

Innovation and Firm Dynamics with Patent Wars

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Abstract

Ideas are hard to protect. Court patent enforcement is uncertain and can be shaped by asymmetries in firms' intellectual property protection (IPP) investments, such as their patent portfolios or IP legal teams. I develop and empirically assess a framework where these IPP investments shape entry barriers by simultaneously inhibiting rival business stealing, and facilitating profitable entry into rival product markets. As a result, large, innovative, 'superstar' firms endogenously arise from greater IPP investment incentives tied to their existing market power and IPP investment cost advantages. However, the aggregate impact of these superstars for economic growth and consumer welfare may be negative due to excessive stifling and copying of new entrants' innovations. Using a novel firm-matched patent portfolio and IP litigation event dataset, I find evidence supporting this mechanism and that existing patent enforcement may inhibit creative destruction and harm consumer welfare.

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1 Introduction

Creative destruction, the revolutionary process by which new innovative ideas replace the products of stagnating incumbents, is a primary determinant of economic growth. Such innovations are costly to discover, but, without intellectual property protection (IPP), often easy for others to copy. Thus, patents which give the owner a monopoly, that is, the legal right to block any product which directly infringes on their intellectual property, may be essential for growth.

Enforcement of intellectual property ownership however is neither automatic or clearcut, with the lines distinguishing two ideas blurry and often requiring expertise to fully apprehend. Adjudication is done by judges and juries, who have little domain-specific knowledge on the ideas in question. Consequently, court outcomes of patent litigation suits are uncertain, with higher IPP investments (like stronger IP legal teams, more cohesively constructed patent portfolios and sophisticated legal arguments) able to sway IP litigation outcomes, both in and out of court, away from what technical experts may suggest.¹ Since a common defense strategy in IP litigation suits is to countersue for infringement on ones own IP, the same IPP investments can be used for both (i) defense: raising entry barriers for rivals attempting to innovate on their own existing products and (ii) offense: lowering the product entry barriers into rival product spaces. This dual role of IPP investments induces an economies of scale which correlates with the value of firms existing product markets. That is, the greater market power a firm has in a product space the more the value in defending those profits from competition increases with rival innovation threats and hence greater IPP incentives. Consequently, IPP asymmetries endogenously vary across firms facing different levels of competitive pressures in their product markets as well as between pre-revenue startup firms and profit-generating incumbents.

In this paper I ask to what extent does asymmetric IP enforcement shape product entry barriers, stifle startup entry and create superstar firms? To answer this question, I build and estimate an endogenous growth model with creative destruction, IPP investments and strategic IP litigation. I empirically assess the theory and estimate the model on a new firm-matched dataset of (dynamic) patent portfolios, legal counsel, federal IP litigation suits and firm balance-sheets. In a framework where it is ambiguous whether strengthening or weakening IP enforcement is growth-enhancing, I find that pooled across industries, lowering the unconditional probability of court infringement rulings

¹Amidst a large surge of patent litigation in the 19th century, an opinion article by a prominent electrical engineer in a technical periodical stated, "There are not many leading counsel who add to their professional ability the power of mastering thoroughly the scientific facts upon which they are "coached" for the occasion – there are very few who have the technical knowledge necessary to deal effectively with unexpected issues which may arise – in a complicated patent case...It has now been alleged (both in the Times and in many technical journals) that in such a suit the more wealthy of the two parties (the one may be a company with a capital of a million, the other a struggling mechanic or instrument maker) is generally found to have retained by means of a general retainer all the leading counsel who have distinguished themselves in electrical patent law, and that they, with the aid of the most experienced scientific experts available, succeed in misleading the Judges by causing them to misunderstand patent specifications and consequently give unjust judgments." The Electrician July 23, 1886, George Forbes

by 10% raises economic growth by 0.4 percentage points while raising consumption equivalents an extra 0.1 percentage point due to reduced monopoly distortions. These results qualitatively extend to an extreme policy of entirely eliminating patent protections completely across all sectors would boost growth by 1.8 p.p., lower average product sales concentration by 4.98 p.p., and culminate in a 5.2 p.p. increase in consumption equivalents. In contrast, making enforcement perfect by blocking all derivative (copied) products and allowing the entry of all innovative products results in a near doubling of economic growth but confers half of the consumer welfare gains of eliminating patent protections due to the pricing distortions from patent-protected monopolies.

My theory builds off the canonical, firm-based creative destruction growth model of Klette and Kortum (2004), where firms grow by innovating upon and stealing rivals' product monopolies. In contrast to the standard model where every innovation translates to the obsolescence of the incumbent firm's product, here I allow incumbents' to litigate the entry of the new product for infringement to either (i) obtain a stochastic court ruling which may preclude the entry of the new innovation or (ii) bargain royalties with the innovator to receive compensation for their IP embedded in the new innovation. I assume that court rulings are a function of the relative IPP investments of the incumbent and new innovator, so that asymmetries in IPP investments across the two influence the probability that the incumbent will win a product ban of the innovator.

Reflecting the common occurrence of litigation suits and countersuits in inter-firm patent disputes, IPP investments serve two purposes for the firm. First, they complement firm's R&D by protecting quasi-monopoly rents of past innovations from rival innovation and copying. Second, they help differentiate and defend their new innovations from infringement claims brought against them by incumbent litigation. The fact that IPP investments both help defend existing products and the successful entry into new product spaces introduces a feedback effect between firm's profits (i.e. market power) in existing product spaces to their innovative capacity. Consequently, with the addition of copying activity, this interplay dynamically results in high market power/ high valuation, high IPP-intensive and high growth firms.² Although these high market power firms are more innovative, their net effect on innovation may be negative due to the higher entry barriers for new, pre-revenue innovators. Furthermore, persistent asymmetries in IPP investment efficiencies, like retaining a high quality in-house IP legal team integrated with the R&D activity of the firm, can result in predatory behavior of stronger IPP incumbents 'litigating to kill' weaker rivals' innovations (akin to the killer acquisitions examined in Cunningham et al. (2021)).

Due to the tradeoffs between (i) monopoly distortion and innovation incentives and (ii) superstars higher innovative capacity vs predatory, entry inhibition behaviour, the aggregate effects of court IP enforcement on competition, aggregate growth and consumer surplus depends on the distribution of monopolies, incumbent firm types and potential entrant masses. As such, micro-measures

²High market power does not necessarily translate to higher measured markups or sales concentration at the firm level. For instance, firms own copying activity of others products lowers firm level markups and local sales concentration. This is consistent with the expansion of scope documented by Hoberg and Phillips (2021).

of firm IPP investments and their realized effects on firm dynamics are crucial to credibly inform the model's predictions on policy reforms. I construct such a dataset linking (using fuzzy string matching) Compustat firm balance sheet data with: (i) patent litigation data from the Federal Judicial Center, supplemented with the (2019) USPTO patent litigation docket dataset; (ii) patent ownership data for Compustat firms from USPTO (patent) Assignment Dataset, as well as data on registered licensing agreements. The constructed patent ownership dataset provides a contribution relative to existing alternatives (including the NBER patent dataset of 2006 which only tracks ownership at the time of patent grant) by updating the parent subsidiary relationships using Capital IQ corporate tree and M&A data, thereby giving more accurate representations of patent ownership. With these enhancements, my coverage of patent ownership increases to over 40% of US granted utility patents, compared to the NBER patent dataset's 32% coverage.³ I also construct a measure of firm's in-house lawyer capital which I use to distinguish low and high IPP investment efficiency firms. I do so by using transaction data on the legal representative filing patents on behalf of each firm and supplement with executive compensation data on chief legal officers.⁴

I measure and empirically assess the ability of IPP investments to influence firm market power and innovation dynamics for large incumbent firms found in Compustat. Using a variance decomposition of patent grant values based on identified lawyer characteristics who initially filed (and presumably constructed) the patent, I find that the lawyer input contributes about 5% to patent grant value variation and that having well-integrated in-house lawyers constructing the IP of the firm is associated with 48% higher patent grant values. I then give evidence consistent with asymmetric IPP investments and high quality in-house IP legal teams able to influence court IP rulings in their favour using matched IPP for both plaintiffs and defendants. Third, I leverage the identity of judges in federal litigation cases to provide exogenous variation on winning a patent litigation with judge 'biases' and document its effect on firm markups and R&D intensity. Finally, studying the differential dynamics of plaintiffs / defendants following litigation events against un-litigated counterparts, I find that patent litigation defendants' markups increase relative to non-litigated defendants in the year following a patent litigation suit.

I combine the model and the documented empirical associations of IPP investments, firm innovation and market power in a structural estimation of a 2003 firm cross-section. Identification of the model is based on variation in innovation incentives, survival patterns and across different intensities of IPP investments, as well as firm dynamics around litigation events and variations

³This increase in coverage is not due to differences in fuzzy string matching algorithms, as my algorithms are found to be more conservative in identifying patents on the NBER patent dataset pre-2001. Hence coverage is increasing due to expanding the set of subsidiaries owned by large public firms.

⁴That is, firm's use of in-house lawyers or including a chief legal officer (CLO) amongst their top 5 most paid executives suggest paying of a fixed cost to retain lawyers, thereby reducing the marginal cost of patenting activity. Furthermore, a primary role of chief legal officers is to reduce costs and increasing efficiencies in the legal concerns of the firm, as well as, according to Bruce Sewell (former Apple CLO), to "use [legal] risk as a competitive advantage." Source: <https://venturebeat.com/2019/06/10/apples-former-top-lawyer-1-billion-budget-enabled-high-risk-strategies/>.

in court judgements. The estimated model suggests a high threat of copying, with high entry barriers for frontier expanding innovations and mild sensitivity of court IP enforcement to IPP investment asymmetries. It also suggests that substantial asymmetries exist between incumbent firms with in-house IP departments and those who hire IP lawyers externally and on an ad hoc basis. These asymmetries result in 1 percent of innovations being inefficiently blocked, while close to three-quarters of the rents from quality improving innovation are retained by incumbents through IP litigation settlement. Empirically, based on un-targeted moments, the estimated model can rationalize 81% of the difference in sales between the average sales in the top 1% and the top 1 percentile, whereas the nested Klette and Kortum (2004) model accounts for less than 4% of the difference in the right tail.

The organization of the rest of the paper is as follows. In the remainder of this section, I discuss how this paper contributes to the existing literature. Section 2, provides a simplified version of the framework to illustrate the core mechanism. Section 3 describes the full quantitative framework used in the structural estimation. Section 4 describes the data and empirically assess the observable links between firm IPP investments, patent litigation and firm dynamics. Section 5 discusses the identification and structural estimation of the model. Section 6 quantifies the potential importance of IP protection activities in generating the superstar firm phenomenon and conducts quantitative evaluation of various counterfactual policy reforms. Section 7 concludes.

1.1 Related literature

This paper contributes and complements a number of distinct different strands of literature. First, this paper seeks to contribute to the academic and policy debate over recent aggregate trends in intangibles, market power, productivity, investment and the labour share documented by, amongst others, De Loecker et al. (2020), Bloom et al. (2020), Crouzet and Eberly (2018), and Akcigit and Ates (2021). On the one-side, Autor et al. (2020), Ayyagari et al. (2020), Crouzet and Eberly (2021) and De Ridder (2019) argue in support of a superstar firm hypothesis that rising productivity differences between the top performers and the rest in an industry have led to a generally efficient re-allocation of sales. On the other side, Gutiérrez and Philippon (2017), Gutiérrez and Philippon (2019), Cunningham et al. (2021) and Kamepalli et al. (2020) document anti-competitive / anti-trust behaviour with very different implications for policy. My paper tries to be agnostic and speak to both sides of the debate with IPP investments and litigation endogenously creating superstars and anti-competitive behaviour. It also helps rationalize: (i) the puzzling trends in the elasticities of investment to Tobin's Q examined by Crouzet and Eberly (2018), Peters and Taylor (2017), Gutiérrez and Philippon (2019), and Ward (2020),⁵ (ii) the documented expansion of scope by Hoberg and Phillips (2021) (through increased copycat activity of well-IP protected superstar firms), (iii) the rise of 'factorless' income Karabarounis and Neiman (2019), and (iv) the contrasting trends of increasing aggregated concentration and declining local concentration documented by

⁵Eisfeldt and Papanikolaou (2013) examines the role of organization capital as measured by SG&A expenditures in explaining cross-sectional variation in firm value.

Rossi-Hansberg et al. (2021) and Benkard et al. (2021). Moreover, it provides a micro-foundation (and actionable reforms) for the increased aggregate entry barriers (hence heightened firm valuation ratios and ‘investment gaps’) structurally inferred by Gutierrez et al. (2021), Corhay (2020), increased costs of innovation / declining R&D productivity (the latter documented by Bloom et al. (2020)) inferred by Cavenaile et al. (2019) and is consistent with the positive association of firm valuation ratios and market power found by Cho et al. (2021).

Second, this paper contributes to the literature more broadly assessing the effects of intellectual property protection on firm performance and innovation by providing a quantitative, general equilibrium assessment of the entry barrier distortions arising from asymmetric IP enforcement. To my knowledge, there are only two other quantitative assessments of IP litigation. Antill and Grenadier (2020) assesses the net effects of third-party financing of IP litigation suits, while Abrams et al. (2019) examines the net effect of non-practicing entities (patent trolls) in facilitating or inhibiting innovation. My paper builds and complements recent theory on IP litigation considering the dynamic and general equilibrium implications of patent litigation disputes. Leading examples include Bessen and Meurer (2006), Farrell and Shapiro (2007), Choi and Gerlach (2017) and Lemus and Temnyalov (2017) which examines on the micro-level how inadvertent infringement and litigation dispute activity between pairs of rivals affects innovation incentives. Choi and Gerlach (2017) study how the size of firm patent portfolios can affect firm R&D incentives, while and Lemus and Temnyalov (2017) study how non-practicing entities have a comparative advantage in extracting royalties from firms due to the negligible risk of countersuits jeopardizing the sale of their own products.⁶

The empirical assessment of links between patents, competition and innovation is expansive (see for instance surveys by Boldrin and Levine (2013) and Sampat (2018)) however, empirical assessments of the impacts of IP litigation disputes has only recently proliferated with more comprehensive data collection. Recent examples include Galasso and Schankerman (2018), Mezzanotti (2021) who use assignment of judges in patent appeals and a supreme court ruling to estimate the effect of litigation on incumbent firm R&D activity. Akin to Galasso and Schankerman (2018), I utilize judge assignment in federal IP litigation suits to obtain plausibly exogenous variation in winning an infringement suit. However, the focus of my paper is not on average treatment effects on incumbent firms, but to infer the effects of IP enforcement on both incumbent and latent entrant firm R&D incentives. Relatedly, Lee et al. (2019) empirically document differences in firm dynamics between firms involved in a patent litigation suit as a plaintiff and defendant using a hand-collected sample of S&P 500 inter-firm litigation suits. My paper complements theirs by examining a wider pool of litigants and studying the conditional dynamics involving asymmetric patent portfolios and legal departments. Appel et al. (2018), Caskurlu (2019) and Cohen et al. (2019) examine the excess risks and costs imposed on startups from patent trolls, with Cohen et al. (2019) in par-

⁶This theory, combined with the majority of patents asserted by NPEs originating from operating firms, suggests that the NPEs can be interpreted as outsourced patent litigation activity by the operating firms.

ticular offering evidence that patent trolls more frequently target firms with smaller legal departments, which supports my theory of in-house IP firms as being stronger opponents in litigation suits. Of course a mechanical driver of firm concentration is M&A which has been substantially linked to IP including work by Bena and Li (2014) and Fresard et al. (2020) which suggest that IP are important determinants of acquisitions and firm consolidation. Although not an investigation into IP litigation, Kempf and Spalt (2021) empirically assess the impact of non-IP, class action lawsuits on subsequent firm patenting activity and dynamics.

Due to the challenge in measuring innovation, much of the empirical literature conflates patents with innovative output. While patents by definition represent novel technical improvements to existing registered intellectual property, as highlighted by Boldrin and Levine (2013) the correlation between patents and other measures of innovation is relatively weak. Argente et al. (2020) empirically assess the link between patent filings and product creation in the consumer goods market and finds that over half of new product creation comes from non-patenting firms, and increases to 65% on a quality adjusted basis. Similarly, Reeb and Zhao (2020) examines the link between new product announcements across non-financial firms within industries and firm patenting activity, and finds evidence that differences in patenting output reflect differences in IP protection strategy of patenting or using trade-secrets as opposed to differences in new product creation. Nevertheless substantial heterogeneity in the scope and quality of patents has been documented and linked to firm value including work by Kogan et al. (2017), and Kogan et al. (2020). I further decompose their implied patent grant values using information on the legal counsel who filed the patents and the patents subsequent use in patent litigation suits, documenting a substantial association of patent grant values with court awarded royalties and the lawyers who constructed / filed the initial patent applications.

This paper builds on and contributes to the applied growth, creative destruction literature following the seminal works of Aghion and Howitt (1992) and Grossman and Helpman (1991) and later quantitative assessments such as Klette and Kortum (2004), Lentz and Mortensen (2008) and Acemoglu et al. (2018). Recent applications of this class of models have been used to examine the interplay of market power and growth dynamics. For instance, Akcigit and Ates (2021) decompose recent innovation dynamics and in their framework find that declining knowledge diffusion across firms can best rationalize observed patterns consistent with the rise of IP litigation examined in my paper. A second closely related paper is De Ridder (2019) who examines the implications of a different aspect of intangible capital, IT, on firm dynamics, market power and growth. In his model, IT / intangibles offer a cost shifting from marginal to fixed costs, giving those with high fixed costs an ability to undercut less IT-intensive innovators, thereby increasing entry barriers. Besides measuring different important aspects of intangible capital and having substantially different implications for policy, our mechanisms differ in that IPP investments are dynamic forward looking choices with feedback between market power and innovation as opposed to the static intangibles fixed cost choice. In another example, Olmstead-Rumsey (2019) decomposes the role of heterogeneous inno-

vations across incumbents, but treat the innovation types as exogenous draws. Finally, this paper relates to the interaction of search frictions and bargaining over ideas and their influence on growth as studied by Silveira and Wright (2010), and Chiu et al. (2017).

My paper also provides a technical contribution to the endogenous growth literature by examining dynamic efficiency tradeoffs of firm investments into expanding market power and obtain a cross-sectional distribution of firm markups generated by firm dynamics. Peters (2020) studies the impacts of firm investment in expanding market power which increases the monopoly distortion through higher markups over marginal costs, whereas in my setting, investments in IP protection inhibit the entry of new products (thereby stifling knowledge spillovers), but reduces the business stealing externality arising from creative destruction through surplus splitting royalties. In addition to examining the effects of investment on different externalities, my paper provides a technical contribution of obtaining the firm-size, markup distribution using advances in applied math on Quasi-Birth Death Processes (for details, see Bean and Latouche (2010), Baumann and Sandmann (2010), Kharoufeh (2011) and Cordeiro et al. (2019)). This technical contribution allows for the study of non-trivial joint dependence between market power and firm-size distributions, yielding a fatter firm-size distribution and better capturing the higher intensity of innovative activities, and declining innovation.

2 A baseline model of IPP induced entry barriers

I present in this section a stripped down version of the quantitative framework to illustrate the basic mechanism linking IPP investments to asymmetric entry barriers across firms.

Time is continuous and runs forever. There is a unit measure of horizontally differentiated products operated by distinct incumbent firms and a fixed pool of startups of mass K_S . Each incumbent has a quality leading product and earns flow profits π . Incumbents and startups can invest $c_\iota(\iota)$ in R&D for rate of discovery of ι . Upon discovery of a quality improvement on an incumbent product, the incumbent of that product litigates the innovator for infringement and wins a court product ban on the new innovator with probability $1 - a(\cdot)$. If the innovator wins, they replace the incumbent as the quality leader in that product and earn π in this additional market. For simplicity in this section, assume that this new product is sold / spun-off into a separate operating firm so that each incumbent is only operating a single product line. Suppose the probability that the court will allow entry of the new innovator is given by $a(\ell - \hat{\ell}) = a_0 + a_1(\ell - \hat{\ell}) + \varepsilon$ where ℓ is the IPP investments of the innovator and $\hat{\ell}$ is the IPP investments of the incumbent, where $a_0, a_1 > 0$ and $\mathbb{E}[\varepsilon] = 0$.⁷ Thus, if the innovator invests relatively more in IPP they are able to increase upwards the probability of their innovation fending off an infringement suit and winning the product market. Let $c_\ell(\ell)$ denote the convex costs of IPP investment. Finally, let δ denote the rate at which some rival firm innovates

⁷Note, this definition of $a(\cdot)$ holds for any $\ell, \hat{\ell}, \varepsilon$ such that $a(\cdot) \in [0, 1]$.

on a given incumbent, which in equilibrium will equal the aggregate rate of innovation across incumbents and startups, so if $\eta = \iota_S K_S$, then $\delta = \iota \cdot 1 + \eta$.

Observe that with this court ruling form, $1 - a_0$ is the unconditional probability of court deeming patent infringed (given symmetric firms) and giving the plaintiff the right to preclude entry of the innovator. Consequently, $-a_0$ can be interpreted as a strengthening of the validity and court enforcement of incumbent IP.

The investment problem facing an incumbent firm f at a given instant is:

$$r\Pi = \max_{\tilde{\iota}, \tilde{\ell}} \pi - c_{\iota}(\tilde{\iota}) - c_{\ell}(\tilde{\ell}) + \tilde{\iota}\mathbb{E}[a(\tilde{\ell} - \hat{\ell})]\Pi' - \delta\mathbb{E}[a(\hat{\ell} - \tilde{\ell})]\Pi$$

where Π is the present-value of the incumbent firm's product and Π' is the value of a new product they acquire.

The optimal incumbent firm investment decisions are then given by

$$\Gamma(\ell)\Pi' = c'_{\iota}(\iota) \quad (1)$$

$$[\iota\Gamma'(\ell) - \delta\Lambda'(\ell)]\Pi = c'_{\ell}(\ell) \quad (2)$$

where $\Gamma(\ell) = \mathbb{E}[a(\ell - \hat{\ell})]$ and $\Lambda(\ell) = \mathbb{E}[a(\hat{\ell} - \ell)]$.

