta}{\delta} a_1(\ell_E - \ell)$.

Thus, plugging these results into the FOCS yields for the incumbents:

$$a_0 \cdot \Pi = c'_\iota(\iota)$$

$$a_1[\iota + \delta] \cdot \Pi = c'_\ell(\ell)$$

and

$$[a_0 - a_1(\ell - \ell_E)] \cdot \Pi = c'_\iota(\iota_E)$$

$$a_1 \cdot \iota_E \cdot \Pi = c'_\ell(\ell_E).$$

Suppose by contradiction that $\iota \leq \iota_E$ then $c'_\iota(\iota) < c'_\iota(\iota_E)$ but then the LHS of the R&D FOCs imply that $a_0 - a_1(\ell - \ell_E) > a_0$, and thus $\ell < \ell_E$. But since $\delta > \iota_E + \iota$, the LHS of the incumbent IPP investment FOC $a_1[\iota + \delta] \cdot \Pi > a_1 \delta \cdot \Pi > a_1 \iota_E \Pi =$ LHS of entrant IPP FOC, but the RHS of the IPP investment FOC with $\ell < \ell_E$ requires $c'_\ell(\ell_E) > c'_\ell(\ell)$. Contradiction.

Theorem 2.1(ii):

Taking $\theta = (a_0, a_1)$ as exogenous parameters, the equilibrium conditions are summarized by the FOCs (5)- (8), equilibrium firm value Π, and the general equilibrium conditions, $\delta = \iota + K_E \iota_E$, $\eta = i_E K_E$.

Total differentiating the FOCs and Π yields the following system of equations

$$d\iota \cdot c''_\iota(\iota) = da_0 \Pi + d\Pi a_0$$

$$d\ell \cdot c''_\ell(\ell) = da_1(\iota + \delta)\Pi + d\Pi a_1(\iota + \delta) + d\iota a_1 \Pi(1 + \frac{\partial \delta}{\partial \iota}) + d\iota_E K_E a_1 \Pi$$

$$d\iota_E \cdot c''_\iota(\iota_E) = da_0 \Pi - da_1(\ell - \ell_E)\Pi + d\Pi[a_0 - a_1(\ell - \ell_E)]$$

$$d\ell_E \cdot c''_\ell(\ell_E) = da_1 \iota_E \Pi + d\iota_E a_1 \Pi + d\Pi a_1 \iota_E$$

Substituting in the $d\Pi$ equation into the FOCs, stacking these equations together yields a 4 x 1 system $\Delta F(y; \theta)$ with endogenous objects, $y = (\iota, \ell, \iota_E, \ell_E)$.

Appealing to the Implicit Function Theorem, yields the solution for the comparative statics:

$$[\frac{dy}{da_0}, \frac{dy}{da_1}] = -dF_y^{-1} dF_x$$

where

$$dF_x = \begin{pmatrix} \Pi - \frac{\Pi\, a_0\,(\delta-\iota)}{r^*} & \frac{K_E\,\Pi\, a_0\, \iota_E\,(l-\ell_E)}{r^*} \\ -\frac{\Pi\, a_1\,(\delta+\iota)\,(\delta-\iota)}{r^*} & \Pi\,(\delta+\iota) + \frac{K_E\,\Pi\, a_1\, \iota_E\,(\delta+\iota)\,(l-\ell_E)}{r^*} \\ \Pi - \frac{\Pi\,(a_0-a_1\,(l-\ell_E))\,(\delta-\iota)}{r^*} & \frac{K_E\,\Pi\,\iota_E\,(a_0-a_1\,(l-\ell_E))\,(l-\ell_E)}{r^*} - \Pi\,(l-\ell_E) \\ -\frac{\Pi\, a_1\, \iota_E\,(\delta-\iota)}{r^*} & \Pi\,\iota_E + \frac{K_E\,\Pi\, a_1\, {\iota_E}^2\,(l-\ell_E)}{r^*} \end{pmatrix}$$

and

$$dF_y^{-1} = \begin{pmatrix} -\frac{r^*}{\Pi\, a_0 + c_\iota''(\iota)\, r^*} & 0 & 0 & 0 \\ \frac{K_E\,\Pi^2\, a_0\, a_1 - K_E\,\Pi^2\, {a_1}^2\, l + K_E\,\Pi^2\, {a_1}^2\, \ell_E + \Pi\, a_1\, c_\iota''(\iota_E)\,\delta - 2\,\Pi\, a_1\, c_\iota''(\iota_E)\, r^* + \Pi\, a_1\, c_\iota''(\iota_E)\,\iota}{c_\iota''(\iota_E)\, c_\ell''(\ell)\,(\Pi\, a_0 + c_\iota''(\iota)\, r^*)} & -\frac{1}{c_\ell''(\ell)} & -\frac{K_E\,\Pi\, a_1}{c_\iota''(\iota_E)\, c_\ell''(\ell)} & 0 \\ \frac{\Pi\,(a_0 - a_1\, l + a_1\,\ell_E)}{c_\iota''(\iota_E)\,(\Pi\, a_0 + c_\iota''(\iota)\, r^*)} & 0 & -\frac{1}{c_\iota''(\iota_E)} & 0 \\ \frac{\Pi^2\, {a_1}^2\,\ell_E - \Pi^2\, {a_1}^2\, l + \Pi^2\, a_0\, a_1 + \Pi\, a_1\, c_\iota''(\iota_E)\,\iota_E}{c_\iota''(\iota_E)\, c_\ell''(\ell_E)\,(\Pi\, a_0 + c_\iota''(\iota)\, r^*)} & 0 & -\frac{\Pi\, a_1}{c_\iota''(\iota_E)\, c_\ell''(\ell_E)} & -\frac{1}{c_\ell''(\ell_E)} \end{pmatrix}$$

After some matrix algebra and simplification, the results follow.

□