The investment problem of a potential entrant is the same as an incumbent with the exception that they don't have any incumbent profits to protect π , thus, their optimal decisions are given by

$$\Gamma_E(\ell_S)\Pi' = c'_{\iota}(\iota_S) \quad (3)$$

$$[\iota_S\Gamma'_E(\ell_S) + 0]\Pi = c'_{\ell}(\ell_S) \quad (4)$$

where $\Gamma_E(\ell_S) = \mathbb{E}[a(\ell_S - \hat{\ell})]$ is the expected probability of winning a litigation suit as a defendant upon innovating on a random rival with IPP investments $\hat{\ell}$.

Given the court ruling sensitivity is linear in ℓ , we have immediately that $\Gamma'(\ell) = \Gamma'_E(\ell) = -\Lambda'(\ell) = a_1$. In addition, as all quality leading products yield the same profits, $\Pi' = \Pi$, which then re-arranging the firm's value function yields

$$\Pi = \frac{\pi - c_{\iota}(\iota) - c_{\ell}(\ell)}{r + \delta\Lambda(\ell) - \iota\Gamma(\ell)}$$

provided $r + \delta\Lambda(\ell) - \iota\Gamma(\ell) > 0$.

Further, as costs are homogenous, and using the assumed functional form of the ruling function, we get that the expected successful entry of a new innovation for existing incumbents is $\Gamma(\ell) = a_0$ (since the innovator and incumbent both invest the same amount in IPP ℓ). On the flip-side, as an incumbent being innovated (realization of a δ shock), the counterparty may be either another incumbent or entrant, with probability $\frac{\eta}{\delta}$ that it is the latter. Thus, the probability of successfully litigating / blocking a new innovator through court is $\Lambda(\ell) = a_0 + \frac{\eta}{\delta}a_1(\ell_S - \ell)$.

In contrast, for startups, the successful entry probability is $\Gamma_E(\ell_S) = \mathbb{E}[a(\ell_S - \hat{\ell})] = a_0 + a_1(\ell_S - \ell)$ which may be above or below the incumbents

probability depending on their relative levels of IPP investment. Because startups earn the same product value Π upon successful innovation, but currently lack any existing products of their own their level of IPP investments, it turns out that their IPP investments ℓ_S will be less than an incumbent who has value in IPP both in defending their own and litigating rivals innovation. Thus, $\Gamma_E(\ell_S) < \Gamma(\ell)$.

From the optimal investment policies, we can see that innovation incentives for incumbents and pre-revenue startups are distorted relative to standard innovation models. Further, intuitively since startups earn the same product value Π upon successful innovation, but currently lack any existing products of their own their level of IPP investments, their IPP investments ℓ_S will be less than an incumbent who has value in IPP both in defending their own and litigating rivals innovation. In other words, court IP enforcement which is sensitive to asymmetric IP investments implies entry barriers are endogenously higher for startups than incumbents. This implies that patent rights and court IP enforcement will have asymmetric impacts on the innovation incentives of incumbents and startups. In particular, an increase in court IP enforcement (a decrease in a_0) will unambiguously increase entrant innovation but may increase or decrease incumbent R&D investment depending on the size of the court enforcement asymmetry in protecting their incumbent products from startups relative to their discount rate.

Proposition 1. Denote $x = a_1(\ell - \ell_S)$. If $a_0 \in (0, 1)$, $a_1 > 0$, $a_0 - a_1(\ell - \ell_S) \in (0, 1)$, $r + \delta a_0 - \eta x > 0$ and c_ι, c_ℓ strictly increasing and convex, then

1. Incumbent investment will be strictly greater than startups, $\ell_S < \ell$ and $\iota_S < \iota$
2. Entrant R&D is strictly decreasing in the strength of IP enforcement, $-a_0$, while incumbent R&D increases if $r > \eta x$ and decreases otherwise

$$\frac{d\iota_S}{da_0} = \frac{r(\Pi x + c'_\iota(\iota)r^*) + \Pi\eta x(a_0 - x)}{c'_\iota(\iota_S)r^*(a_0 + r^*c'_\iota(\iota)\Pi^{-1})} > 0,^8 \quad (5)$$

$$\frac{d\iota}{da_0} = \frac{r - \eta x}{a_0 + r^*c'_\iota(\iota)\Pi^{-1}}, \quad (6)$$

where $r^* = r + (\delta - \iota)a_0 - \eta a_1(\ell - \ell_S)$, $x = a_1(\ell - \ell_S)$.

Proof. Proof given in Appendix C.1. □

These results suggests that the determination process of IP enforcement through the legal system can generate significant barriers to entry and favour some firms over others. Consequently, whether strengthening or weakening IP protections is better for growth depends on whether there is more latent innovative capacity amongst startups being blocked by the IP system or more IP incentives for incumbents. In light of recent trends of declining entry rates and some suggestion that more disruptive innovations come from new startups,

⁸Note that $a(\ell_S - \ell) = (a_0 - x)$ and as $a(\cdot)$ is a probability $a(\cdot)$ is restricted to be between 0 and 1.

based on this simplified environment it may seem that eliminating court IP enforcement (e.g. $a_0 = 1$ and $a_1 = 0$) is optimal for growth.

A crucial component of IP protection missing is the threat of copied innovations diluting the quasi-monopoly rents requisite to incur the sunk costs of R&D. Concretely, taking λ^* to denote the aggregate rate of successfully copied innovation, the value of an innovation Π becomes

$$\Pi = \frac{\pi - c_i(\iota) - c_\ell(\ell)}{r + \delta\Lambda(\ell) - \iota\Gamma(\ell) + \lambda^*}.$$

Hence, if copycat threats are sufficiently high, so $\lambda^* \rightarrow \infty$, court IP enforcement is essential to for innovation based growth to occur. Thus, optimal policy for court IP enforcement depends on the level of copycat threats λ and the degree of distortions. In the remainder of the paper, I will use new firm-matched data on IPP investments and litigation events and a quantitative model of IPP enforcement to determine the net effects of IP enforcement on competition and aggregate growth.

3 A quantitative framework of IPP activity

I present the theory of how IP investments and court IP enforcement shapes product entry barriers and outline the quantitative framework used for the structural estimation and quantification exercise. A complete description of the model and derivations are deferred to Appendix B and in Appendix C.

3.1 Adapting the simple model to the quantitative framework

The quantitative framework presented in the remainder of this section adapts the model from Section 2 into a general equilibrium, endogenous growth model. As in Klette and Kortum (2004) it allow firms to accumulate n frontier products under their span of control dynamically over time. Diverging from Klette and Kortum (2004), I introduce heterogeneity in the product competition across products with the possibility that a frontier product of a firm has been copied by a rival, which given price competition destroys the monopoly profits of the incumbent. The addition of copying therefore introduces a second dynamic state variable for a firm, m , which denotes the number of frontier products which remain un-copied, $m \leq n$.

Copying is taken to be a similar, but cheaper R&D investment technology, $c_\gamma(\cdot)$ than expanding the frontier, $c_i(\cdot)$. This copying technology when successful gives the copier the same quality product as the incumbent producer, with a short-term positive payoff obtained by undercutting the incumbent and stealing $\phi \cdot \pi$ before price competition with the incumbent drives profits for both firms to zero in this product permanently thereafter. Copied firms, though losing the monopoly profits, still retain the R&D capital including the IP and R&D team associated with their frontier product. Thus, copied firms still may retain some value beyond the cashflows for being the latest frontier innovator of a product.

Upon a product being innovated or copied, the incumbent firm has the opportunity to demand a trial in court or try to negotiate a settlement from the rival who innovated or copied their product. The innovator in turn can choose to agree to a settlement or force a trial as well. Should either elect for trial, a stochastic ruling judgement is made. As I assume that the courts can only imperfectly distinguish copying from innovation, the stochastic ruling function for the plaintiff is given by $J(\cdot)$ if facing an innovation, and $J_c(\cdot)$ if copied. That is with probability $J(\cdot)$, the court blocks the new innovators' product, while given a copied innovation, $J_c(\cdot)$ is the probability the incumbent blocks / fully recoups any damages from the copied innovation. As in the illustrative model, these probabilities depend on the relative difference in IPP investments of the innovating and incumbent product line ℓ_i, ℓ_j , e.g. $J(\hat{\ell} - \ell) = 1 - a(\ell - \hat{\ell})$.⁹ ¹⁰ If both firms agree to settle, Nash bargaining occurs where the outside option of the bargaining between the incumbent and innovator is to go to trial and obtain the same stochastic court judgement. For simplicity, Nash bargained royalties $b(\cdot)$ are paid as a perpetuity.¹¹

Firms are endowed with heterogeneous IPP investment costs $c_{\ell_\tau}(\cdot)$, $\tau \in \{O, I\}$ which applies across all their frontier product lines n , where I denotes in-house legal cost type, and O denotes outside counsel legal cost type, with the in-house legal cost type assumed to have a lower marginal cost than cheaper than the outside counsel cost type for all ℓ in some interval $[0, \bar{\ell}]$.

Besides incumbent firms already endowed with past disruptive innovations / R&D teams, there is a fixed mass K_S of potential entrants, who conduct both types of R&D, invest in IPP protection for their in-progress R&D. They are initially endowed with the high (outside counsel)marginal cost IP technology $\tau = O$, but upon successful entry as a quality leader in a product market may stochastically draw the low cost IP technology $\tau = I$.¹² The probability that upon entry they gain the lower cost, in-house IP technology is given by $\phi_I \in [0, 1]$.

3.2 Firm innovation and value

As opposed to in Section 2, a firm upon innovating will retain the new product so that at a given point in time a firm will consist of n intermediate goods that they are latest innovator to have improved the quality of (and hence own the intellectual property rights and the capital underlying the quality advantage).

⁹Note, this assumption of product level IPP is maintained primarily for tractability, but also captures the imperfect overlap of IPP across different product spaces.

¹⁰For convenience, I switch from $a(\cdot)$ to $J(\cdot)$ where $J(\cdot) = 1 - a(\cdot)$.

¹¹As will be shown in the model solution, the horizon of the payments are immaterial in equilibrium. That is, the model could be reformulated to have lump-sum, instantaneous payments with no effect to the results.

¹²This assumption about startups having the high cost is meant to capture an economies of scale / age feature that revenue generating incumbents (e.g. public firms or multinationals) typically have more regulatory and other legal concerns and hence will more likely retain an in-house legal department. Furthermore, due to their age, longer / stronger relationships with hired IP lawyers will generate cost savings. This could be endogenized by allowing for another investment which with a poisson shock allows them to transition, but would complicate the model further without much gain for the purpose at hand.

Following Klette and Kortum (2004) with each frontier product the firm retains another R&D / IP legal team so that their total investment costs scale linearly in n . Firms are assumed to be endowed with a common IPP investment cost type τ across their product lines, where $\tau \in \{O, I\}$. Allowing for the possibility of financial transfers in litigation and profits from product for which they are not the frontier producer of, firm's receive flow 'royalties' $r\rho$. Each of their $i = 1, \dots, n$ products may be in one of two competition states, $m_i \in \{0, 1\}$ where $m_i = 0$ represents perfect competition and $m_i = 1$ represents monopoly. Denote $[m_i]_n = (m_1, \dots, m_n)$ as the vector of the n monopoly states. When in monopoly state firms earn flow profits π on that that product, while in the competitive state they earn zero flow profits from that product.

Firms have three different investment activities, innovation ι , copying γ and IPP ℓ with convex cost functions $w \cdot c_z(\cdot)$ and $z \in \{\iota, \gamma, \ell_\tau\}$ where w is the variable price (wage if in labor terms) per unit of investment common across each investment activity. With a successful and non-blocked innovation, firms gain a new monopoly product so that their new size is $n + 1$ and $[m_i]_{n+1} = [m_i] \cup \{m_{n+1} = 1\}$. Conversely, should a rival successfully innovate on product i of the firm, the firm loses that product with its associated monopoly status, m_i . In contrast, with unblocked copying, the firm gets a short-term profit $\phi\pi$ from undercutting the frontier producer which increases $\rho' = \rho + \phi\pi$. On the flip-side, a firm copied permanently loses the monopoly profits from their frontier innovation (i.e. $m'_i = 0$ for i copied).

The firm Hamilton-Jacobi Bellman is given by

$$rV(n, \{m_i\}, \rho, \tau) = \max_{\iota_i, \gamma_i, \ell_i} \sum_i m_i \pi + r\rho - w \sum_i [c_\iota(\iota_i) + c_\gamma(\gamma_i) - c_\ell(\ell_i; \tau)] \quad (7)$$

$$+ \mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)] + \dot{V} \quad (8)$$

where,

$$\begin{aligned} \mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)] = & \\ & \sum_i \iota_i \cdot D_i(n, \{m_k, \ell_k\}, \rho, \tau) + \sum_i \gamma_i \cdot C_i(n, \{m_k, \ell_k\}, \rho, \tau) \\ & + \sum_i \delta \cdot P_i(n, \{m_k, \ell_k\}, \rho, \tau) + \sum_i \lambda \cdot m_i \cdot Q_i(n, \{m_k, \ell_k\}, \rho, \tau). \end{aligned}$$

The first line in (25) captures the static flow profits from each of the n products of which $m = \sum_i m_i$ are monopolies and $n - m$ are copied. The second line captures the expected net change in their firm value given their investments (taking as given the distribution of IPP investments ℓ , firm types, and the aggregate rate of rivals innovating on their products δ or copying λ). The expected growth in firm value $\mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)]$ is decomposed for each product line $i = 1, \dots, n$ into (1) D_i , the expected net value of innovating on a new product and becoming a defendant in a suit, (2) C_i , the expected net value of copying a rival's product and becoming a copycat defendant, (3) P_i , the expected net value (loss) of being innovated on by a rival giving them the opportunity to litigate as a plaintiff, and (4) Q_i the expected net value (loss)

from being a plaintiff when their product has been copied. The final term is the time-derivative of the value function.

As in Klette and Kortum (2004), the built-in homotheticity in the investment cost functions allows optimal firm decisions to be decentralized at the product line level. Since all products conditional on their competition status are homogeneous, firm values and investments are naturally a linear aggregation of the products under their portfolio plus whatever non-frontier product revenue streams they are generating. Restricting attention to stationary equilibria, where product level investment policies and values are constant given their state, and using a guess and verify approach, with a guess that firm value is given by $V_\tau(m, n, \rho) = \rho + m \cdot \Pi_{\tau 1} + (n - m) \cdot \Pi_{\tau 0}$, implies that the firms expected change in value facing a litigation on an innovation or copied product is only a function of the incumbent and innovating product values Π and their IPP investments. That is, for instance for an innovating defendant, the net expected value of innovating as a defendant is $D_i = S[V_\tau(m + 1, n + 1, \rho - b) - V_\tau(m, n, \rho)] = \Pi_{\tau 1} \cdot Pr(entry) - \mathbb{E}[b]$ where the probability of entry and expectation over royalty transfers b are taken conditional on the IPP investments, firm IPP cost type and product competition status of the innovating product team i .

Shown formally in the Appendix B, firm expected net IP litigation values on each side of an IP suit for an innovation depends only on the IPP cost type, the product competition status of the R&D team involved in the litigation suit prior to the innovation (or copying) and the level of IPP investments made for that R&D team, so that the net expected values in IP litigation for innovation can be expressed as

$$\underbrace{P_{\tau j}(\ell)}_{\text{Expected Plaintiff Gain}} = -\Pi_{\tau j} \Psi_{\tau j}^P(\ell) + T_{\tau j}^P(\ell), \quad \underbrace{D_{\tau j}(\ell)}_{\text{Expected Defendant Gain}} = \Pi_{\tau 1} \Psi_{\tau j}^D(\ell) - T_{\tau j}^D(\ell)$$

and similarly the expected net IP litigation values on on each side of an IP suit for a copied product can be expressed as

$$\underbrace{Q_{\tau j}(\ell)}_{\text{Copied Incumbent Gain}} = -(\Pi_{\tau 1} - \Pi_{\tau 0}) \Psi_{\tau 1}^Q(\ell) + T_{\tau j}^Q(\ell), \quad \underbrace{C_{\tau j}(\ell)}_{\text{Copying Defendant Gain}} = \phi \pi \Psi_{\tau j}^C(\ell) - T_{\tau j}^C(\ell)$$

where Ψ^D denotes the anticipated retention share of the product innovation for the innovating defendant, Ψ^P the retention share of the incumbent product value after being innovated on, etc. Similarly T^D (T^P) denotes the anticipated additional royalty compensation paid (received) based on the incumbent's (innovator's) product value from IP litigation. For the case of an innovating defendant, the anticipated retention share of the product innovation for the defendant is given by

$$\Psi_x^D = \sum_{\hat{x}} \theta_{\hat{x}}^D \left\{ (1 - \varphi(\hat{x}, x)) \cdot \left(1 - \frac{J(\ell_{\hat{x}} - \ell_x)}{2}\right) + \varphi(\hat{x}, x) \cdot (1 - J(\ell_{\hat{x}} - \ell_x)) \right\} \quad (9)$$

and the expected transfer is

$$T_x^D = \frac{1}{2} \sum_{\hat{x}} \theta_{\hat{x}}^D \cdot (1 - \varphi(\hat{x}, x)) \cdot J(\ell_{\hat{x}} - \ell_x) \cdot \Pi_{\hat{x}} \quad (10)$$

with θ_x^D is the probability of an innovator innovating on a type x incumbent, $\varphi(\cdot, \cdot)$ the equilibrium probability that a trial occurs between an incumbent and innovator conditional on their states x, \hat{x} . Values of Ψ^P, Ψ^Q, Ψ^C and T^P are defined similarly and given explicitly in Appendix B.

Restricting attention to stationary equilibria, where product level investment policies and values are constant given their state, and using a guess and verify approach, the value of a firm takes the form (11) stated in the following proposition.

Proposition 2. *In a stationary equilibrium, the value function of a firm with n frontier products, m monopolies and IPP cost type τ is given by*

$$V_{\tau}(m, n, \rho) = \rho + m \cdot \Pi_{\tau 1} + (n - m) \cdot \Pi_{\tau 0} \quad (11)$$

where the value of a frontier product for IPP cost type τ and competition status $j \in \{0, 1\}$ is decomposed into

$$\Pi_{\tau j} = \underbrace{U_{\tau j}}_{\text{Tangible value}} + \underbrace{R_{\tau j}}_{\text{Royalty value}} + \underbrace{O_{\tau j}}_{\text{Copycat option value}} \quad (12)$$

with

$$U_{\tau j} = \frac{\pi \cdot j - \text{costs}_{\tau j}}{r_{\tau j}}, \quad R_{\tau j} = \frac{\delta T_{\tau 1}^P - \iota_{\tau 1} T_{\tau 1}^D}{r_{\tau j}}, \quad O_{\tau j} = \begin{cases} \frac{\gamma \Psi_{\tau 1}^C \phi \pi + \lambda \Psi_{\tau 1}^Q \Pi_{\tau 0}}{r_{\tau 1}} & j = 1 \\ \frac{\iota_{\tau 0} \Psi_{\tau 0}^D \Pi_{\tau 1}}{r_{\tau 0}} & j = 0 \end{cases},$$

where costs are given by $\text{costs}_{\tau m} = w[c_{\iota}(\iota_{\tau m}) + c_{\gamma}(\gamma_{\tau m}) + c_{\ell}(\ell_{\tau m}; \tau)]$, effective discount rates $r_{\tau 1} = r + \delta \Psi_{\tau 1}^P + \lambda \Psi_{\tau 1}^Q - \iota_{\tau 1} \Psi_{\tau 1}^D$ and $r_{\tau 0} = r + \delta \Psi_{\tau 0}^P$.

With this result, we have that the value of a firm can be linearly decomposed into the value of their m monopoly products and their $n - m$ monopoly products. Further, the present value of being the latest frontier innovator on a product is pinned down by the firm IPP cost type τ and the monopoly status j and itself can be decomposed into three terms given in (12). The first component is the direct profits from production net the costs of production and costs associated with R&D and IP protection investments (SG&A). The second component, in the terminology of Karabarbounis and Neiman (2019), is the expected factorless income component arising from the net expected royalties earned upon a rival innovating vs the net extra costs they anticipate having to pay when they innovate. The third term consists of the option value associated with the copycat technology, where they gain the copycat profits if they innovate on a rival and, in the case of a monopoly product line, transition to the non-monopoly value if a copycat innovation occurs upon them.

Each of these components is scaled by the effective discount rate of the firm r_x which is given by the interest rate plus the net expected destruction rate of their product by a rival innovating, or in the case of a monopoly product, copying it. This net destruction rate depends on their expected outcomes in IP litigation, and is increasing in the aggregate innovation intensity of their rivals, and decreasing in their own innovation intensity.

Equipped with the firm value function, direct calculations yield the expected value of entering IP litigation as a defendant / plaintiff with a copied or innovated product. Thus, the optimal frontier investment incentives are given by

$$\Psi_{\tau j}^D(\ell)\Pi_{\tau 1} - T_x^D(\ell) = wc'_i(\iota_{\tau j}) \quad (13)$$

which is similar to the FOC we found in Section 2 but with the addition of the separate term T^D that depends not on the innovators value of the new product line $\Pi_{\tau 1}$ but instead on the incumbents value of the line $\Pi_{x'}$ and their anticipated anticipated probabilities of winning in court. Similarly, optimal copying activity takes into account the expected benefit

$$\Psi_x^C(\ell)\phi\pi - T_x^C(\ell) = wc'_\gamma(\gamma_{\tau j}). \quad (14)$$

Finally, optimal IPP investments are given by

$$\iota_{\tau, m} \frac{\partial \mathbb{E}[D_{\tau j}(\ell)]}{\partial \ell} + \gamma_{\tau, m} \frac{\partial \mathbb{E}[C_{\tau j}(\ell)]}{\partial \ell} + \delta \frac{\partial P_{\tau j}(\ell)}{\partial \ell} + \lambda \frac{\partial Q_{\tau j}(\ell)}{\partial \ell} = wc'_\tau(\ell). \quad (15)$$

Based on these optimal product level investment policies, aggregating product level investment policies within a firm characterizes firm level investment. Further, as straightforward arguments yield $\Pi_{\tau 1} > \Pi_{\tau 0}$ and $c_{\ell_L}(\ell) < c_{\ell_H}(\ell)$ for all $\ell \in (0, \bar{\ell})$ where $\bar{\ell}$ is some upper-bound on feasible ℓ , we have the result that firm investment intensities are strictly increasing in the fraction of monopolies $\frac{m}{n}$ and decreasing across IPP cost types τ .

Corollary 3. *In a stationary equilibrium, firm investment policies are given by $\iota_\tau(n, m)$, $\gamma_\tau(n, m)$, $\ell_\tau(n, m)$*

$$z_\tau(n, m) = m \cdot z_{\tau 1} + (n - m) \cdot z_{\tau 0}, \quad z \in \{\iota, \gamma, \ell\}$$

where $\iota_x, \gamma_x, \ell_x$ are given by (65) - (67).

Furthermore, provided $\ell_x \in (0, \bar{\ell})$, investment intensities (e.g. $\frac{\iota_\tau(n, m)}{n}$) are strictly increasing in monopoly share $\frac{m}{n}$ and decreasing in $\tau \in \{L, H\}$, i.e. $z_{\tau 1} > z_{\tau 0}$ and $z_{Lj} > z_{Hj}$.

3.3 Equilibrium IP litigation and product distribution

Having established that firm IPP cost type and the frontier product competition status are sufficient statistics for investment policies at the product level, it

remains to characterize the joint stationary distribution of product competition and IPP cost type. Without loss of generality we can decompose this joint distribution into, K_τ the probability of a firm with IPP cost type τ being the incumbent frontier producer of a product and ν_τ , the probability that conditional on that IPP cost type, the product is yielding monopoly profits to the incumbent.

With IP litigation, aggregate inflows and outflows across these IPP cost types and product competition statuses are mediated by the selection of innovations / copying activity which are blocked in court. Equilibrium IP litigation decisions to go to trial or bilaterally settle depend only on the innovation type, the relative values of the contested product line, and the IPP investments for the two firms. In the case of copying for instance, if successful the incumbent loses the monopoly product line with value $\Pi_{\tau 1}$ but gains the value of a copied product line $\Pi_{\tau 0}$, while the copier gains only $\phi\pi$. Thus, provided $\Pi_{\tau 1} - \Pi_{\tau 0} > \phi\pi$, the joint surplus for settlement between the two firms is negative, so that a trial will necessarily result in equilibrium. With innovations, the value of the contested product is the same for the two symmetric firms and so settlement is weakly optimal (strictly optimal with any fixed cost of trial which for simplicity are set to zero despite being empirically relevant). Consequently, in equilibrium trials on innovation only occur in equilibrium if there is an asymmetry in the values of the contested product such that the incumbent's value is in fact higher than the new innovator.

These IP litigation decisions affect the inflow and outflow of products across types and competition statuses. Denote by stars the effective product entry rates of that innovation type and $\theta_{x'}^P, \theta_{x'}^D, \theta_{x'}^C, \theta_{x'}^Q$ the matching probabilities to a rival product team of type x' given their IP litigation event type. The effective innovation (or copying) success for a product team with state x of a firm is given by

$$\iota_x^* = \iota_x \cdot \Gamma_x^D(\ell_x), \quad \gamma_x^* = \gamma_x \cdot \Gamma_x^C(\ell_x)$$

where product entry probability given type $x \in (\{O, I\} \times \{0, 1\}) \cup E$

$$\Gamma_x^D(\ell_x) = 1 - \sum_{\hat{x}} \varphi(\hat{x}, x) \cdot J(\ell_{\hat{x}} - \ell_x) \theta_{\hat{x}}^D, \quad \Gamma_x^C(\ell_x) = 1 - \sum_{\hat{x}} J^C(\ell_{\hat{x}} - \ell_x) \theta_{\hat{x}}^C.$$

On the flip-side, the type contingent product destruction rates for innovation and copying are

$$\delta_x^* = \delta \cdot \left[1 - \sum_{\hat{x}} (1 - \varphi(x, \hat{x}) \cdot J(\ell_x - \ell_{\hat{x}})) \cdot \theta_{\hat{x}}^P \right], \quad \lambda_\tau^* = \lambda \cdot \sum_{\hat{x}} \theta_{\hat{x}}^Q \cdot (1 - J(\ell_{\hat{x}} - \ell_x)).$$

In contrast, aggregate rates of innovating and copying is the same decomposition of innovation across incumbents and startups as seen in Section 2 but now varies across different types of incumbents,

$$\delta = \sum_{\tau} [\iota_{\tau 1} \nu_\tau + (1 - \nu_\tau) \iota_{\tau 0}] K_\tau + \iota_S K_S, \quad \lambda = \sum_{\tau} [\gamma_{\tau 1} \nu_\tau + (1 - \nu_\tau) \gamma_{\tau 0}] K_\tau + \gamma_S K_S. \quad (16)$$

Observe that a limitation of Klette and Kortum (2004) is that in their model aggregate R&D and productivity are perfectly correlated. This contrast with the data where aggregate corporate R&D / R&D intensities have grown persistently higher, while productivity growth has slowed (see for instance Bloom et al. (2020) for a recent examination of these trends). Here, with the addition of IPP investments and IP litigation we have a wedge between R&D inputs and output so that higher R&D still creates the same amount of innovative output, ι but is impeded so that only $\iota^* \leq \iota$ contributes to productivity. This mechanism differs from others like Bloom et al. (2020) who have proposed decreasing returns to scale in R&D processes. Furthermore, both the copying and IPP investment activity could plausibly be included in R&D expenditures in which case rising R&D from these sources would have no productivity correlations or in fact even negative correlations.

For steady state, the aggregate in-flows and outflows of frontier products in a particular product competition status and retained by IPP cost type τ must be equated. For monopoly products, outflows can occur either by (i) a different IPP cost type successfully innovating and taking over the incumbent firm's product or (ii) a rival destroying the monopoly by copying. In contrast for competitive products, outflows only occur when some firm innovates on the product transforming it into a monopoly status. Symmetrically, inflows for monopoly products occur when either (i) a different IPP cost type successfully inherits a product monopoly from a monopoly incumbent, or (ii) the same IPP cost type transforms a competitive product into a monopoly, while inflows into competitive products occur only within the same IPP cost type. Balancing inflows and outflows, yields the following equilibrium stationary distribution of product competition and IPP cost types.

Proposition 4. *The stationary distribution of products across incumbents is given by IPP cost type mass, K_τ and type-contingent monopoly share, ν_τ where*

$$K_\tau = \frac{\phi_\tau \eta^*}{\nu_\tau \delta_{\tau 1}^* + \lambda_\tau^* \nu_\tau - \iota_{\tau 1}^* \nu_\tau - \iota_{\tau 0}^* (1 - \nu_\tau)}, \quad \nu_\tau = \frac{1}{\frac{\lambda_\tau^*}{\delta_{\tau 0}^*} + 1} \quad (17)$$

where the aggregate product entry rate of startups is $\eta^* = \iota_S \cdot \Gamma_S^D \cdot K_S$.

Here we see that the product competition distribution conditional on IPP cost type, described by the probability of a monopoly product, ν_τ , is pinned down by the monopoly churn rate ($\frac{\lambda_\tau^*}{\delta_{\tau 0}^*}$), i.e. the ratio of the rate of successfully copied monopolies to innovation on competitive products. The higher the relative rate of copying the lower the equilibrium monopoly product distribution. The IPP cost type mass K_τ is given by the inflow of new entrants into IPP cost type $\phi_\tau \eta^*$ scaled up by a survival selection effect which is increasing in the average innovation rate of that product type $\iota_{\tau 1}^* \nu_\tau + \iota_{\tau 0}^* (1 - \nu_\tau)$ relative to the expected destruction rate of monopolies $\nu_\tau (\delta_{\tau 1}^* + \lambda_\tau^*)$.

3.4 Aggregating firm IPP activities into balanced growth

In this subsection, describe the preference and production of the final consumption good which aggregates the intermediate products and innovations by the firms analyzed above.

The economy consists of a unit continuum of differentiated goods. Consumers have symmetric Cobb-Douglas preferences across the goods so that their expenditure on each good is the same. Household's can borrow or lend at interest rate r_t and maximize their path of consumption given their present-value flow of labour income and profits from firms,

$$U_t = \max_{C_s} \int_t^\infty \log C_s e^{-\rho(s-t)} ds \quad (18)$$

s/t $P_t C_t = E_t$, $\int_s C_s ds = \int_s w_s L ds$. Optimal consumption expenditure must then solve the differential equation $\frac{\dot{E}}{E} = r_t - \rho$. Following Grossman and Helpman (1991), I choose numeraire so that $E_t = 1 \forall t$, which implies $r_t = \rho$.

I set the numeraire so that household expenditure is constant at one ($E_t = P_t C_t = 1$.) Since time is continuous there is thus a unit flow of expenditure on each good.¹³ The consumption good is an aggregate of a unit measure of intermediate goods with productivity/quality $A_t(j)$ as follows:

$$\log C_t = \int_0^1 \log(A_t(i)x_t(i)) di. \quad (19)$$

Demand for individual products (varieties) $x(i)$ is Cobb-Douglas, and treat different vintages of a given variety as perfect substitutes. Consequently, demand for a given variety i , vintage j is

$$x_{tj}(i) = \begin{cases} \frac{E_t}{p_{tj}(i)} & p_{tj}(i) = \min p_{tj'}(i) \\ 0 & o/w. \end{cases} \quad (20)$$

The productivity / quality of the intermediate goods follow a Poisson jump process, where the quality A is given by the number of disruptive innovations (jumps), N_{it} on a quality increment $q > 1$, $A_{it} = q^{N_{it}+1}$.

Intermediate good producers price compete against other vintages / variants of their product. A given product markets (for variety i), can be in two different states: monopoly (m) or competitive (c) states. In a monopoly state, there is a single intermediate good quality leader with quality $A_{it} = q^{N_{it}}$, while in the competitive state there are at least two firms with the same quality. Price competition in the monopoly state results in the unique quality leader charging $p(i) = wq$ which makes this rival indifferent to competing in the market and generates monopoly profits for the quality leader of $\pi = 1 - \frac{1}{q}$.

¹³See Klette and Kortum (2004), Lentz and Mortensen (2005) and Grossman and Helpman (1991) for more details.

Let ν denote the fraction of product markets which have a single quality leader and $1 - \nu$ the fraction which have competing quality leaders. The total equilibrium expected quantity of intermediate goods supplied is

$$\log X_t = \int_0^1 \log x(i) di = \nu \log x_m + (1 - \nu) \mathbb{E}_t[\log x_c(i)] = \nu \log\left(\frac{1}{wq}\right) + (1 - \nu) \mathbb{E}[\log\left(\frac{1}{w}\right)]. \quad (21)$$

As the level of aggregate consumption is $\log C_t = \int_0^1 \log(A_t(i)x_t(i)) di$ we have that the growth rate of aggregate consumption, δ_C , is given by

$$\frac{d\mathbb{E}[\log C_t]}{dt} = \mathbb{E} \left[\frac{d \log A_t}{dt} + \frac{d \log X_t}{dt} \right] = \log q \cdot \delta^*, \quad (22)$$

where δ^* is the endogenous product destruction rate.

This endogenous product destruction rate δ^* in the absence of litigation, as is the case in Klette and Kortum (2004), is equal to the aggregate innovation rate across incumbents and startups, δ . With litigation, $\delta^* = \delta \cdot A$ where $A \in [0, 1]$ is the share of innovations which successfully enter the product market (i.e. are not blocked by the courts). Thus, in the illustrative model, $A = \mathbb{E}[a(\ell - \hat{\ell})] = a_0 + \iota_S K_S a_1 (\ell_S - \ell)$. Similarly, define $\lambda^* = \lambda \cdot A_c$ where $A_c \in [0, 1]$ is the share of copied products which successfully enter the product market (i.e. are not blocked by the courts).

Finally, there is a fixed pool of labour supply L . Labour demand across the unit measure of products is $\int_0^1 L_i di$ with product i 's labour demand given by $L_i = y + c_i(\iota) + c_\gamma(\gamma) + c_{\ell_\tau}(\ell)$, and startup labour demand of $L_S = K_S [c_i(\iota_S) + c_\gamma(\gamma_S) + c_{\ell_\tau}(\ell_E)]$. Thus, the equilibrium wage solves $L = \int_0^1 L_i di + L_S$.

We can now turn to aggregate consumption and evaluating consumer welfare in this economy. In Corollary 5, I characterize the representative households aggregate consumption in terms of the equilibrium distribution and innovation activities of intermediate good firms and startups.

Corollary 5. *Expected consumption of the representative household is given by*

$$E[\log C_t] = \delta^* \cdot t \cdot \log q + \log\left(\frac{1}{w}\right) - \sum_{\tau} K_{\tau} \cdot \nu_{\tau} \cdot \log q$$

where

$$\delta^* = \sum_{\tau} \{ \delta_{\tau 1}^* \nu_{\tau} + (1 - \nu_{\tau}) \delta_{\tau 0}^* \} K_{\tau}. \quad (23)$$

The expected consumption highlights a classic tradeoff in innovation and competition policy where higher monopoly shares results in greater deadweight loss from monopoly pricing while from Theorem 3 the more monopoly products a firm has the higher their innovation incentives. Like Peters (2020) churn statistics in his endogenous markups model, the monopoly churn statistic here (which determines ν_{τ}) governs the . Thus, the relative rate of blocking copying activity can change the degree of mis-allocation and shape the innovation incentives.

Finally, to complete the equilibrium characterization, aggregate labour demand for product state x is given by $L_{\tau 1} = \frac{1}{wq} + c_l(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_{\ell_\tau}(\ell_{\tau 1})$, $L_{\tau 0} = \frac{1}{w} + c_l(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_{\ell_\tau}(\ell_{\tau 1})$ and for startups, $L_E = 0 + c_l(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_{\ell_\tau}(\ell_{\tau 1})$.

A stationary equilibrium is a set of allocations

$$\{x_m, x_c, L_{\tau j}, \iota_{\tau j}, \gamma_{\tau j}, \ell_{\tau j}, \iota_S, \gamma_S, \ell_E, \delta_{\tau j}^*, \lambda_\tau^*, \lambda, \delta, C\},$$

litigation decision rules $\varphi_{xx'}$, and prices $\{w, r, p_j, b(x, x')\}$ for $j \in \{0, 1\}$, $\tau \in \{L, H\}$ such that

1. all aggregate variables grow at a constant rate
2. consumers choose C_t, y_{it} to maximize lifetime utility (18)
3. firms choose p_{it} and $\{\iota_{\tau m}, \gamma_{\tau m}, \ell_{\tau m}\}$ to maximize firm value (25)
4. startups optimally solve startup problem with $\iota_S, \gamma_S, \ell_E$
5. innovation litigation rules $\varphi_{xx'}$ are a sub-game perfect Nash equilibrium outcome in the IP litigation game
6. bargained royalties $b(x, x')$ solve (29)
7. markets clear and beliefs are consistent, i.e.
 - (a) w solves labour market clearing condition
 - (b) aggregate product / monopoly destruction and litigation rates are consistent with the distribution of firm innovation / copying rates, e.g. λ and δ are given by (16)
 - (c) anticipated matching rates $\theta_{\tau m}^C, \theta_{\tau m}^Q, \theta_{\tau m}^D, \theta_{\tau m}^P$ are given by (64), (62), (51), (49) and $\theta_{\tau E}^P, \theta_{\tau E}^Q$ by (50), (63)
 - (d) K_τ, ν_τ , balance aggregate inflows / outflows (given by Theorem 4)

Observe that like Lentz and Mortensen (2008), this model nests Klette and Kortum (2004) and the extension by Lentz and Mortensen (2005) by eliminating IP litigation and copying activity (e.g. setting $J(\cdot) = 1, \phi\pi = 0$). Equilibrium existence is guaranteed in Lentz and Mortensen (2005) setting provided the investment costs $c_l(0) = c'_l(0) = 0$ and $c''_l > 0$ and aggregate labour supply L is sufficiently large. Imposing similar restrictions on the cost functions for copying and IPP investment, and restricting attention to linear functions of $J(\cdot), J_c(\cdot)$ (to guarantee a unique optimal ℓ response by firms) and fixed IP litigation outcomes, by continuity of the equilibrium relationships described above an equilibrium will also be assured existence here for small perturbations from their setting.¹⁴

¹⁴For general parameters, with $J(\cdot), J_c(\cdot)$ linear, and fixed litigation outcomes (or no IPP type heterogeneity) an analogous proof to Lentz and Mortensen (2005) can be applied with the addition of one more equation / unknown of λ^* which is driven one for one with ϕ the value of copying. Allowing for endogenous litigation strategies / decisions and IPP type heterogeneity and observing that the optimal IP litigation decision is locally constant (continuous) except at points of $\Pi_x = \Pi_{x'}, x \neq x'$, suitable restrictions in the parameter space can exclude these points and yield an equilibrium.

3.5 The cross-sectional firm size and monopoly distribution

The model equilibrium is characterized at the product line level, however, when moving to the data, most observables are firm level aggregates. As such, for the structural estimation and an empirical assessment of the theory, it is useful to characterize the distribution of equilibrium activity across firms rather than products. Two main complications arise. First, the evolution of the the number of monopolies and number of frontier products retained by the firm is a bivariate Markov process and so can't be directly solved as a birth-death process. Second, products the firm themselves have copied will affect observables like sales, but based on the model, won't affect investment incentives. In this case, must track the number of products copied, and further since the destruction rate of these copied products will vary conditional on the incumbent IPP type who was copied, the relevant firm states are $\tau, n, m, n_{c,I}, n_{c,O}$.

The stationary firm type, size, monopoly and copycat product distribution $M_\tau(n, m, n_{c,I}, n_{c,O})$ can be decomposed into the firm type \times size \times monopoly distribution $M_\tau(n, m)$ and the conditional distribution $M_\tau^c(n_{c,I}, n_{c,O} | n, m)$. This is because the quasi-birth-death process of the firm is independent of any copied products $n_{c,I}, n_{c,O}$. The firm type \times size \times monopoly distribution $M_\tau(n, m)$ is pinned down by solving for the quasi-birth-death (QBD) process stationary firm-size monopoly distribution which can be computed given the equilibrium objects $\delta_{\tau m_i}^*, l_{\tau m_i}^*, \gamma_{\tau m_i}^*, \lambda_\tau^*, K_\tau, \nu_\tau$ and the entry type prob. parameters ϕ_τ .

Of course, given these same objects and the distribution $M_\tau(n, m)$ we can then obtain the distribution of $n_{c,I}, n_{c,O}$ which follows a simple birth-death process conditional on the τ, n, m . The aggregate destruction rate δ^* and λ^* are also pinned down by the same objects. This conditional distribution of copied products $M_\tau^c(n_{c,I}, n_{c,O} | n, m)$ implies a stochastic network of firm product rivals, defined by those whose frontier product has been copied or is copying. This network of product overlap creates a notion of firm specific product rivals, and metrics of local product concentration HHI.

Now since τ is fixed for the lifetime of the firm, I can solve separately for the distribution $M_\tau(n, m)$ which in general is a level dependent quasi-birth death process with tridagonal phase.¹⁵ That is, the infinitesimal generator matrix of the system (characterizing the balance equations for steady state) for a given τ is (dropping the τ subscript) given by

$$Q^\tau = \begin{pmatrix} B_0^\tau & A_0^\tau & 0 & \dots & & \\ C_1^\tau & B_1^\tau & A_1^\tau & 0 & \dots & \\ 0 & C_2^\tau & B_2^\tau & A_2^\tau & 0 & \dots \\ & & & \dots & & \end{pmatrix}$$

where A_n is a matrix of dimension $n \times n + 1$ governing jumps from state n to $n + 1$ with columns corresponding to movement from monopoly state m to m' , B_n is a matrix of dimension $n \times n$ governing no switches in the level n (but possibly switches in the phase m), and C_n is the $n \times (n - 1)$ matrix governing jumps from n to $n - 1$.

¹⁵The quasi-birth death process is a generalization of a birth-death process allowing for a second dimension, for details see for instance Kharoufeh (2011).

First, since a jump to $n + 1$ occurs with a gain of a new monopoly $m + 1$ every time is given by

$$(A_n^\tau)_{ij} = \begin{cases} m\iota_{\tau 1}^* + (n - m)\iota_{\tau 0}^*, & i = m, j = m + 1, m = 0, \dots, n \\ 0 & \end{cases}.$$

Second, to fall backwards in size to $n - 1$ only occurs with a successful frontier shock on either a monopoly or non-monopoly line,

$$(C_n^\tau)_{ij} = \begin{cases} m\delta_{\tau 1}^* & i = m, j = m - 1, m = 1, \dots, n \\ (n - m)\delta_{\tau 0}^*, & i = m, j = m, m = 0, \dots, n \\ 0 & \end{cases}.$$

Third, to stay at the same size n , either (i) no shock occurs, or (ii) a copycat innovation shock succeeds on a monopoly product,

$$(B_n^\tau)_{ij} = \begin{cases} m\lambda_\tau^* & i = m, j = m - 1, m = 1, \dots, n \\ -m(\delta_{\tau 0}^* + \lambda_\tau^*) + (n - m)\delta_{\tau 0}^*, & i = m, j = m, m = 0, \dots, n \\ 0 & \end{cases}.$$

As the last piece to characterize the QBD process, the initial forward (entry) transition is

$$(A_0^\tau)_{ij} = \begin{cases} \iota_S^* \phi_\tau K_S & , i = 0, j = 1 \\ 0 & \end{cases}$$

and the initial local transition is the opposing

$$(B_0^\tau)_{ij} = \begin{cases} -\iota_S^* \phi_\tau K_S & , i = 0, j = 0 \\ 0 & \end{cases}.$$

Finally, the type conditional copycat distributions are more standard, separable birth-death processes which are destroyed when the copied producer loses their frontier product at rate $n_{c,\tau'}\delta_{\tau' 0}^*$ and born at rate $m\gamma_{\tau 1}^*\theta^C(\tau') + (n - m)\gamma_{\tau 0}^*\theta^C(\tau')$. See Klette and Kortum (2004) for details of its computation.

The existence of a stationary distribution for this quasi-birth death process is akin to that of the standard linear birth death model (i.e. that the death rates exceed the birth rates), given by Neut's 'mean drift condition: $KFe < KBe$ where e is a column vector of ones and K is the stationary distribution of the irreducible Markov chain such that $KQ = 0, Ke = 1$. Unfortunately, unlike the standard linear birth death model, obtaining a closed form solution for the distribution is more challenging and seems beyond the scope of this paper.¹⁶ Thus, in order to compute the stationary firm-size \times monopoly distribution by type τ , I resort to numerical methods. To do so, I use the matrix continued fractions and the numerical algorithm recently developed by Baumann and Sandmann (2010).

¹⁶For a discussion on conditions for existence, and a basic algorithm for computation see <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1068.1327&rep=rep1&type=pdf>. See also a review <https://epubs.siam.org/doi/pdf/10.1137/19M1289625>.

3.6 Model discussion

Having characterized equilibrium across products, it is of interest to highlight a few implications of IP litigation not observed in Klette and Kortum (2004) or close extensions of it. First, as already discussed, with IP litigation we can account for imperfect or even negative correlation of aggregate R&D and productivity growth that Klette and Kortum (2004) cannot account for. Second, with IPP investments the model generates a positive correlation between markups and R&D within firms which Klette and Kortum (2004), Lentz and Mortensen (2005) or even Peters (2020) with heterogeneous within firm markups captures. Third, Klette and Kortum (2004) and variants predict perfect correlation between startup entry rates and aggregate incumbent firm markups / stock values, while these two trends have diverged over the past few decades. With the endogenous asymmetries of IPP, my model can accommodate this fact.

In addition, copying introduces a well-defined notion of sales HHI amongst product rivals can be defined. Namely, a firm's average product-level HHI given n frontier products, m monopoly products and n_c copied products of rivals, is given by

$$HHI^*(n, m, n_c) = 1^2 \cdot \frac{m}{n + n_c} + s_0^2 \cdot \frac{n - m}{n + n_c} + s_c^2 \cdot \frac{n_c}{n + n_c}$$

where s_0, s_c are the shares of sales retained by the copied frontier producer and copycat respectively.¹⁷ That is, the first term for instance gives that in m of the product markets, the sales share of the firm is 1 as the monopolist, and the denominator captures the $n + n_c$ product spaces in which the firm is currently competing in. Observe that this quantity is intimately connected to the relative effective effort in copying and innovation. For illustration setting $n_c = 0 = s_c = s_0$, $\mathbb{E}[E[HHI^*(n, m, n_c)|n, m, n_c]] = \mathbb{E}[\frac{m}{n}] = \nu_\tau$ so firm sales concentration is tied to the monopoly share, which in turn from (79) pins down the effective monopoly churn rate ($\frac{\lambda_\tau^*}{\delta_\tau^*}$).

Another interesting implication is that with either royalties or copying extension, Gibrat's law is violated although with copying it depends on $\frac{m}{n}$ and so is not monotonic in firm size. The result is summarized in the next theorem. Proof and related discussion given in Appendix C.2.

Proposition 6. *With royalties alone, up to an approximation error, Gibrat's law is violated, expected firm stock value growth is no longer independent from firm size n_0 . Further since $\delta > \iota$ then expected firm stock value growth may be increasing in firm size n_0 .*

¹⁷I distinguish between the theoretical product level HHI^* here and the observed/ empirical counterpart used of firm-level HHI which includes not only the $n + n_c$ product markets of the firm in question, but also contaminated with the rival firm's product sales in different products. Greater details provided in the estimation.

4 Measuring IPP and their links to firm dynamics

4.1 Data construction

The primary data for this paper combines patent litigation, and patent ownership transfers data with balance sheet, and executive compensation data for public firms. The patent litigation data is obtained from the civil suits database of Federal Judicial Center (FJC) Integrated Database (obtained from WRDS) supplemented with the (2019) USPTO patent litigation docket dataset described in Marco et al. (2017) and Schwartz et al. (2019). Patent transaction data is obtained from the USPTO (Re-) Assignment Dataset which tracks the ownership of patents across time since the time of initial filing, and covers the universe of patents filed in the US (described by Graham et al. (2018)). The resulting dataset firm-matched dataset differs from the initial NBER patent dataset and a number of extensions like that of Kogan et al. (2017), in that it updates parent-subsidary relationships that have evolved since the initial NBER patent dataset construction using 2001 corporate relationship structures.

An initial impediment to utilizing these rich micro-datasets is a lack of firm level identifiers for these administrative datasets. Furthermore, in the midst of a surge of M&As and expanding size of public firms, accurate characterization of firms patent portfolio and litigation exposures should take into account evolving parent-subsidary relationships. To tackle these data challenges, I follow a similar broad approach to Hall (2005) by using fuzzy string matching between the patent assignment database and utilizing firm ownership / merger information to track transfers of patents.¹⁸ To assist in linking patents to their ultimate owners, I use Capital IQ corporate tree data to fuzzy string match not just the ultimate parent to any patent assignment transfers involving them, but also those of any subsidiary. Unfortunately, the corporate tree data required manual downloading, so I at this stage only downloaded 262 corporate trees yielding 78,229 former or existing subsidiaries.¹⁹ The 262 ultimate parent companies of the top 100 S&P 500 firms in terms of market cap (July 21, 2020), the top 50 patent holders as of 2017 (<https://www.ificlaims.com/ultimate-owners-2018.htm>) and other S&P 500 firms in more innovative GIC-subsectors.²⁰ As these corporate trees only reflect the current situation, I use capital IQ M&A data to determine the data which these firms became subsidiaries of the parent.

¹⁸Due to the many variations in the recorded name of the firm in the assignment dataset, I do another fuzzy string matching exercise to disambiguate firms which differ only by a typo / formatting issue or mislabel of inc, etc. I do two procedures, 1) the same fuzzy string matching routine as with the other datasets, 2) sorting alphabetically, looking at pairwise string distance metrics and grouping firm names together which are in close proximity. The procedures I follow can be seen as refinements of the method done by Mann (2018).

¹⁹Note the coverage of Capital IQ only starts in the 1990s, so other datasources are needed to determine corporate parents in earlier time-periods.

²⁰The GIC subsectors I included are: Commodity chemicals / materials, diversified chemicals, ag chemicals, speciality chemicals, application software, comm eq, data processing, electric components, electric eq, electric manufacturing, IT infrastructure, IT consulting, Semi conductors, systems software, tech hardware, aerospace / defense, ag machinery, electric eq components, industrial conglomerates, bio tech, health eq, health supplies, life science tools, pharma, drug retail, auto parts, household appliances, advertising, cable and satellite, ITS, interactive home entertainment, Wireless Telecommunication Services.

The patent assignment dataset provides well over 11.5 million patent assignment transactions, recorded from 1970-present and with respect to over 7.5 million patents. Of which 92% are identified as either ownership assignment transactions or mergers, with the remaining 8.1% being name changes. Between the tracking of parent-subsidiaries and refinements to the string matching developed since the original NBER dataset was constructed, my firm-matched patent coverage increases over 30% relative to the original.

In the fully linked dataset, across the full-sample, I linked 7,606 unique Compustat to holding at least one patent over 75,289 firm-year observations. The max matched patent stock to a single firm-year in sample is IBM in 2018 with 120,626 patents, while according to IBM's own blog, they have obtained since 1920, over 150,000 U.S. patents.

The number of unique Compustat parent plaintiffs linked to patent litigation cases which terminated between 1979 - 2019 is 1,939, 3,125 were defendants, 1,403 firms were both defendants and plaintiffs, and total coverage is 3,661 unique Compustat firms across 14,735 firm \times year observations. When restricted to the sample period from 1996 - 2016 yields 2,542 unique Compustat linked parent defendants, 1,515 unique Compustat linked parent plaintiffs, and 1,403 were both, giving 3283 unique Compustat firms with a completed patent litigation suit in-sample across 10,979 firm \times year observations. Finally, restricting to the set of firms which existed in the 2003 Compustat cross-section of firms and to the sample up to 2016, reduces to 1,885 unique firms across 7,103 firm \times years.

Finally, in addition to the firm-patent and patent litigation matching, in an attempt to identify the legal representatives for firm's patent transactions using the name and address of the legal representative listed on the transaction document. I match the name of the legal representative organization to the firm as well as to a list of public law firms given by Capital IQ as well as legal representatives for other patent transactions and firms to identify if the legal representative is in-house or an external representative.

4.2 Lawyer and litigation activity's contribution to patent value

The value of a patent depends not only on the quality of the patented idea but in the precise legal language and crafting of the patent document to maximize the breadth in cases in which it will be held valid against a counterparty in an infringement case. In this section, I empirically assess the contribution and variation of lawyer input to patent value using identified characteristics of the legal entity the identified legal representative is associated with and legal representative fixed effects.

To measure the contribution of lawyers to the strength or quality of patent protections, I merge my patent and legal representative data to Kogan et al. (2017) patent grant value data, and assess the contribution of the identity and characteristics of the lawyer input to the initially filed patent in explaining the variation of the inferred patent grant values from stock market responses.²¹ The

²¹I restrict attention to only the first legal representative of a patent filed in order to capture the lawyer input on the construction of the patent.

lawyer characteristics I use besides named legal representative and firm fixed effects are (i) an in-house lawyer indicator, (2) the 2020 revenue of the public law firm the lawyer works for and (3) the revenue per employee of the public law firm of the legal representative, where for the latter two I set the value to zero for non-public law firms.

Table 1 presents the results. Controlling for the named legal representative and firm (not ultimate parent) fixed effects as well as year and USPTO patent technology class, I find that patents constructed and filed by in-house lawyers are associated with between 43-58% higher patent values upon announcement of patent grants. This semi-elasticity is comparable to a roughly 16% increase in forward citations. In contrast, patents filed by an externally hired lawyer of a large public law firm are lower than a private external law firm consistent with the view that private boutique law firms which specialize and are retained by a small set of clients provide higher quality work than the large public counterparts. This negative effect of large public law firms is counter-balanced by higher quality public law firms exemplified by high revenue per employee. However, these two effects lose statistical significance when restricted to the sub-sample of patents which were filed by the ultimate public parents.

As patents are ultimately valued for their perceived enforceability in court, I link the patents with patent litigation events associated with these transactions using the (2019) USPTO patent litigation docket supplement which includes patent numbers matched to litigation cases. I find that patents which are ultimately named by a public firm defendant are associated with a 5.6% higher patent grant value, but this effect is drowned out after controlling for being used as a plaintiff or is involved in license agreements. Court awarded royalties have a significant semi-elasticity with patent values, where a one thousand dollar increase in royalties ever awarded to the patent is linked to a 11.5% increase in patent grant value. Relatedly, but in addition, patents which are named by a plaintiff of having been infringed in a patent suit are tied to 7.8% higher patent grant values.

Finally, I assess the total contribution of the lawyer input to patent grant values by doing a variance decomposition of the firm fixed effects and the pooled lawyer input including legal rep fixed effects, the in-house indicator and the public law firm covariates. I find that 4.8% of the total variation in patent grant values is attributed to the lawyer input. As with Abowd et al. (1999), this lawyer contribution is likely biased downwards due to low mobility of legal representatives across firms (although many law firms do have relationships with numerous corporations in the cross-section and over-time). Supporting this, I find the correlation between the in-house firms and firm fixed effects is 36.1% suggesting substantial positive assortativeness between in-house legal departments and high quality firms.

Altogether these results suggest that integration of lawyers with the R&D team and the implied specialization that results provides substantial value. "More and more, large international corporations are building small law firms within their own borders...Things like intellectual property, maintaining the IP portfolio, doing a lot of the patent prosecution, handling a lot of the licensing, is I think more efficiently handled by people who are experts internally, who not only have the skills to do the licensing, but also have the knowledge of

what is at the core of the corporations needs in this space. What kinds of rights they need in order to do what they want to do. That’s hard to communicate to outside counsel, and is much easier for in-house counsel,” Bruce Sewell, former Chief Legal Officer of Apple, Columbia law school interview, 2019.²²

4.3 Rival IPP investments and their influence on court outcomes

An important component of the theoretical link between court IP enforcement and distortions in firm innovation incentives and entry barriers is that asymmetric IPP investments are able to influence court IP enforcement outcomes. To empirically validate this link, I use a proxy for ℓ the sum of the patents assigned (either by being internally filed or acquired) to the plaintiff and defendant over the year of the litigation and the previous three years and regress this log count against a plaintiff judgement. I restrict to patent litigation cases which have parent firms matched to patent portfolios on both sides of the litigation. Further, due to limitations on data availability for some series in later exercises on firm dynamics, I restrict to the sample period between 1996 to 2016 leaving me with 18,866 patent litigation parent company case pairs.

The results are given in Table 2. Regressing the plaintiff and defendant count separately, I find in column (1) that both contribute significantly and largely counterbalance each-other, with a 10% percent increase in the plaintiff’s accumulated patents increasing the average probability of the plaintiff winning by about 6.1% while for a defendant a 10% increase in their accumulated patents over the past 3 years reduces the probability of the plaintiff winning by 4.6%. Consistent with the theory (where a_1 is the sensitivity to the difference in IPP investments), taking the difference between these log counts of the plaintiff, in column (2) I find that the effect of the difference is the mid-point between the estimated coefficients in column (1) for the two counterparties individually.²³ Furthermore, comparing the standard errors, we see that it is even more precisely estimated than those in column (1), suggesting the difference is the relevant factor in plaintiff winning likelihood. Moving to columns (3) - (5) with various other controls the net difference remains statistically and economically significant.

Using the average share of assigned patents represented by an in-house lawyer in the past three years to the plaintiff and defendant respectively, I find that the in-house share of the plaintiff is statistically significant with a 10 p.p. increase in in-house patents increasing the probability of the plaintiff winning by .4%. Defendant’s in-house patents are not found to be relevant. In addition, asymmetry in the plaintiff and defendant in their status as a public firm is not found to be significant suggesting either that asymmetries between the size of firms is not relevant or that the selection effects weed out all but the stronger private firms in these litigations. I also find that the concentration

²²Interview available at <https://youtu.be/-wuf3KI76Ds?t=1658>.

²³Note that in the model $a()$ was the probability of the defendant winning the litigation suit, while here we are considering $1 - a()$.

of the plaintiff's patent technology within classes is associated with a higher winning probability.

Now observe that the last two columns of this table used judge fixed effects. This data is not available in the FJC dataset. Instead, judge initials presiding over each case were obtained from the USPTO patent docket supplement from the tailing strings in the raw case numbers following court conventions. Besides being relevant controls, these judge fixed effects will be useful in subsequent empirical exercises as plausible exogenous variation on favourable litigation outcomes for the plaintiff. From the last specification, I extract the judge fixed effects 'bias'. In total, there are 345 individual judge fixed effect estimates. A summary of their 'biases' are given in Table 3. Overall there is substantial dispersion in judge 'biases', with a standard deviation of 0.21.

As the final exercise for this subsection, in Table 4 I examine how these effects have changed over time as well as examine alternative measures of IPP investments coming from the lawyer input in patent construction. Column (1) contains the results pooled across all bilaterally matched litigation suits in sample since 1978. Ignoring for the moment any concerns about selection in cases which ultimately make it to trial, the constant maps to $J_0 = 1 - a_0$ in the model. I find that pooled across the sample $\hat{J}_0 = 1 - \hat{a}_0 = 5.9\%$ while pre-2000s the unconditional level of enforcement was 3.3 p.p. higher, and following the America Invents Act (AIA) of 2011, the unconditional level of enforcement fell by 1.4 p.p. These three effects together suggest that unconditional court patent enforcement has declined over time. It is important to note however that patents and patent litigation have 'exploded' since court reforms in the early 80s and reached peak levels around 2011 before falling somewhat.

Next I consider simultaneously estimates of $J_1 = a_1$ from (a) my primary measure of IPP investments, the sum of assigned patents to the plaintiff / defendant over the past three years, and (b) an alternative measure based on the sum of lawyer fitted values from column (1) specification of Table 1. Both have numerically similar coefficient estimates for the full sample and are statistically significant. However, the former is in logs while the latter is in levels suggesting economically distinct effects.²⁴ For generality, I also consider the addition of a quadratic term J_2 to the court ruling function capture in the two quadratic regressor terms. In the full sample, these non-linear effects are found to be insignificant.

Moving to column (2) of Table 4 I consider changes in court IP enforcement sensitivity to IPP investment asymmetries of the plaintiff / defendant prior to 2000s. Although only significant at the 10% level, I find that the point estimate effectively undoes the entire pooled effect for the patent measure. This implies that court IP enforcement susceptibility to influence, a_1 was insignificant during the boom growth period of the 90s but has increased in the period since where productivity growth has declined.

²⁴Observe that given the larger support and variation in patents to fitted values of the legal input, and the fact that the fitted legal values do not statistically hold across all four samples, my primary measure of IPP investments appears stronger and more robust. In addition, and relatedly, the fitted legal inputs are only available for patents granted to public firms and so is more restrictive than my patent count measures which consist of patents obtained by public parent companies and any of their private subsidiaries.

Finally, in columns (3) and (4) of Table 4 I study sub-samples of the second half of the sample, from 1996- 2007 and 2008 - 2016 respectively. The former is broadly consistent with the full sample, although the legal fitted value becomes insignificant. The latter is qualitatively distinct with the quadratic term on the legal fitted value becoming statistically significant. This corresponds in the model to the addition of a quadratic term to the court ruling function ($a(\ell - \hat{\ell}) = a_0 + a_1(\ell - \hat{\ell}) + a_2(\ell - \hat{\ell})^2$, $a_2 > 0$) which can be shown to be associated with a strategic complementarity or arms race dynamic.

4.4 Effects of IP litigation on market power and innovation

In this last subsection, I link the patent and litigation data to Compustat firm balance-sheets in order to assess the impacts of IP litigation on firm market power and innovation dynamics. I restrict attention to non-financial, and not heavily regulated or government sponsored industries of utilities, tobacco and defense as classified by Fama-French 48 industries.

To provide a plausibly causal interpretation of estimated treatment effects on public incumbent firms, I follow a strategy similar to Galasso and Schankerman (2018) and use estimated judge biases from specification (5) of Table 2 as exogenous variation for a plaintiff favourable ruling. To be interpretable, I standardize the judge fixed effects, so that one unit corresponds to a standard deviation increase in the fixed effect. I first consider the instrumented effect of a favourable plaintiff ruling on firm's R&D expenditure (XRD) scaled by physical capital (PPNT). Commensurate with classic Q theory and in-line with my model where Π is the scale free, average Tobin's Q (equity value / capital) in the absence of court IP enforcement or other frictions should capture firm investment incentives.

The results are given in columns (1) - (3) of Table 5. I find that favourable court rulings to the plaintiff spur higher R&D intensity for the plaintiff, and this is robust to both year and firm fixed effects.²⁵ Lagged Tobin's Q has a positive and significant effect on R&D intensity, however with the inclusion of firm fixed effects its elasticity on R&D is the same magnitude as a one-standard deviation increase in judge bias from last years litigation. In columns (2) and (3) I include the lagged / logged difference in patents assigned over the past three years and indicators of whether the firm is currently a defendant or plaintiff. While past asymmetric IP investments have a statistically significant effect in column (2) this disappears with firm fixed effects. Neither of the litigation indicators in the contemporaneous year have any effect on R&D.

Second, in columns (4) - (6) I conduct the same exercise but consider the influence of winning a patent litigation suit on firm markups. As noted by De Loecker et al. (2020), markups are up to an industry specific constant given by revenues over marginal costs, and so following Crouzet and Eberly (2018) I define markups in Compustat as simply sales over cost of goods (SALE/COG) and examine only percent changes so that the industry specific constant / level

²⁵Note that the sample size here is quite small driven by only those litigations covered by the USPTO docket supplement and including matched public plaintiffs / defendants with non-missing balance sheet data for the 4 years around the litigation are included.

is immaterial. I find that a one standard deviation higher probability of winning a plaintiff litigation suit increases markups by 8.5% with year fixed effects. However, I find that with the addition of additional controls and firm fixed effects the effect on markups is washed out.

In Table 6 I consider the same two dependent variables but try to estimate the differential dynamics of plaintiffs / defendants following litigation events against un-litigated counterparts. In column (1) with just year fixed effects, we see that firms participating in patent litigation events are associated with higher markups (greater market power) than those who aren't. Controlling for industry in column (2), selection of firms involved in IP litigation on either side is tied to higher markups. In contrast, with the addition of firm fixed effects in column (3) we see that being a defendant last year in IP litigation has a positive and significant association with markups. This implies that defendants in these IP litigation suits are not simply cheap copycat infringers who undercut the original innovators, but are offering new products which have some degree of product differentiation.

Moving to the differential effect on R&D intensity in columns (4) - (6) we see that R&D intensity falls on average for those who have been subject to IP litigation suits or, after controlling for industry, are found on both sides of litigation in a likely patent litigation war. However, none of these dynamics are robust to firm fixed effects.

5 Estimation

If the same patents and other IP protection investments both support firms own innovation and help preclude competition from rivals, then differences in firm market power will be associated with different levels of IP protection and hence innovation.

The model has sharp predictions about a one to one relationship between IP protection and market power ($\frac{m}{n}$) due to the public good nature of IP investments. However, the model's notion of market power does not map one to one to HHI or markups since firm's own incentives and effectiveness to copycat activity also increase with higher IP investments, ℓ . Consequently, identification of the investment cost functions and $a_0, a_{0,c}$ will come from investment, profits and expense moments conditional on τ and quantiles of the IP protection investments scaled by physical capital.

5.1 US firm IPP and litigation data

The theory predicts that firms with greater market power will have more IP protection and so will grow faster than firms with less profits. However, because royalty payments increase with innovations and incentives to copy also increase with market power ($\frac{m}{n}$), markups / HHI do not necessarily immediately increase with more growth.

The model is estimated on an unbalanced panel of 3,105 Compustat firms and a second event panel of IP litigation cases matched to counterparties. As in

Lentz and Mortensen (2008), I will restrict to firms which appear in the initial cross-section and follow their dynamics over time. I also restrict to firms who have positive R&D or at least one patent in 2003. In the estimation, the initial observed cross-section will be interpreted as the steady state, whereas the years following will deviate from steady state due to the selection of survival across firm types / market power. Gradual exit of the initial cross-section will be leveraged for identification and since entry into the Compustat dataset suffers from a selection of older and larger public firms (which has suffered by an additional selection bias of declining listing propensities) and is not necessary for identification I omit it from the panel (outside of any litigation which may occur between an entrant and incumbent). However, by including the surviving cross-section (of 2007), any dynamic processes changing the composition of survivors will be captured in the estimation.

For the patent litigation cases data, I restrict to cases with a single parent plaintiff and defendant, cases filed in 1996 and completed by 2016. This leaves me with a sample of 16,876 unique litigation cases, of which, restricting to matched Compustat firms on both sides consists of 3,126.²⁶ I use the data range 1996 to 2016 for the patent litigation data, rather than restricting to the same period of the data, due to the relative sparsity of litigation cases.²⁷

5.2 Targeted moments

Descriptives of the steady state and surviving cross-sections used for the estimation are given in Table 7. The first three columns pertain to the 2003 cross-section of US R&D / IP intensive firms sorted into terciles based on their IPP intensity. The second three columns pertain to the surviving subset of firms in 2007 from the 2003 cross-section. I measure IPP intensity as the number of patents assigned to the Compustat parent (directly or to one of its subsidiaries) in the past 3 years scaled by physical capital. All variables besides indicator variables (fraction in-house / survivors) are de-trended for industry / year and re-centered at the 2003 unconditional mean.²⁸ Since patent count is just one measure of IPP and imperfectly tracks to the full IPP of the firm, I will not use it for the estimation beyond its use to rank the firms IP protection investment levels.

I identify in-house firms as those with average share of patents filed done by an in-house lawyer above 8% or if the firm includes a CLO amongst their top 5 most paid executives.²⁹ HHI is taken to be the Hoberg-Phillip (2016) product rivals with a minimum distance score of $\geq 1/12$ and firms required to be in the sample, and set to 1 for firms which have product rival scores, but not above

²⁶Note that this is about twice the number of cases as covered by Lee et al. (2019) who cover the interval between 2000 - 2006 using hand-collected data from PACER.

²⁷I start in 1996 due to a limitation on the Hoberg-Phillips product similarity data and for the sake of consistency with the diff in diff regressions I run around litigation cases with the product rival HHI.

²⁸Due to the de-trending, average IP protection intensity for the lowest tercile in both 2003 and 2007 is below zero reflecting a substantial number of firms without any patents assigned in that interval.

²⁹Note while the latter is based on execucomp data and hence restricted to roughly the S&P 1500 firms, the in-house lawyer segment is identified from patent transactions and so faces no such sample selection.

the cutoff level or in the existing firm set. As was discussed earlier, to reflect total firm value, Tobins Q is measured as equity value + long-term debt - current assets, $(CSHO * PRCC_F + DLLT + DLC - ACT) / PPENT$.³⁰ In addition to the cross-section moments in this table, I use the 1996 - 2016 sample estimates of the court regression intercept, and the estimates of the judge bias regressions (Q and judge iv coefficients) as additional moments to target for the estimation.

5.3 Model estimator

The model is estimated off two correlated datasets: (1) an unbalanced firm panel given by ψ_{it}^P consisting of annual flow revenues and expenditures on production (COGS), R&D, sales and general administrative expenditures (SG&A),³¹ well physical capital, total labour compensation (wage), sales (revenue), firm stock value³², patent flows, in-house status, product rival similarity measures,³³ and (2) a litigation event data set of individual observations ψ_{ijt}^L consisting of a plaintiff i , defendant j and time t with statistics on the public / private litigant status of the defendant / plaintiff, when available (i.e. public firm) pairwise firm characteristics in the year prior and following the litigation, litigation dispute choice and outcome, as well as a judge specific bias (fixed effect) drawn in the litigation case.

Let $\psi_i = \{\psi_{it}^P, \psi_{it}^L\}_{t=0, \dots, T}$ and $\psi = \{\psi_i\}_{i=1, \dots, N}$. The model is estimated by indirect inference. Define $\Psi(\psi)$ as the vector of auxiliary parameters. I produce a simulated panel $\tilde{\psi}^S(\Theta)$ based on the parameters Θ . The model simulation draws the initial firm distribution from the steady state, and then simulates dynamics by drawing the destruction shocks to each incumbent along with the identity of the rival who innovated upon them.

The simulated auxiliary parameters are then given by

$$\tilde{\Psi}^S(\Theta) = \frac{1}{S} \sum_{s=1}^S \Psi(\tilde{\psi}(\Theta)^s)$$

where S is the number of simulated repetitions.

The estimator is the choice of model parameters which minimizes the weighted distance between the data and auxiliary parameters, with $g(\Theta) = (\tilde{\Psi}^S(\Theta) - \Psi(\psi))$

$$\hat{\Theta} = \arg \min_{\Theta} g(\Theta)' W g(\Theta)$$

³⁰Belo et al. (2013) also use net capital PPENT for the replacement cost of capital, including in Tobins Q, due to a better mapping to the model. Notice that for a firm in the model, $\frac{PPENT}{sales} = \frac{\kappa[m+(n-m)s_0+n_c s_c]}{[m+(n-m)s_0+n_c s_c]}$.

³¹This is mapped in the model as expenditures on IPP protection $w \cdot c_\ell$ as well as any royalties R paid out to rivals.

³²Stock value in the data includes market value of equity plus book debt minus current assets to conform to the model.

³³Product similarity measure ($\{\text{product similarity}_{ijt}\}_{j \in \text{rivals}}$) is computed based on the network of copying and copied products, with a pairwise firm similarity score given by the average of binary indicators of a common product.

for some appropriately chosen weight matrix W .³⁴

5.4 Model parameters and endogenous objects

Given a parameter vector Θ , simulation of the model generates time paths for ψ_i including a litigation panel. The firm type distribution ϕ_τ is taken to be a two-point discrete distribution. The cost function is parameterized as

$$c_z(z) = \frac{1}{c_{0,z}} z^{c_{1,z}+1}, \quad z \in \{\iota, \gamma, \ell_\tau\}, \tau \in \{L, H\}$$

and based on our earlier empirical results for the full sample, I fix $J_2 = 0$. Consequently, the model is characterized by the following set of parameters $\Theta^{full} = \{\Theta^{fixed}, \Theta\}$. The set of the parameters endogenously estimated based on the simulation is

$$\Theta = \{q, \phi_H, J_0, J_{0,c}, J_1, \{(c_{0j}, c_{1j})\}_{j \in \{\iota, \gamma\}}, \{c_{0\ell}^\tau, c_{1\ell}^\tau\}, K_E, \kappa, s_0, s_c, \pi^c\}$$

where q is the quality increment, $\phi_L = 1 - \phi_H$ is the entry share of low IP capital, J_0, J_1 are the IP litigation judgment parameters for frontier innovation (where $J(\cdot) = 1 - a(\cdot)$), and $J_{0,c}, J_1$ are the parameters governing court IP enforcement for copycat infringement cases, c_{0j}, c_{1j} are the scalar and exponential R&D / copying investment cost terms, $\{c_{0\ell}^\tau, c_{1\ell}^\tau\}$ are the IPP investment cost terms for each type τ, κ if the unit cost of capital per unit of output, s_c is the copycat share of market after entry (= share of flow profits at time of entry), s_0 is the incumbent share of the market after a copycat entry, $1 - s_c - s_0$ is assumed to be the share of a competitive fringe, π^c is the copycat profits which is a share \leq of the uncopied monopolist,³⁵ and K_E is total fixed mass of potential entrants. The set of exogenously fixed parameters is

$$\Theta^{fixed} = \{r, w, L, \sigma_{judge}, \sigma_{idio,trial}, \{\sigma_{\tau,\ell}\}\}$$

where r is the interest rate, w is the wage, σ_{judge} , and $\sigma_{idio,trial}, \{\sigma_{\tau,\ell}\}$ all are taken directly from the data, and L is total labour supply,

The endogenous objects generated from the model are:

$$X = \left\{ w, g, \delta, \lambda, \lambda_E^*, \{\iota_{\tau m}, \gamma_{\tau m}, \ell_{\tau m}, \lambda_{\tau m}^*, \delta_{\tau m}^*\}_{\tau \in \{L, H\}, m \in \{0, 1, E\}}, \right. \\ \left. B_{trial}^{disruptive}(\tau, m, \tau', m'), \{v_\tau, K_\tau\}_\tau, M_\tau(n, m, n_{c,I}, n_{c,O}) \right\}.$$

For the estimation, I assume that all output is produced by a unit of variable (labour) input with price w and one unit of fixed capital, which is obtained / destroyed with the innovation and has replacement cost κ . While in the model, the distribution of sales of a product across firms is immaterial after copycat entry (since profits are zero), when mapping to the data, we must impose some

³⁴Since all targeted moments are normalized (unit free), an identity weight-matrix is used.

³⁵For consistency with the model setup, I impose the constraint of $\pi^c \leq \pi$ although relaxation of this requirement is an interesting future exploration.

structure. I assume that after copycat entry, s_0 is the share of sales retained by the original frontier innovator, s_c is the share retained by the first copycat and s_{fringe} is the share of sales retained on aggregate by a competitive fringe of producers who crowd in after the first copycat success. Thus, for a firm, sales is given by

$$sales_{f,t} = sales_f(n, m, R, \tau, n_c) = m \cdot 1 + (n - m)s_0 + n_c s_c + R\{R > 0\},$$

and capital is given by $\kappa(m + (n - m)s_0 + n_c s_c)$.³⁶

Assuming lump-sum transient royalties (for simplicity),³⁷ a firm's product rival sales concentration is given by:

$$HHI_{f,t} = \left(\frac{sales_f}{\sum_{j \in S_{f,t}} sales_j + sales_f} \right)^2 + \sum_{i \in S_{f,t}} \left(\frac{sales_i}{\sum_{j \in S_{f,t}} sales_j + sales_f} \right)^2$$

where $S_{f,t}$ is the set of firms which firm f has a product overlap with and each firm's sales at a point in time. The product rivals are identified based off a minimum positive threshold pairwise firm product similarity score, set to 1/12, with product overlap being the sum of product for which either firm has copied the other.

Firm's market value is decomposed as $V_{f,t} = V(n, m, R, \tau, n_c) = R + m\Pi_{\tau 1} + (n - m)\Pi_{\tau 0}$. The firm's stock return, $G_{t+\Delta}^V = \frac{V_{t+\Delta} - V_t}{V_t}$ can be locally approximated by taking m and n to evolve as independent Poisson processes with rates $\delta_{\tau 1} + \lambda_{\tau} - \iota_{\tau 1}$,

$$\mathbb{E}[G_t^V | n_0, m_0, \tau] = \frac{\left[\frac{m_0}{n_0} (\Pi_{\tau 1} - \Pi_{\tau 0}) (e^{-(\delta_{\tau 1} + \lambda_{\tau} - \iota_{\tau 1})t} - 1) + \frac{n_0 - m_0}{n_0} \Pi_{\tau 0} (e^{-(\delta_{\tau 0} - \iota_{\tau 0})t} - 1) \right] + \frac{1}{n_0} \mathbb{E}[R_t | n_0, m_0]}{\frac{m_0}{n_0} (\Pi_{\tau 1} - \Pi_{\tau 0}) + \frac{(n_0 - m_0)}{n_0} \Pi_{\tau 0}}. \quad ^{38}$$

Finally, $R\&D = w \cdot (m [c_i(\iota_{\tau 1} + \gamma_{\tau 1})] + (n - m) [c_i(\iota_{\tau 1} + \gamma_{\tau 1})])$ and $SG\&A = w \cdot (m \cdot c_{\ell}^{\tau}(\ell_{\tau 1}) + (n - m) \cdot c_{\ell}^{\tau}(\ell_{\tau 0})) + R\{R < 0\}$. Finally, Tobins $Q = \frac{V}{PP\&E}$, where $V = R + m\Pi_{\tau 1} + (n - m)\Pi_{\tau 0} + \pi_c \{n'_c - n_c\}$.

Given the endogenous model objects, the steady state firm-size monopoly distribution is computed using properties of a QBD stationary distribution. For the forward simulation, I divide each annual period into Δ_T sub-periods, and for each incumbent firm draw (i) whether a destruction shock occurs (δ or λ) given the total destruction rate $n\delta + m\lambda$ (ii) determine the type of destruction

³⁶I'm implicitly taking κ to be a by-product of the innovation and copying process, that upon innovation the capital $\kappa \cdot 1$ is obtained, while after a successful copycat entry, there is an instant depreciation of $\kappa(1 - s_0)$, while successful copying themselves results in accumulated physical capital of $s_c \kappa$.

³⁷Relaxing this is an interesting and likely empirically relevant exercise, but computationally costly as it requires an additional simulation per set of parameters to obtain the stationary distribution of royalties across firms.

³⁸Notice that Gibrat's law (firm level growth rates being independent of firm size) is weakly violated in this model, even abstracting from the expected royalties term for any $t > 0$, except when $n_0 = m_0$ or $m_0 = 0$. However, the growth is increasing in firm size.

shock and the monopoly state of the product it is on (iii) I then draw the identity of the rival firm based on the matching probabilities Θ^P, Θ^Q , (iv) draw the judge and (v) determine the outcome of the trial if it occurs by a random draw. Firm statistics are then obtained by averaging the per-sub-period statistics across the year (i.e. over Δ_T).

5.5 Identification

The interest rate is set at $r = 0.01$ (in line with the closing US one year t-bill rate of 1.24%). Since the model abstract from heterogeneous inputs to production and connects investment and production inputs through a market clearing wage, the unit cost of labour and capital will be identified simply as the sales-weighted average labour expense to employees (XLR/EMP), $w = 70.61$. While the wage w is an equilibrium object, as with LM (2008) identifying the wage upfront saves from having to do a costly second fixed point algorithm, and the parameter L (mass of labour) maps one to one with w given all the other parameters / equilibrium objects.

The stationary firm type, monopoly and copycat product distribution, $M_\tau(n, m, n_{c,I}, n_{c,O})$ can be decomposed into the firm type \times monopoly distribution $M_\tau(n, m)$ and the conditional distribution $M_\tau^c(n_{c,I}, n_{c,O} | n, m)$. This is because the quasi-birth-death process of the firm is independent of any copied products $n_{c,I}, n_{c,O}$. The firm type \times monopoly distribution $M_\tau(n, m)$ is pinned down by solving for the QBD process stationary firm-size monopoly distribution which can be computed given the equilibrium objects $\delta_{\tau m_i}^*, \iota_{\tau m_i}^*, \gamma_{\tau m_i}^*, \lambda_\tau^*, \iota_E^* K_E$ and the entry type prob. parameters ϕ_τ . Of course, given these same objects and the distribution $M_\tau(n, m)$ we can then obtain the distribution of $n_{c,I}, n_{c,O}$ which follows a simple birth-death process conditional on the τ, n, m . Equivalently, given the other parameters, the mass of incumbent firm types M_τ maps one to one with the initial entry probability type ϕ_τ .³⁹ These quasi-birth-death process parameters are directly reflected in the survival patterns of firms across the terciles and the resulting distributional effects on HHI and average (scaled) investment costs. In particular, the startup entry mass K_E will drive variation in the levels of the effective destruction rates of incumbent firms, and the amount of copycat rivalry between incumbent firms.

As highlighted in the simplified model environment, firm per-product value Π , is driven by the average firm's level of quality markups q , the monopoly share of products, and magnified by the effective entry barriers governed by $J_0, J_{0,c}, J_1$ and their relative IPP. In addition, as it is a ratio against physical capital, the level of κ also inversely scales Π to yield the level of average Tobin's Q (referred hereon as Q). Consequently, variation across IPP terciles captures variation in average effective entry barriers and monopoly shares across firm. That is, increasing Tobin's Q across the IPP terciles arises solely due to $J_1 > 0$. The magnitude of the increase in Q across terciles depends not only on J_1 but also the selection effects driving firm sorting into the IPP intensity bins.

³⁹This conditional distribution of copied products $M_\tau^c(n_{c,I}, n_{c,O} | n, m)$ implies a stochastic network of firm product rivals, defined by those whose frontier product has been copied or is copying. This network of product overlap creates a notion of firm specific product rivals, and metrics of local product concentration HHI.

To further help disentangle the level of q and κ from entry barriers, I use firm dynamics around litigation, and firm performance correlated to litigation outcomes. In particular, the judge bias regression parameters provide estimates of the average impact of winning a litigation trial (conditional on the selection).⁴⁰ Hence variation in this correlation of Q and defendant litigation events across terciles will be driven by different proportions of defendants as copycats vs frontier innovators, thereby disentangling J_0 and $J_{0,c}$. The correlation of plaintiff status and returns, and plaintiff and in-house provide further variation that helps disentangle the J_0 and $J_{0,c}$ parameters.

Since sales is driven by both copycat products n_c and the monopoly / copied original frontier products of the firm, while R&D efforts are only linked to their own frontier products n , variation of the R&D sales ratio across two of the terciles can pin down s_0 and s_c given the R&D cost function and the firm distribution. Similarly, local product rival sales HHI is determined by the dispersion in sales shares across firms which have copied products of each other. Moving across terciles puts increasing average frontier monopoly product shares and varies the fraction of τ firms and so the loadings on s_0 , q and s_c will vary systematically across the different terciles.

It then remains to identify and estimate the investment cost parameters (8 unknowns). The model has sharp predictions about the relationship between IP protection and market power ($\frac{m}{n}$) due to the (within-firm) public good nature of IP investments. That is a product line without a monopoly has different incentives to protect / defend than a product line with a monopoly.⁴¹ Consequently, as there are three investment decisions, two monopoly states for a product and two types τ of incumbent, we have 12 equations and 8 unknowns. Variation across the within IPP tercile average investment costs scaled and correlation of different aspects of R&D, SG&A with firm outcomes and in-house type are sufficient to pin down the IPP investment cost parameters.

5.6 Estimation results and model fit

The estimated parameters are given in Table 8. Noting that $\gamma, \iota, \ell < 1$, we see that the copycat cost function is strictly cheaper / flatter than the frontier R&D, and that the asymmetries between the low and high IP cost firms parameters are of the same order of magnitude, but the level effect is about 5 times lower. The mass of potential entrants is found to be large, and the share of in-house on entry relatively small (but I find becomes more populated due to selection on survival in sample). The court IP enforcement function has a large slope of 0.3 and the unconditional probability of a plaintiff favourable outcome is markedly higher for a frontier innovation than for a copycat. Notice that these parameters as estimated do not just reflect the ex-post probability of litigation conditional

⁴⁰A naive method of identification of IPP entry barriers $J_0, J_{0,c}, J_1$ would come from the estimated parameters of the court ruling function, however, this only estimates the blocking effect conditional on trial, and ignores the effective blocking already done in cases that didn't go to trial, as well as neglects any effective royalties.

⁴¹However, note that the model's notion of market power does not map one to one to HHI or markups since firm's own incentives and effectiveness to copycat activity also increase with higher IP investments, ℓ .

on the two seeking a judgement, but rather also captures the blocked entry through aggressive litigation and predatory practices prior to the suit being filed. Hence, this likely reflects incumbents have more incentive to heavily litigate more disruptive innovations than say a third generic competitor.

The results of the estimation are given in Table 9 and in the replicated regressions in Table 10. Key to the identification of entry barriers and their tie to IPP activities is the level and slopes of Tobins Q across IPP bins as well as the survivorship dynamics in the 2007 cross-section.⁴² To this end, the model matches the variation of Tobin's Q across IPP terciles in the initial cross-section quite well as well as captures the decay in firm value for the middle IPP cross-section in the surviving sample. On the other hand, the model undershoots the average level of HHI, implying a greater level (and standard deviation) of competition than found in the public Compustat firm sample, but which is likely closer to what is found in the population of firms. The model does capture well the flatness in HHI in the surviving cross-section. In contrast, the association of identified in-house firms with IPP bins is also reasonably approximated with a low prevalence in the low and medium IPP bins and a high prevalence in the high IPP bin, although the model doubles the estimated prevalence. In terms of survivorship, the model captures the higher survivorship of high IPP intensity and in-house firms, both in terms of percent survivors and fraction inhouse, with increasing prevalence of inhouse in the surviving sample relative to initial cross-section. The upward sloping pattern of investment costs in the initial cross-section is captured by the estimation, with the middle IPP bin level of investment costs closely matched, but undershooting by half both the low and high IPP bin levels. For the surviving cross-section, the model fails to capture the increased investment intensity in the data, but does capture the relatively muted variations across the bins.

Thee remaining correlation moments in Table 9 are only targeted for the initial cross-section. The estimation captures the qualitative decay in the correlation of R&D and Tobin's Q as IPP intensities fall (from right to left in IPP bins), but overstates the degree of positive association. The estimated model also matches for high IPP firms qualitatively the relationships of inhouse to various measures of firm performance and IPP intensity, with the exception of plaintiff litigation occurrences where effectively no association is predicted. The correlations of investment components of R&D and SG&A with firm returns matches in sign but broadly undershoots and overshoots the data respectively. The correlation of R&D intensity and returns is fairly flat / weak in the simulated model while positive in the data, hinting at possible a missing financial friction. The correlation of SG&A and returns of the simulated model overshoots the data in the low and middle IPP bins, likely driven by other aspects of SG&A measure (noise) not captured in the model including advertising, miscellaneous non-IP related legal expenditures, salaries / wages and rent. The estimated model captures fairly well the levels of correlation between plaintiff litigation and returns and inhouse for the low IPP firms, but gets the wrong association of defendant litigation events and Tobins Q. The failure to match the correlation of defendant litigation events and Q is likely driven by (1) patent troll litigation

⁴²Reflecting the identity weight matrix used, the Tobins Q moments are heavily emphasized in the estimation.

suits and (2) fixed costs to litigation which destroy value for innovating firms hit with a litigation suit.

We now move to the regression moments targeted in the estimation in Table 10. Recalling that I also targeted the intercept and the judge bias and Q coefficient for the R&D intensity and markup regressions, the remaining estimates were not used in the identification but provide additional validity checks of model fit. The model matches the impact of judge bias and association of Tobins Q to both markups and R&D intensity fairly well. The model overshoots the implied impact of winning a litigation suit as a plaintiff on R&D and undershoots the impact on markups. The Tobins Q R&D elasticity for firms around these patent litigation events is almost perfectly matched while the link of Tobins Q to markups is somewhat overestimated. Finally moving to the court ruling regressions in the third row of Table 10, we see that the model reasonably matches the implied court ruling constant $\hat{J}_0 = .05$ despite the structurally estimated frontier and copycat ruling functions exceeding 0.05. This feature suggests a reasonable capturing of the selection effects of firms going to trial rather than settling. In un-targeted moments, the model qualitatively replicates but substantially overshoots the sensitivity of court rulings to IPP, reflecting possible differences in the cardinal properties of the model and data IPP measures (where in the model / simulation the IPP level is not explicitly targeted to match patents).

As an additional assessment of the estimated model fit, in Table 11 I consider the joint firm dynamics of the plaintiff and defendant around litigation events in un-targeted moments. For each presented statistic, I compute the average of the two firms in the year prior / following the IP litigation event terminating. Since the model has stark predictions of the selection of firms which will litigate each other in which circumstances, I restrict to considering pairs of litigants where the plaintiff and defendant are of different in-house statuses.

The estimated model captures the dynamics of joint firm HHI across the two considered firm pairings, where when an inhouse plaintiff litigates a non-inhouse defendant, their average market power increases, while when the asymmetry runs the other way average market power decreases. In contrast, the model doesn't capture the average drop in joint firm equity value when an in-house plaintiff litigates an outside counsel defendant, but does capture the joint fall when an inhouse defendant is litigated by an outside counsel plaintiff. Together these patterns in both the data and model are overall consistent with innovative products being litigated by asymmetrically IP-protected plaintiffs while when plaintiff's are weakly IP-protected the IP disputes typically involve copied products.

6 Conclusion

I study how intellectual property protection shapes product entry barriers and acts as a separate, complementary input to the innovation process. I provide a mechanism which endogenously links firm's existing market power to innovation incentives, creating the superstar firm phenomenon. I measure the

influence these investments have on firm valuations and firm dynamics using a new firm matched dataset of IPP investments and IP litigation. Using a quantitative framework model of IPP investments and litigation with a (Schumpeterian) endogenous growth model, I illustrate and quantify the entry distortions which arise with IP litigation. I find evidence that the quantitative distortions of court IP decision making are substantial and that there may be actionable reforms which can promote firm entry and aggregate growth while also improving consumer welfare.

The results suggest that current intellectual property protection activities and court patent enforcement may be excessive in their protection of past innovators. In fact, estimation on 21st century US firms suggests total elimination of patents rights would improve both competition and aggregate innovation. Driving this result is the high quality of new innovations, & fundamental challenges in replicating the same technology, so that even after eliminating patent protections the rents from innovating are still sufficiently high to encourage innovative efforts. Moreover, the estimated advantages in IP enforcement for large intangible incumbent firms inhibits startups & other small firms to such an extent that aggregate R&D increases with lower entry barriers. This aligns with the arguments of Boldrin and Levine (2013) & supports the observation made in senate testimony by 13,000 activists of "US inventor" that "patents are useless to the very entities the patent system was created to help... inventors and patent-based startups."⁴³ While implementing the Restoring the America Invents Act of 2011, "to encourage innovation..by improving the mechanism for challenging [patents]" may help mitigate some of the predatory practices alleged with the creation of the Patent Trial Appeals Board, reforming the system to even the playing field in patent enforcement for firms with less sophistication in IP protection may foster innovation. Potential actionable reforms include: (i) providing additional legal counsel support to startups, ii) forcing arms-length legal services for all firms, or iii) making more clear-cut patent examination and approval policies to reduce re-litigation of the validity of new patents.

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⁴³See senate testimony by <https://www.judiciary.senate.gov/imo/media/doc/Morinville%20Testimony.pdf>

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A Tables and Figures

Table 1: Decomposition of patent grant values with lawyer input

Dependent Variable: Model:	Log(Patent Grant Announcement Value)				
	(1)	(2)	(3)	(4)	(5)
<i>Variables</i>					
Inhouse lawyer	0.4793*** (0.1192)	0.5882*** (0.0926)	0.4793*** (0.1192)	0.4793*** (0.1192)	0.4345*** (0.0894)
Log(1+ # forward citations)	0.0319*** (0.0061)	0.0304*** (0.0068)	0.0318*** (0.0061)	0.0318*** (0.0061)	0.0267*** (0.0043)
Public law firm revenue	-0.0091*** (0.0026)	-0.0566 (0.2062)	-0.0091*** (0.0026)	-0.0091*** (0.0026)	0.0012 (0.0030)
Public law firm revenue/employee	7.842*** (1.619)	27.60 (85.43)	7.837*** (1.619)	7.833*** (1.619)	2.978* (1.536)
Held by public defendant			0.0559** (0.0285)	0.0129 (0.0322)	-0.0089 (0.0311)
Royalties awarded total			0.1145*** (0.0338)	0.1100*** (0.0337)	0.1059** (0.0419)
# of litigations vs private defendants			0.0002 (0.0041)	-0.0011 (0.0041)	-0.0012 (0.0039)
# license agreements				-0.1119* (0.0649)	-0.1009* (0.0607)
# private firm licenses given				0.1018 (0.0986)	0.0921 (0.0931)
Used by public plaintiff				0.0780** (0.0320)	0.0706** (0.0311)
<i>Fixed-effects</i>					
Firm	Yes	Yes	Yes	Yes	Yes
Legal representative	Yes	Yes	Yes	Yes	Yes
Year	Yes	Yes	Yes	Yes	
Patent tech class	Yes	Yes	Yes	Yes	
Class X Year					Yes
<i>Fit statistics</i>					
Observations	556,024	419,080	556,024	556,024	556,038
R ²	0.89227	0.89409	0.89227	0.89228	0.90089
Within R ²	0.00419	0.00612	0.00422	0.00425	0.00342

One-way (consolidated_company_pdpco) standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Column 1: Baseline, Column 2: Baseline, only patents filed by NBER parent, Column 3: Baseline + litigation, Column 4: Baseline + litigation + licensing, Column 5: Column 4 + class x year FEs. Kogan et al. (2017) patent grant values matched to patent filing transaction and legal representative that represented the firm. Patent litigation data from federal judicial center patent infringement suits, licensing and patent data from USPTO patent re-assignment dataset. Patent grant dates from 1887 - 2010. Firms exclude utilities and finance.

Table 2: IP investments effects on court outcomes

Dependent Variable: Model:	Plaintiff Judgement				
	(1)	(2)	(3)	(4)	(5)
<i>Variables</i>					
(Intercept)	0.0498*** (0.0025)	0.0527*** (0.0016)			
Log (1 + # Patents Plaintiff)	0.0061*** (0.0008)				
Log (1+ # Patents Defendant)	-0.0046*** (0.0007)				
Log($\frac{1+\#Patents\ Plaintiff}{1+\#Patents\ Defendant}$)		0.0052*** (0.0005)	0.0035** (0.0015)	0.0030* (0.0015)	0.0034** (0.0015)
Avg. Share Patents Inhouse - Plaintiff			0.0422** (0.0169)	0.0459** (0.0202)	0.0450** (0.0206)
Avg. Share Patents Inhouse - Defendant			0.0086 (0.0101)	0.0050 (0.0126)	0.0063 (0.0130)
Plaintiff Public vs Defendant Private			0.0018 (0.0109)	0.0018 (0.0111)	0.0003 (0.0112)
Plaintiff Private vs Defendant Public			-0.0058 (0.0080)	-0.0025 (0.0077)	-0.0023 (0.0074)
Plaintiff Tech Concentration			0.0198** (0.0095)	0.0215** (0.0107)	0.0211* (0.0110)
Defendant Tech Concentration			-0.0073 (0.0085)	-0.0016 (0.0101)	-0.0011 (0.0098)
<i>Fixed-effects</i>					
Year			Yes	Yes	Yes
Judge				Yes	Yes
District					Yes
<i>Fit statistics</i>					
Standard-Errors	Standard	Standard	Year	Judge	Judge
Observations	18,866	18,866	7,681	7,291	7,291
R ²	0.00512	0.00500	0.01066	0.13293	0.14297
Within R ²			0.00552	0.00605	0.00620

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Notes: Sample US federal litigation cases, plaintiff / defendant IP portfolio matched, 1996 - 2016. Column 1: Separate Patents, Column 2: Patent Differences (Baseline), Column 3: Baseline + base controls, Column 4: Column 3 + tech controls + judge fixed effects, Column 5: Column 4 + district fixed effects. Firm matched patent stock data. Firm patent holding characteristics = sum over 3 years prior to filing. Year fixed effects for termination date.

Table 3: Distribution of estimated judge fixed effects

Judge fixed effects	
Min.	-1.20
1st Qu.	-0.07
Median	-0.03
Mean	0.00
3rd Qu.	0.05
Max.	0.98
Std. Dev	0.21

Summary statistics of the estimated judge fixed effects from column (5) of Table 2.

Table 4: Estimated court ruling function across full-sample

Dependent Variable: Model:	Plaintiff wins litigation			
	(1)	(2)	(3)	(4)
<i>Variables</i>				
Constant	0.0593*** (0.0027)	0.0607*** (0.0022)	0.0604*** (0.0033)	0.0552*** (0.0032)
$\text{Log}\left(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}}\right)$	0.0032*** (0.0006)	0.0036*** (0.0006)	0.0029*** (0.0010)	0.0040*** (0.0008)
Fitted legal input differences	0.0032*** (0.0006)	0.0036*** (0.0006)	-0.0002 (0.0009)	0.0059*** (0.0008)
$[\text{Log}\left(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}}\right)]^2$	2.17×10^{-5} (0.0001)		0.0002 (0.0002)	-4.63×10^{-5} (0.0002)
(Fitted legal input differences) ²	0.0001 (0.0001)		-9.77×10^{-5} (0.0002)	0.0003** (0.0001)
Post 2011 AIA	-0.0143*** (0.0038)	-0.0140*** (0.0038)		
Pre-2000	0.0329*** (0.0043)	0.0327*** (0.0042)		
$\text{Log}\left(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}}\right)$ pre-2000		-0.0032* (0.0018)		
Fitted legal input differences pre-2000		-0.0023 (0.0015)		
<i>Fit statistics</i>				
Observations	22,509	22,509	9,279	9,167
R ²	0.00854	0.00878	0.00107	0.01238
Adjusted R ²	0.00827	0.00852	0.00064	0.01195

Normal standard-errors in parentheses

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

Notes: Full litigation sample of matched parent firm portfolios on both sides: 1978 - 2016. # patents = number of patents assigned to plaintiff / defendant over the year of the litigation and 3 prior years. Fitted legal input differences are the sum of the pooled legal input effects for plaintiff / defendant over the past 3 years extracted from the regression in column (1) of Table 1. Column (1) and (2) is on the full sample. Column (3) is the sub-sample from 1996 - 2007 and Column (4) is 2008 to 2016.

Table 5: Impact of judge bias on firm innovation and market power

Dependent Variables: Model:	Log(R&D intensity)			Log(Markups)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Variables</i>						
Avg. last year judge fixed effects as plaintiff	0.0784** (0.0301)	0.0769** (0.0319)	0.0276** (0.0121)	0.0852*** (0.0213)	0.0825*** (0.0216)	0.0061 (0.0107)
Log(Tobins Q)	0.6574*** (0.0275)	0.6370*** (0.0272)	0.0798*** (0.0261)	0.3950*** (0.0186)	0.3867*** (0.0181)	0.1243*** (0.0389)
$\text{Log}\left(\frac{1+\#\text{Patents Plaintiff}}{1+\#\text{Patents Defendant}}\right)$		-0.0604*** (0.0116)	-0.0016 (0.0056)		-0.0109 (0.0070)	0.0027 (0.0055)
Defendant litigation in year		-0.0630 (0.0714)	0.0285 (0.0287)		0.0091 (0.0451)	0.0110 (0.0310)
Plaintiff litigation in year		0.0743 (0.0515)	0.0160 (0.0262)		0.2557*** (0.0435)	0.0024 (0.0369)
<i>Fixed-effects</i>						
Year	Yes	Yes	Yes	Yes	Yes	Yes
Firm			Yes			Yes
<i>Fit statistics</i>						
Standard-Errors	Year	Year	Firm	Year	Year	Firm
Observations	1,672	1,672	1,672	1,824	1,824	1,824
R ²	0.36592	0.37965	0.97018	0.26498	0.27933	0.93235
Within R ²	0.34461	0.35880	0.02975	0.24467	0.25942	0.04890

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

Notes: Estimating the impact of winning IP litigation suits and distortions of asymmetric IP on R&D investment and market power. Firm (parent) matched patent stock data, with balance sheets for Compustat firms, sample period from 1996 - 2016. Sample is restricted to firms with either positive R&D or patent counts, and is ever involved in an FJC filed litigation as a plaintiff itself or by one of its subsidiaries. Year fixed effects correspond to litigation termination date filing year. First three independent variables one year lagged and are averages across all matched litigation suits that firm is involved in in that year as a plaintiff. The first three columns uses a dependent variable of firm R&D intensity measured as $\log R\&D/PPENT$ and the last three are of firm's gross markups measured as total firm revenues over cost of goods (SALES/COGS).

Table 6: Differences in Differences of firm dynamics around litigation events

Dependent Variables: Model:	(1)	Log(Markups) (2)	(3)	(4)	Log(R&D/PPENT) (5)	(6)
<i>Variables</i>						
Plaintiff litigation last year	0.2037*** (0.0316)	0.1165* (0.0605)	0.0208 (0.0139)	0.0710 (0.0587)	-0.0333 (0.0578)	-0.0208 (0.0177)
Defendant litigation last year	0.0634** (0.0269)	0.0548 (0.0410)	0.0220** (0.0097)	0.0398 (0.0611)	-0.0085 (0.0872)	0.0123 (0.0138)
Both sides litigation last year	0.0596 (0.0370)	-0.0087 (0.0421)	-0.0196 (0.0164)	-0.0753 (0.0670)	-0.1915*** (0.0686)	0.0112 (0.0226)
Ever Defendant	0.0192 (0.0187)	0.1241** (0.0486)		-0.5639*** (0.0292)	-0.3096* (0.1781)	
Ever Plaintiff	0.1212*** (0.0075)	0.1128** (0.0487)		-0.0222* (0.0125)	-0.0311 (0.0683)	
<i>Fixed-effects</i>						
Year	Yes	Yes	Yes	Yes	Yes	Yes
Fama-French 48 industry		Yes			Yes	
Firm		Yes	Yes		Yes	Yes
<i>Fit statistics</i>						
Standard-Errors	Year	Fama-French 48 industry	Firm	Year	Fama-French 48 industry	Firm
Observations	31,087	30,761	31,087	24,556	24,324	24,556
R ²	0.01636	0.33579	0.83776	0.01029	0.48421	0.92075
Within R ²	0.01306	0.00916	0.00032	0.00777	0.00704	0.00014

Signif. Codes: ***: 0.01, **: 0.05, *: 0.1

Notes: Estimating differences in differences of firms around litigation suits of firm R&D investment intensity and market power. Sample period from 1996 - 2016 estimated on (parent) firm matched patent stock data with balance sheets from Compustat. Firm patent stocks = averages over 3 years prior to filing. Year fixed effects corresponding to litigation termination date filing year. Sample is restricted to firms with First three independent variables one year lagged. The first three columns uses a dependent variable of gross markups measured as total firm revenues over cost of goods (SALES/COGS) and the last three use firm R&D intensity measured as $\log R\&D/PPENT$. Dependent variables are indicators of the firm (or their subsidiary) being a litigant participant in the year preceding the accounting filing date for that firm.

Table 7: Cross-section summary statistics

	2003			2007		
	L	M	H	L	M	H
# survivors	1037	1031	1037	729	667	783
E[Tobins Q]	21.737	57.387	67.816	13.771	46.771	104.136
E[HHI]	0.864	0.872	0.869	0.874	0.874	0.869
STD[HHI]	0.181	0.183	0.174	0.163	0.170	0.165
Fraction inhouse	0.125	0.102	0.273	0.198	0.180	0.328
E[invest costs / capital]	0.085	0.108	0.263	0.160	0.151	0.136
Corr(Inhouse, Markups)	-0.027	-0.044	-0.006	-0.013	-0.015	0.011
Corr(Inhouse, IPP)	0.020	-0.128	0.115	0.107	-0.007	-0.024
Corr(R&D, Q)	0.471	0.796	0.893	0.432	0.265	0.930
Corr(R&D,SG&A)	0.370	0.791	0.509	0.246	-0.420	0.809
Corr(R&D,returns)	0.045	0.108	0.115	-0.147	0.126	0.015
Corr(SG&A,returns)	0.107	0.135	0.122	-0.138	-0.118	-0.117
Corr(Inhouse,returns)	-0.110	-0.086	-0.123	0.027	-0.018	0.052
Corr(# plaintiff lit., returns)	-0.049	-0.027	-0.090	-0.010	0.027	0.062
Corr(# defendant lit., returns)	-0.038	-0.064	-0.071	0.006	0.036	0.066
Corr(# plaintiff lit., Inhouse)	0.132	0.147	0.194	0.075	0.132	0.119
E[IPP/capital]*	-0.172	0.040	0.251	-0.159	0.009	0.366
E[# plaintiff cases]*	0.084	0.048	0.273	0.024	0.065	0.440
E[# defendant cases]*	0.074	0.103	0.348	0.004	0.119	0.605
E[net income / capital]*	-0.028	0.022	0.067	0.050	0.101	0.083
E[sales %]*	0.041	0.023	0.032	0.051	0.044	0.040
STD[sales %]*	0.182	0.110	0.109	0.210	0.183	0.120

Summary statistics of R&D-intensive, non-financial / utility firms from 2003 sorted by 2003 IP intensity tertiles. IP intensity is defined as number of patents assigned to parent over the past 3 years (ie since 2000 for 2003 sample). All variables (besides identifier indicators) are industry x year detrended over 1996 - 2016 with the 2003 average added back in. IPP, R&D and SG&A are all expressed in intensities with respect to physical capital (measured by PPENT), similarly firm value for Tobins Q and returns is computed as firm market equity plus book debt minus current assets, with Tobins Q scaled by net physical capital (PPENT). Columns 1 - 3 summarize the 2003 cross-section taken to be steady state and the last columns pertain to the surviving set of these same firms in 2007. Starred variables are not used in estimation. Markups are computed as revenue over cost of goods (COGS). Firm-level HHI is computed using Hoberg-Phillips product similarity data, and is computed as sales HHI for all pair-wise rivals restricted to similarities $> \frac{1}{12}$ and who are in the initial 2003 cross-section.

Table 8: Parameter estimates

q	s_0	s_{copy}	φ_{copy}	J_0	J_1	J_0^{copy}	K_E	κ	ϕ_I
5.8736	0.0448	0.0449	0.9993	0.7608	0.0068	0.0997	1.3365	10.1921	0.2291

c_ι	α_ι	c_γ	α_γ	c_ℓ^I	c_ℓ^O	α_ℓ^I	α_ℓ^O
0.0061	1.7732	1.1985	16.3843	0.2513	0.0010	17.4731	12.6830

Estimated model parameters from simulating 3,000 firms, over 10 years with six sub-periods per year. Second row contains the estimated cost parameters, $c_j^\tau(x) = \frac{1}{c_j^\tau} x^{\alpha_j^\tau}$, $j \in \{\iota, \gamma, \ell\}$ where ι is innovation effort, γ is copying effort and ℓ is IPP effort. In addition to non-negativity bounds for all parameters, parameters constrained to ensure $\ell^I \geq \ell^O$ and $\alpha_j^\tau \geq 1$ for convex cost functions. φ_{copy} is the share of instantaneous profits obtained by the copycat relative to the incumbent's $\pi = 1 - \frac{1}{q}$, constrained to be less than or equal to 1.

Table 9: Target and Simulated Moments for 2003 Firm Cross-Section

	2003						2007					
	L		M		H		L		M		H	
	Data	Sim.	Data	Sim.	Data	Sim.	Data	Sim.	Data	Sim.	Data	Sim.
Survivors (%)	1.000	1.000	1.000	1.000	1.000	1.000	0.703	0.925	0.647	0.937	0.755	0.963
E[TobinsQ]	21.737	23.451	57.387	56.963	67.816	67.310	13.771	48.148	46.771	49.727	104.136 ^x	49.519 ^x
E[HHI]	0.864	0.625	0.872	0.637	0.869	0.740	0.874	0.654	0.874	0.640	0.869	0.664
STD[HHI]	0.181	0.232	0.183	0.260	0.174	0.264	0.163	0.249	0.170	0.248	0.165	0.245
Fraction inhouse	0.125	0.113	0.102	0.126	0.273	0.520	0.198	0.114	0.180	0.131	0.328	0.501
E[invest costs]	0.085	0.036	0.108	0.092	0.263	0.110	0.160	0.077	0.151	0.080	0.136	0.080
Corr(inhouse,markups)	-0.027	0.368	-0.044	-0.067	-0.006	-0.103						
Corr(inhouse,IP)	0.020	-0.262	-0.128	0.155	0.115	0.734						
Corr(R&D,Q)	0.471	0.951	0.796	0.993	0.893	0.999						
Corr(R&D,SG&A)	0.370	0.010	0.791	0.000	0.509	0.068						
Corr(R&D,returns)	0.045	-0.055	0.108	0.002	0.115	0.005						
Corr(SG&A,returns)	0.107	0.688	0.135	0.232	0.122	0.066						
Corr(inhouse,returns)	-0.110	-0.010	-0.086	-0.014	-0.123	-0.009						
Corr(# plaintiff lit.,returns)	-0.049	-0.079	-0.027	-0.506	-0.090	-0.600						
Corr(defendant,Q)	-0.031	0.027	-0.039	0.072	-0.022	0.039						
Corr(# plaintiff lit.,inhouse)	0.132	0.278	0.147	-0.053	0.194	-0.021						

Cross-section data targets and simulated moment comparisons from the estimated data. Simulation consists of an unbalanced panel with an initial cross-section of 3,000 firms over 10 years where each year is sub-divided into 6 sub-periods and aggregated to the annual level. 2003 firm cross-section(s) target data (D) and simulated moments (S) for 2003 and 2007. Firms are split in terciles of ℓ after the first sample year, low (L), medium (M) and high (H). Summary stats for these terciles are then computed on both the initial cross-section of 2003 and for the subset of surviving firms from this sample again in 2007. 2007 moments in the sub-divided bottom-right quadrant are un-targeted in the estimation. The high IPP bin 2007 Tobins Q moment (indicated with superscript x) is also excluded from the estimation and all other 2007 Tobin's Q moments are downweighted by a factor of 5. Number of survivors are normalized by the number of firms at the end of the initial year. Investment costs and Tobins Q are scaled using physical capital.

Table 10: Targeted Regression Estimates

	Coef. 1		Coef. 2	
	Data	Sim.	Data	Sim
Judge Impact - R&D	0.078	0.240	0.657	0.643
Judge Impact - Markups	0.085	0.011	0.395	0.524
Court Ruling	0.053	0.013	0.005 ^x	0.585 ^x

Comparison of data target and simulated data regression estimates. Simulated data is from the un-balanced 10-year simulated panel of 3,000 firms. Data estimates for the judge bias regressions on R&D intensity and markups are obtained from Table 5 columns 1 and 4 respectively (i.e. firm year estimates) for the 1996 - 2016 sample. The simulated judge bias is drawn on a per litigation basis from a normal distribution with standard deviation as estimated in Table 3 based on judge fixed effects in the last column of Table 4. These judge bias effects are averaged over the year for firms with multiple plaintiff litigation suits. Data estimates for the court ruling regression is obtained from Column 1 of Table 4.

Table 11: Un-targeted moments, joint litigation pair average differences

	I-Plaintiff vs O-Defendant		O-Plaintiff vs I-Defendant	
	Data	Sim.	Data	Sim
HHI	0.180	0.010	-0.108	-0.069
Stock	-0.036	0.012	-0.140	-0.014

Joint firm average differences around litigation events for plaintiff and defendant conditional on in-house type of plaintiff / defendant. All of these moments are un-targeted. Data sample consists of all US IP litigations between 1996 - 2016 with bilateral matched public parent companies on either side of the litigation. All moments are computed as average of the plaintiff and defendant in the year following minus the year prior to the litigation. Columns 1 / 2 is restricted to averages across litigations involving an in-house plaintiff facing a non-inhouse defendant firm for data / simulated sample respectively. Similarly, columns 3 / 4 is restricted to averages across litigations involving a non-inhouse (public) plaintiff against an in-house (public) defendant. In-house status in the data is based on the in-house patent filing classification described previously or with an identified chief legal officers as a top 5 paid executive. Stock is firm value computed as market equity plus book debt minus current assets. HHI is computed using Hoberg-Phillips product similarity data and 'close' rivals. SG&A intensity is sales and general administrative expenses scaled by physical capital.

Table 12: Un-targeted R&D and market power regressions

Dependent Variables:	R&D		Log(R&D)	
	Data	Sim	Data	Sim
<i>Variables</i>				
Markups	0.0995 (0.0846)	0.0001 (0.0002)		
Log(Markups)			0.0496** (0.0219)	0.0078*** (0.0014)
Log(SG&A)			0.7991*** (0.0173)	1.8673*** (0.0383)
Log(Sales) x log(Markups)			0.7666*** (0.0187)	1.7044*** (0.04574)
			0.0339*** (0.0054)	0.0041*** (0.0012)
<i>Fixed-effects</i>				
Firm	Yes	Yes	Yes	Yes
<i>Fit statistics</i>				
Observations	55,968		32,640	32,640
R ²	0.83846		0.95183	0.95211
Within R ²	2.16 × 10 ⁻⁵		0.40814	0.41156

Clustered (Firm) standard-errors in parentheses

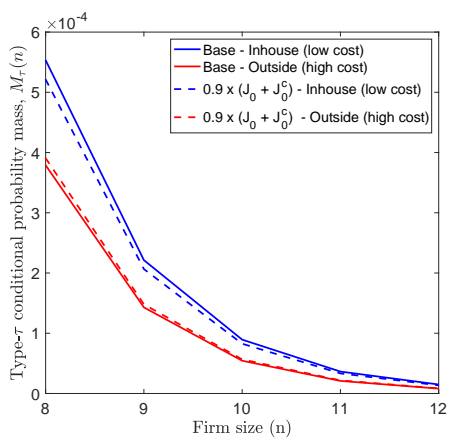
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Data vs simulated regressions of R&D vs markups on the full firm panel. Markups (gross) is computed as total revenues (SALE) over cost of goods (COGS). SG&A is sales and general administrative expenditure. R&D and SG&A levels are deflated. Simulated data consists of 3,000 simulated firms over 10 years in an unbalanced panel.

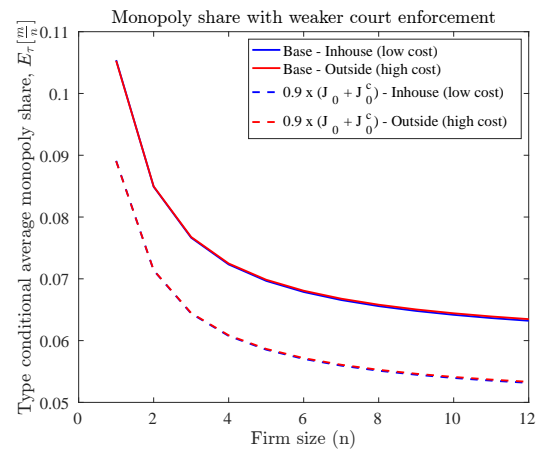
Table 13: Counterfactual policy interventions

	Baseline		Counterfactuals - Baseline			
	\hat{J}_0	Weaken courts $0.9 \times \hat{J}_0$	Court IPP sensitivity, $J_1 \times 2$		Court IPP sensitivity, $J_1 = 0$	
			\hat{J}_0	Weaken courts $0.9 \times \hat{J}_0$	No patents $\hat{J}_0 = 0$	Perfect enforcement Klette & Kortum
Growth (%)	3.879	0.395	-0.003	0.393	1.782	2.918
Consumption equivalents (%)	0.000	0.503	-0.003	0.499	5.232	2.448
E[HHI]	66.231	-0.043	0.765	-0.312	-4.498	33.769
Log(Consumption level)	-4.360	0.016	0.000	0.016	0.066	-1.668
E[Q]	49.627	-4.011	-0.208	-4.177	-24.898	-47.478
STD[Q]	23.580	-2.767	0.061	-2.920	-14.396	-23.580
E[Gross profits]	21.442	-1.606	0.294	-1.687	-7.960	596.725
Copycat share (%)	98.550	-0.140	0.001	-0.140	-0.619	-98.550
Innovation block (%)	0.996	-0.251	0.000	-0.251	-0.996	-0.996
Copycat block (%)	59.882	-5.979	0.073	-5.903	-59.882	NaN
Startup growth share (%)	57.693	-0.036	-0.028	-0.057	-0.038	-0.492
Startup copying share (%)	57.158	0.011	-0.047	-0.024	0.044	NaN

Notes: Column 1 contains economy statistics for the baseline estimated model. Columns 2 - 6 are counterfactual experiments expressed in differences to the baseline estimated model (Column 1). $\hat{J}_0 = (\hat{J}_0, \hat{J}_{0,copy})$ denotes the estimated levels of the IP litigation court enforcement constants J_0 and $J_{0,c}$ for innovation and copying respectively. Column 2 considers a 10% decrease in the probability of court blocking in a litigation suit (equally for copying or innovation). Columns 3 - 4 consider counterfactuals where, J_1 , the court sensitivity to asymmetric IPP is increased 30-fold, while Columns 5 - 6 consider sensitivity of zero. Column 3 keeps the estimated unconditional constants J_0 . Column 4 considers the same 10% decrease in court blocking but with the increased asymmetric IPP sensitivity, Column 5 eliminates patent protections / court IP enforcement entirely leaving only the copycat R&D extension, and finally Column 6 presents the nested Klette & Kortum economy. Growth is expected consumption growth, $E[HHI]$ is average product rival sales concentration, $\log(\text{Consumption level})$ is the level of log consumption ignoring the growth component, $E[Q]$ is average Tobins Q, $E[\text{gross profits}]$ is the average profitability, frontier block % is the share of frontier innovation blocked by incumbents, copycat block percentage is the % of copying which is blocked by the courts, startup growth contribution is the share of consumption growth generated by innovations of startups, while startup copying contribution is startup share of all successfully copied products. Note, HHI and gross profits are levels $\times 100$, so that HHI falls in the range $[0, 100]$.



(a) Type-contingent firm-size right-tail

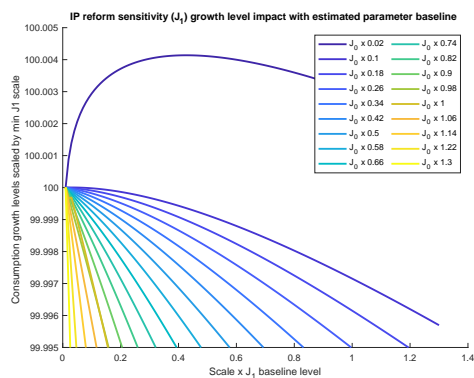


(b) Estimated model type-contingent average monopoly share over size

Figure 1: Impact on firm-size / monopoly distribution with increased court IP enforcement

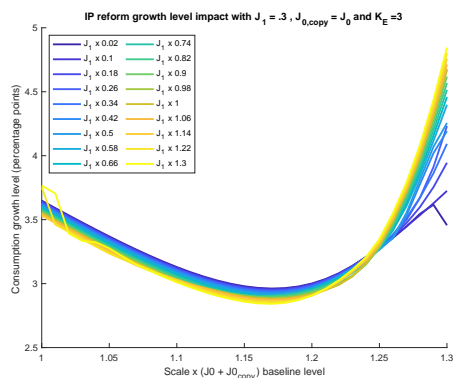
Left panel: firm-size distribution right tail with 10% decrease in court IP enforcement intercepts, Right panel: average monopoly share by type un-adjusted for survivorship. Note the average level of monopoly share of low IPP cost firms is below that of the high IPP cost firms due to survivorship selection. That is, firms upon entry have $\frac{m}{n} = 1$ thus with age, $E[\frac{m}{n} | age]$ will decline.

Figure 2: Strengthening court IP enforcement with the estimated model



Impact on consumption growth from varying court IPP sensitivity and court IP intercepts with the estimated parameters. Each line is a scaled version of the court IP intercepts $J_0 + J_0^{copy}$ with J_1 varying in scale relative to the estimated baseline.

Figure 3: Strengthening court IP enforcement under alternative parameterization



Impact on consumption growth from varying court IPP sensitivity and court IP intercepts under an alternative parameterization with higher K_S , J_1 and $J_0 = J_0^{copy}$ (all other parameters from estimation).

B Appendix - Model Details

B.1 Firm problem

As opposed to in Section 2, a firm upon innovating will retain the new product so that at a given point in time a firm will consist of n intermediate goods that they are latest innovator to have improved the quality of (and hence own the intellectual property rights and the capital underlying the quality advantage). Following Klette and Kortum (2004) with each frontier product the firm retains another R&D / IP legal team so that their total investment costs scale linearly in n . Firms are assumed to be endowed with a common IPP investment cost type τ across their product lines, where $\tau \in \{L, H\}$. Allowing for the possibility of financial transfers in litigation and profits from product for which they are not the frontier producer of, firm's receive flow 'royalties' $r\rho$. Each of their $i = 1, \dots, n$ products may be in one of two competition states, $m_i \in \{0, 1\}$ where $m_i = 0$ represents perfect competition and $m_i = 1$ represents monopoly. Denote $[m_i]_n = (m_1, \dots, m_n)$ as the vector of the n monopoly states. When in monopoly state firms earn flow profits π on that that product, while in the competitive state they earn zero flow profits from that product.

Firms have three different investment activities, innovation ι , copying γ and IPP ℓ . With a successful and non-blocked innovation, firms gain a new monopoly product so that their new size is $n + 1$ and $[m_i]_{n+1} = [m_i] \cup \{m_{n+1} = 1\}$. Conversely, should a rival successfully innovate on product i of the firm, the firm loses that product with its associated monopoly status, m_i . In contrast, with unblocked copying, the firm gets a short-term profit $\phi\pi$ from undercutting the frontier producer which increases $\rho' = \rho + \phi\pi$. On the flip-side, a firm copied permanently loses the monopoly profits from their frontier innovation (i.e. $m'_i = 0$ for i copied).

The firm Hamilton-Jacobi Bellman is given by

$$rV(n, \{m_i\}, \rho, \tau) = \max_{\iota_i, \gamma_i, \ell_i} \sum_i m_i \pi + r\rho - w \sum_i [c_i(\iota_i) + c_\gamma(\gamma_i) - c_\ell(\ell_i; \tau)] \quad (24)$$

$$+ \mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)] - \dot{V} \quad (25)$$

where, letting $\hat{x} = (\hat{n}, \{\hat{M}, \hat{\ell}\}, \hat{\rho}, \hat{\tau})$ denote the rival firms type, we have

$$\begin{aligned} \mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)] = & \\ & \sum_i \iota_i \cdot \mathbb{E}[D_i(n, \{m_k, \ell_k\}, \rho, \tau, \hat{x})] + \sum_i \gamma_i \cdot \mathbb{E}[C_i(n, \{m_k, \ell_k\}, \rho, \tau, \hat{x})] \\ & + \sum_i \delta \cdot \mathbb{E}[P_i(n, \{m_k, \ell_k\}, \rho, \tau, \hat{x})] + \sum_i \lambda \cdot m_i \cdot \mathbb{E}[Q_i(n, \{m_k, \ell_k\}, \rho, \tau, \hat{x})] \end{aligned}$$

where the IP litigation value functions D_i, C_i, P_i, Q_i are defined in Section B.2.

The first line in (25) captures the static flow profits from each of the n products of which $m = \sum_i m_i$ are monopolies and $n - m$ are copied. The

second line captures the expected net change in their firm value given their investments (taking as given the distribution of IPP investments ℓ , firm types, and the aggregate rate of rivals innovating on their products δ or copying λ). The expected growth in firm value $\mathbb{E}[\Delta V(n, \{m_i, \ell_i, \iota_i, \gamma_i\}, \rho, \tau)]$ is decomposed for each product line $i = 1, \dots, n$ into (1) D_i , the expected net value of innovating on a new product and becoming a defendant in a suit, (2) C_i , the expected net value of copying a rival's product and becoming a copycat defendant, (3) P_i , the expected net value (loss) of being innovated on by a rival giving them the opportunity to litigate as a plaintiff, and (4) Q_i the expected net value (loss) from being a plaintiff when their product has been copied.

B.2 IP litigation and firm value

Upon a product being innovated or copied, the incumbent firm has the opportunity to force a trial in court or try to negotiate a settlement from the rival who innovated / copied their product. The innovator in turn can choose to agree to a settlement or force a trial as well. Should either elect for trial, a stochastic ruling judgement is made, where with probability $J(\cdot)$ if innovation, $J_c(\cdot)$ if copied the court blocks the new entrants' product and returns whatever profits were lost by the incumbent. As in the illustrative model, these probabilities depend on the relative difference in IPP investments of the innovating and incumbent product line ℓ_i, ℓ_j , e.g. $J(\hat{\ell} - \ell) = 1 - a(\ell - \hat{\ell})$.⁴⁴

Therefore, in the case of innovation for instance, the value of the defendant innovating and going to trial with product i 's team, and facing incumbent with $\hat{\ell}$ is

$$\tilde{D}_i(\text{trial}, n, [m_k]_n, \{\ell_k\}, \rho, \hat{x}) = a(\hat{\ell} - \ell_i) [V(n + 1, \{[m_k]_n, 1\}, \rho, \tau) - V(n, [m_k]_n, \rho, \tau)]. \quad (26)$$

Similarly, for the plaintiff with product j innovated upon is given by

$$\tilde{P}_j(\text{trial}, n, \{m_k, \ell_k\}, \rho, \hat{x}) = a(\hat{\ell} - \ell_j) [V(n - 1, [m_k]_n \setminus m_j, \rho, \tau) - V(n, [m_k]_n, \rho, \tau)].$$

Observe that at the point of going to trial, all that matters to each firm about their rival is their rival's level of ℓ .

Should both choose to negotiate, Nash bargaining occurs between the incumbent and product entrant where the product entrant pays a royalty (financial transfer) in exchange for avoiding court. The value of settlement for an innovating defendant is simply the net value of the new monopoly product minus the royalties b paid to the incumbent,

$$\tilde{D}_i(\text{settle}, n, \{m_k, \ell_k\}, \rho, \hat{x}) = V_\tau(n + 1, \{[m_i]_n, 1\}, \rho - b) - V_\tau(n, [m_i]_n, \rho, \tau) \quad (27)$$

and symmetrically, the value to the plaintiff for settlement is the gain from the transfer, b but with the loss of the innovated product j

⁴⁴Note, this assumption of product level IPP is maintained primarily for tractability, but also captures the imperfect overlap of IPP across different product spaces.

$$\tilde{P}_j(\text{settle}, n, \{m_k, \ell_k\}, \rho, \hat{x}) = V(n-1, [m_k]_n \setminus m_j, \rho + b) - V_\tau(n, [m_k]_n, \rho, \tau). \quad (28)$$

Let $x = (\ell, m_i, \tau)$ denote the state of the product line i of the defendant which innovated and $\hat{x} = (\hat{\ell}, \hat{m}_j, \hat{\tau})$ the state of the product line j of the plaintiff which was innovated upon. The Nash bargained settlement royalties between plaintiff and defendant with team j and i respectively solves:

$$b(\hat{x}, x) = \arg \max_{\tilde{b}} \left(\mathbb{S}_P(\tilde{b}, x, \hat{x}) \right)^{0.5} \left(\mathbb{S}_D(\tilde{b}, x, \hat{x}) \right)^{0.5} \quad (29)$$

where in the case of an innovation the settlement surplus is for the defendant and plaintiff

$$\mathbb{S}_P(j, i, \tilde{b}, x, \hat{x}) = P_j(\text{settle}, \hat{x}, x) - P_j(\text{trial}, \hat{x}, x) \quad (30)$$

and

$$\mathbb{S}_D(j, i, \tilde{b}, \hat{x}, x) = D_i(\text{settle}, \hat{x}, x) - D_i(\text{trial}, \hat{x}, x). \quad (31)$$

Thus the ex-ante value of innovating and becoming a defendant is

$$\begin{aligned} D_i(n, \{m_k, \ell_k\}, \rho, \hat{x}) = & \\ & (1 - \varphi(\hat{x}, x_i)) \tilde{D}_i(\text{settle}, n, \{m_k, \ell_k\}, \rho, \hat{x}) \\ & + \varphi(\hat{x}, x_i) \tilde{D}_i(\text{trial}, n, [m_k]_n, \{\ell_k\}, \rho, \hat{x}) \end{aligned}$$

Symmetrically replacing P with Q and D with C yields the individual surpluses with copycat innovation.

B.3 Startup problem

Besides incumbent firms already endowed with past disruptive innovations / R&D teams, there is a fixed mass μ of potential entrants, who conduct both types of R&D, invest in IPP protection for their in-progress R&D. They are initially endowed with the high marginal cost IP technology $\tau = H$, but upon successful entry as a quality leader in a product market may stochastically draw the low cost IP technology $\tau = L$.⁴⁵ That is, the entrant does not know if they have the low or high IP cost type until after finishing the litigation with the

⁴⁵This assumption about startups having the high cost is meant to capture an economies of scale / age feature that revenue generating incumbents (e.g. public firms or multinationals) typically have more regulatory and other legal concerns and hence will more likely retain an in-house legal department. Furthermore, due to their age, longer / stronger relationships with hired IP lawyers will generate cost savings. This could be endogenized by allowing for another investment which with a poisson shock allows them to transition, but would complicate the model further without much gain for the purpose at hand.

incumbent firms. Besides not being initially endowed with any products, the problem is the same as the incumbents above:

$$rV_0 = \max_{\iota, \gamma, \ell} -wc_\iota(\iota) - wc_\gamma(\gamma) - wc_\tau(\ell) + \mathbb{E}[\Delta V_0(\ell, \iota, \gamma, k, \hat{x})] \quad (32)$$

where

$$\mathbb{E}[\Delta V_0(\ell, \iota, \gamma, \hat{x})] = \iota \mathbb{E}[D_0(\ell, \hat{x})] + \gamma \mathbb{E}[C_0(\ell, \hat{x})]$$

and $\mathbb{E}[D_0(\ell, \hat{x})]$ is the same as for the incumbent but taking $n = 0, m = 0$ as baseline.

B.4 Firm Value Decomposition

Will conjecture that firm value takes the following form:

$$V(n, [m_i]_n, \rho, \tau) = \rho + \sum_i \Pi_{\tau m_i} \quad (33)$$

That is, firm value is comprised of the present value of being the quality leading innovator in n products, with retained monopoly in $\sum_i m_i$ products, and a non production based term ρ which simply depends on the history of past innovations involving the firm.

With this guess, the value of a trial is as follows:

Disruptive innovation trials:

The value of the plaintiff from a trial facing a disruptive innovation is given by

$$P_i(\text{trial}, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\bar{J}(\ell_i, \hat{\ell}) \Pi_{\tau m_i} \quad (34)$$

On the flip-side, the defendant with a disruptive innovation in a trial obtains (regardless of whether a monopoly existed in the product market prior to their innovation):

$$D_i(\text{trial}, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \bar{J}(\hat{\ell}, \ell_i) \Pi_{\tau 1}. \quad (35)$$

where here m_i is the innovator's original product line monopoly status which conditional on ℓ_i is immaterial for the defendant's payoffs.

Copycat innovation trials:

Trials in the presence of copycat innovations have the same form as above, where the only difference is that the plaintiff doesn't lose their R&D team (i.e. n remains fixed) and the innovator only gains short-term profits $\phi\pi$.

$$Q_i(\text{trial}, m_i = 1, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\bar{J}(\ell_i, \hat{\ell}) [\Pi_{\tau 1} - \Pi_{\tau 0}] \quad (36)$$

$$C_i(\text{trial}, m_i, \ell_i, \tau, \hat{m} = 1, \hat{\ell}, \hat{\tau}) = \bar{J}(\hat{\ell}, \ell_i) \phi \pi \quad (37)$$

where there are no profits to be made for the copycat if they should copy a non-monopoly product.

Disruptive innovation settlement:

Similarly, with the guess, the value of settlement for a plaintiff given a disruptive innovation on a monopoly becomes

$$P_i(\text{settle}, b, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Pi_{\tau m_i} + b \quad (38)$$

while the value of settlement for a defendant given a disruptive innovation is

$$\tilde{D}_i(\text{settle}, b, m_i, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \Pi_{\tau 1} - b \quad (39)$$

Copycat innovation settlement:

In the case of copycat innovation, the settlement values are

$$Q_i(\text{settle}, b, m_i = 1, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Pi_{\tau 1} + b$$

and

$$C_i(\text{settle}, b, m_i, \ell_i, \tau, \hat{m} = 1, \hat{\ell}, \hat{\tau}, b) = \phi \pi - b.$$

B.5 Litigation equilibrium outcomes

We can now solve for the equilibrium outcomes of litigation between a given defendant and plaintiff and the two innovation types, disruptive and copycat. Since a trial is the outside option for both parties, standard results of Nash bargaining implies bargaining is weakly preferred provided the total trade (settlement) surplus is positive.⁴⁶

From the analysis in the previous section, we see that only the monopoly status of the product market i which was innovated on for the plaintiff and the monopoly status of the product line which did the innovating matters (all the other products are independent). From here-on, we will drop the i subscript and instead use simply m for the status of the i th product market of the firm in question and \hat{m} for the rival firm.

Given the guess of the value functions, the total settlement surplus in the case of a copycat is:

$$S = -(\Pi_{\tau 1} - \Pi_{\tau 0}) + \bar{J}(\ell_i, \hat{\ell})(\Pi_{\tau 1} - \Pi_{\tau 0}) + \phi \pi - \bar{J}(\hat{\ell}, \ell_i) \phi \pi$$

⁴⁶With positive trial costs ξ then any weak preference here would be strict preference for settlement. To keep the model as simple as possible, I have abstracted from these costs here since qualitatively nothing changes for ξ not too large to induce some firms to entirely forfeit a litigation proceeding.

$$= -J(\ell_i, \hat{\ell}) [(\Pi_{\tau_1} - \Pi_{\tau_0}) - \phi\pi] < 0$$

where the last inequality holds for any ϕ not too large. This gives us our first result.

Lemma 7. *Provided $\Pi_{\tau_1} - \Pi_{\tau_0} > \phi\pi$ trial is the equilibrium outcome of copycat innovation on a monopoly market, (and the outcome is irrelevant in the case of $m = 0$). The interim values of the copycat plaintiff and defendant are respectively:*

$$C(\ell, \hat{m}, \hat{\ell}) = \pi\phi\hat{m}\bar{J}(\hat{\ell}, \ell) \quad (40)$$

and

$$Q(\ell, m, \tau, \hat{\ell}) = -\bar{J}(\ell, \hat{\ell})(\Pi_{\tau_1} - \Pi_{\tau_0}). \quad (41)$$

Moving to the case of a disruptive innovation, the settlement surplus for the plaintiff is

$$S_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -\Pi_{\tau m} + \tilde{b} - \left(-\bar{J}(\ell_i, \hat{\ell})\Pi_{\tau m} \right)$$

which simplifies to

$$S_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = -J(\ell_i, \hat{\ell})\Pi_{\tau m} + \tilde{b}. \quad (42)$$

Similarly for the defendant,

$$S_D(\tilde{b}, \ell_i, m, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = J(\hat{\ell}, \ell_i)\Pi_{\tau_1} - \tilde{b}. \quad (43)$$

Thus, the total surplus from settlement for a disruptive innovation is

$$S(m, x, \hat{x}) = S_P(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) + S_D(\tilde{b}, m, \ell_i, \tau, \hat{m}, \hat{\ell}, \hat{\tau}),$$

that is,

$$S(m, x, \hat{x}) = J(\ell, \hat{\ell})[\Pi_{\hat{\tau}_1} - \Pi_{\tau m}]. \quad (44)$$

Given two types τ , there are 6 cases with disruptive innovation. In the first two cases, $\tau = \hat{\tau}$ and plaintiff has a monopoly ($m = 1$) then the surplus is zero (but is strictly positive if trial costs are positive) hence settlement is the equilibrium outcome. In the case where the plaintiff has a monopoly, but is of a lower type than $\hat{\tau}$, assuming monotonicity in τ so that $\Pi_{\tau m} \geq \Pi_{\tau' m}$ for $\tau > \tau'$, settlement surplus is strictly positive, hence settlement again. In the fourth case, the plaintiff is of a higher type than the defendant, in which case the total surplus is negative, and hence trial is the equilibrium outcome. Finally, the last two cases involve asymmetric τ and plaintiff has no monopoly. Conjecturing that the value of a monopoly product for the low type is higher than the value of not owning a monopoly for the high type, we obtain settlement is optimal in this case as well.

Lemma 8. Assume that $\Pi_{\tau 1} > \Pi_{\tau 0}$ and $\Pi_{\tau' 1} \geq \Pi_{\tau 0}$ for $\tau' < \tau$. With disruptive innovation, the unique equilibrium litigation outcomes for two firms where plaintiff is of type τ with monopoly product status m and defendant is of type $\hat{\tau}$ is to settle in all meetings except when the plaintiff firm is of the low marginal cost type $\tau = 1$, the defendant is of the high marginal cost type $\hat{\tau} = 0$ and the plaintiff's product under threat is a monopoly $m = 1$.⁴⁷ The equilibrium interim payoffs of the defendant is given by

$$D(\ell, \tau, \hat{m}, \hat{\ell}, \hat{\tau}) = \begin{cases} \bar{J}(\hat{\ell}, \ell) \Pi_{\tau 1} & \hat{\tau} = 1 > \tau = 0, \hat{m} = 1 \\ \Pi_{\tau 1} - b(\hat{m}, \hat{\ell}, \hat{\tau}, \ell, \tau) & \text{else} \end{cases} \quad (45)$$

while the interim value of a plaintiff with ℓ, τ, m facing a defendant in state $\hat{\ell}, \hat{\tau}$ is given by

$$P(\ell, \tau, m, \hat{\ell}, \hat{\tau}) = \begin{cases} -\bar{J}(\ell, \hat{\ell}) \Pi_{\tau m} & \tau = 1 > \hat{\tau} = 0, m = 1 \\ -\Pi_{\tau m} + b(m, \ell, \tau, \hat{\ell}, \hat{\tau}) & \text{else} \end{cases} \quad (46)$$

where the bargained royalties in the case of settlement are

$$b(m, \ell, \tau, \hat{\ell}, \hat{\tau}) = \frac{1}{2} J(\ell, \hat{\ell}) [\Pi_{\tau m} + \Pi_{\hat{\tau} 1}]. \quad (47)$$

Thus far, we have only considered the case of two incumbent firms meeting in litigation. If we assume the entrant knows their IP type upon innovation, but not prior, then the forward looking settlement / trial decisions are identical to the incumbent case, but since the entrant did not know their type ex-ante, their IP protection investment will lie between the non-monopoly high marginal cost and the non-monopoly product low marginal cost IP protection levels. Hence the size of transfers / the probability of a favourable judgment will differ for the case with an entrant defendant.

Lemma 9. With an entrant's innovation, provided the entrant knows their incumbent type τ upon innovation, then under the same assumptions as above, the same equilibrium outcomes of trial or settlement occur conditional on the realized type of the entrant, however, the royalties differ in value, and are given by

$$b_S(m, \ell, \tau, \ell_S, \tau_S) = \frac{1}{2} J(\ell, \ell_S) [\Pi_{\tau m} + \Pi_{\tau_S 1}]. \quad (48)$$

When it comes to structurally estimating the model, it will in fact be convenient to assume that entrant's only discover their incumbent type τ after the innovation / litigation with the incumbent, so their value itself is uncertain.

⁴⁷On the other hand, if $M_{01} + R_{01} < R_{10}$ then trial occurs also (for the same $\tau = 1, \hat{\tau} = 0$) when the plaintiff's product under threat is a non-monopoly product.

B.6 Ex-ante value of litigation

Finally, to determine the optimal innovation and IP protection levels, we must take expectations over the type of counterparty the firm will face in each. Commensurate with our guess of the value-function, this implies that all policy variables for a given product line i only depend on τ, m_i .

Denote K_τ as the mass of incumbents of type τ , $v_{\tau m}$ as the probability of that firm type product being a monopoly status m , $K_E = \mu$ as the mass of potential entrants. Then the probability of a plaintiff meeting a given type $\hat{m}, \hat{\tau}$ disruptive innovator is

$$\theta_{\hat{\tau}\hat{m}}^P = \frac{\iota_{\hat{\tau}\hat{m}} v_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\sum_{\tau'} \sum_{m'} \iota_{\tau'm'} v_{\tau'm'} K_{\tau'} + \iota_S K_S} \quad (49)$$

and meeting an entrant of type τ is:

$$\theta_{\hat{\tau}E}^P = \frac{\iota_E \phi_{\hat{\tau}} K_S}{\sum_{\tau'} \sum_{m'} \iota_{\tau'm'} v_{\tau'm'} K_{\tau'} + \iota_S K_S} \quad (50)$$

while the probability of a defendant meeting a given type plaintiff $\hat{m}, \hat{\tau}$ is simply

$$\theta_{\hat{\tau}\hat{m}}^D = \frac{\delta v_{\hat{\tau}\hat{m}} K_{\hat{\tau}}}{\sum_{\tau'} \sum_{m'} \delta v_{\tau'm'} K_{\tau'}} = v_{\hat{\tau}\hat{m}} K_{\hat{\tau}}. \quad (51)$$

where the last equality follows from there being a unit measure of products,

$$\sum_{\tau', m'} v_{\tau'}(m') K_{\tau'} = 1.$$

Using the above, (and applying symmetric reasoning for the defendant) yields the following lemma.

Lemma 10. *Assuming $\Pi_{\tau 1} > \Pi_{\tau 0}$ and $\Pi_{01} \geq \Pi_{10}$, the equilibrium expected net payoff given a disruptive innovation is as follows.*

Disruptive plaintiff

$$E[P|\ell, m, \tau] = -\Pi_{\tau m} \Psi_{\tau m}^P(\ell) + T_{\tau m}^P(\ell) \quad (52)$$

where the effective probability of losing the product market for a plaintiff is

$$\Psi_{\tau m}^P(\ell) = \sum_{\tau'} \sum_{m' \in \{0,1,E\}} \theta_{\tau'm'}^P \left\{ B_{\text{settle}}(\tau, \tau', m) \left(1 - \frac{J(\ell, \ell_{\tau', m'})}{2} \right) + B_{\text{trial}}(\tau, \tau', m) \bar{J}(\ell, \ell_{\tau', m'}) \right\} \quad (53)$$

while the expected gain of royalties (obtained in settlement) from the sharing of the rival innovators surplus is

$$T_{\tau m}^P(\ell) = \sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^P B_{\text{settle}}(\tau, \tau', m) \frac{J(\ell, \ell_{\tau' m'})}{2} \Pi_{\tau' 1} \quad (54)$$

Disruptive defendant

$$E[D|\ell, m, \tau] = \Pi_{\tau 1} \Psi_{\tau m}^D(\ell) - T_{\tau m}^D(\ell) \quad (55)$$

$$\Psi_{\tau m}^D(\ell) = \sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^D \left\{ B_{\text{settle}}(\tau', \tau, m') \left(1 - \frac{J(\ell_{\tau' m'}, \ell)}{2} \right) + B_{\text{trial}}(\tau', \tau, m') \bar{J}(\ell_{\tau' m'}, \ell) \right\} \quad (56)$$

$$T_{\tau m}^D(\ell) = \sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^D B_{\text{settle}}(\tau', \tau, m') \frac{J(\ell_{\tau' m'}, \ell)}{2} \Pi_{\tau' m'} \quad (57)$$

Lemma 11. *If $\Pi_{\tau 1} - \Pi_{\tau 0} > \phi\pi$ for all τ . The ex-ante payoffs of copycat innovation are*

Copycat plaintiff

$$E[Q|\ell, m, \tau] = -(\Pi_{\tau 1} - \Pi_{\tau 0}) \Psi_{\tau m}^Q(\ell) \quad (58)$$

$$\Psi_{\tau m}^Q(\ell) = \sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^Q \bar{J}(\ell, \ell_{\tau' m'}) \quad (59)$$

Copycat defendant

$$E[C|\ell, m, \tau] = \phi\pi \Psi_{\tau m}^C(\ell) \quad (60)$$

where

$$\Psi_{\tau m}^C(\ell) = \sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^C m' \bar{J}(\ell_{\tau' m'}, \ell) \quad (61)$$

$$\theta_{\hat{\tau} \hat{m}}^Q = \frac{\gamma \hat{\tau} \hat{m} \nu_{\hat{\tau} \hat{m}} K_{\hat{\tau}}}{\sum_{\tau'} \sum_{m'} \gamma_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \gamma_S K_S} \quad (62)$$

and

$$\theta_{\hat{\tau} E}^Q = \frac{\gamma_S \phi_{\hat{\tau}} K_S}{\sum_{\tau'} \sum_{m'} \gamma_{\tau' m'} \nu_{\tau' m'} K_{\tau'} + \gamma_S K_S} \quad (63)$$

while the probability of a defendant meeting a given type plaintiff \hat{m} , $\hat{\tau}$ is

$$\theta_{\hat{\tau} \hat{m}}^C = \frac{\lambda \nu_{\hat{\tau} \hat{m}} K_{\hat{\tau}}}{\lambda \sum_{\tau'} \sum_{m'} \nu_{\tau' m'} K_{\tau'}}. \quad (64)$$

B.7 Optimal investment policies

Having solved for the expected value of innovation for the firm $\mathbb{E}[\Delta V]$, I can now determine the optimal R&D and legal protection investment policies, ι, γ, ℓ for both types of incumbents as well as startups.

Incumbent optimal policies

$$\mathbb{E}[D|\ell_{\tau m}, m, \tau] = wc'_\iota(\iota_{\tau, m}) \quad (65)$$

$$\mathbb{E}[C|\ell_{\tau m}, m, \tau] = wc'_\gamma(\gamma_{\tau, m}) \quad (66)$$

$$\begin{aligned} \iota_{\tau, m} \frac{\partial \mathbb{E}[D|\ell, m, \tau]}{\partial \ell} + \gamma_{\tau, m} \frac{\partial \mathbb{E}[C|\ell, m, \tau]}{\partial \ell} \\ + \delta \frac{\partial \mathbb{E}[P|\ell, m, \tau]}{\partial \ell} + \lambda \frac{\partial \mathbb{E}[Q|\ell, m, \tau]}{\partial \ell} = wc'_\tau(\ell) \end{aligned} \quad (67)$$

By inspection, we see that the optimal policies of ℓ, ι, γ depend as conjectured only on m, τ .

Proposition 12. Assume $c_\iota(\iota) = \frac{1}{a_\iota} \iota^{\alpha_\iota + 1}$ and $c_\gamma(\gamma) = \frac{1}{a_\gamma} \gamma^{\alpha_\gamma + 1}$, $\alpha_\gamma < \alpha_\iota$, $a_\gamma > a_\iota$, and $c_\tau(\ell) = \frac{1}{a_\ell(\tau)} \ell^{\alpha_\ell(\tau) + 1}$. Also let $\xi \rightarrow 0$, then optimal firm policies are given by

$$\iota_{\tau m} = c_\iota(\iota)^{-1'} \left(w^{-1} \mathbb{E}[D|\ell_{\tau m}, m, \tau] \right) \quad (68)$$

$$\gamma_{\tau m} = c_\gamma(\gamma)^{-1'} \left(w^{-1} \mathbb{E}[C|\ell_{\tau m}, m, \tau] \right) \quad (69)$$

$$\ell_{\tau m} = c_\ell(\ell, \tau)^{-1'} \left(\frac{w^{-1} \partial \mathbb{E}[\Delta V]}{\partial \ell} \right) \quad (70)$$

where

$$\frac{\partial \mathbb{E}[\Delta V|x]}{\partial \ell} = \iota_\tau \left(\frac{\partial \mathbb{E}[D|x]}{\partial \ell^D} \right) + \delta \frac{\partial \mathbb{E}[P|x]}{\partial \ell^P} + \gamma_\tau \frac{\partial \mathbb{E}[C|x]}{\partial \ell^D} + \lambda \frac{\partial \mathbb{E}[Q|x]}{\partial \ell^P}.$$

B.8 Solving for value functions

Given the above, I am now equipped to solve for $\Pi_{\tau m}$ (and thus the value function V). Plugging in the expected values of IP litigation events into the HJB yields:

$$\begin{aligned}
rV &= \sum_i m_i \pi + r\rho - w \sum_i [c_l(\iota_{\tau m_i}) + c_\gamma(\gamma_{\tau m_i}) + c_\ell(\ell_{\tau m_i}; \tau)] \\
&\quad + \sum_i \left\{ \iota_{\tau m_i} (\Psi_{\tau m_i}^D \Pi_{\tau 1} - T_{\tau m_i}^D) - \delta (\Psi_{\tau m_i}^P \Pi_{\tau m_i} - T_{\tau m_i}^P) \right\} \\
&\quad + \sum_i \gamma_{\tau m_i} \Psi_{\tau m_i}^C \phi \pi - \lambda m_i \Psi_{\tau 1}^Q (\Pi_{\tau 1} - \Pi_{\tau 0})
\end{aligned}$$

which plugging in the guess of V on the LHS of FirmProblem and matching coefficients of \hat{m}, \hat{n} :

$$\begin{aligned}
r\Pi_{\tau 1} &= \pi - w[c_l(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_\ell(\ell_{\tau 1}; \tau)] \\
&+ \iota_{\tau 1} (\Psi_{\tau 1}^D \Pi_{\tau 1} - T_{\tau 1}^D) - \delta (\Psi_{\tau 1}^P \Pi_{\tau 1} - T_{\tau 1}^P) - \lambda \Psi_{\tau 1}^Q \Pi_{\tau 1} + \gamma_{\tau 1} \Psi_{\tau 1}^C \phi \pi + \lambda \Psi_{\tau 1}^Q \Pi_{\tau 0}
\end{aligned}$$

and

$$r\Pi_{\tau 0} = -w[c_l(\iota_{\tau 0}) + c_\gamma(\gamma_{\tau 0}) + c_\ell(\ell_{\tau 0}; \tau)] + \iota_{\tau 0} (\Psi_{\tau 0}^D \Pi_{\tau 1} - T_{\tau 0}^D) - \delta (\Psi_{\tau 0}^P \Pi_{\tau 0} - T_{\tau 0}^P) + \gamma_{\tau 0} \Psi_{\tau 0}^C \phi \pi$$

Solving for $\Pi_{\tau 1}$ and $\Pi_{\tau 0}$ yields

$$\begin{aligned}
\Pi_{\tau 1} &= \frac{\pi - w[c_l(\iota_{\tau 1}) + c_\gamma(\gamma_{\tau 1}) + c_\ell(\ell_{\tau 1}; \tau)] + \gamma_{\tau 1} \Psi_{\tau 1}^C \phi \pi - \iota_{\tau 1} T_{\tau 1}^D + \delta T_{\tau 1}^P}{r + \delta \Psi_{\tau 1}^P + \lambda \Psi_{\tau 1}^Q - \iota_{\tau 1} \Psi_{\tau 1}^D} \\
&\quad + \frac{\lambda \Psi_{\tau 0}^Q \Pi_{\tau 0}}{r + \delta \Psi_{\tau 1}^P + \lambda \Psi_{\tau 1}^Q - \iota_{\tau 1} \Psi_{\tau 1}^D}, \tag{71}
\end{aligned}$$

$$\begin{aligned}
\Pi_{\tau 0} &= \frac{-w[c_l(\iota_{\tau 0}) + c_\gamma(\gamma_{\tau 0}) + c_\ell(\ell_{\tau 0}; \tau)] - \iota_{\tau 0} T_{\tau 0}^D + \delta T_{\tau 0}^P + \gamma_{\tau 0} \Psi_{\tau 0}^C \phi \pi}{r + \delta \Psi_{\tau 0}^P} \\
&\quad + \frac{\iota_{\tau 0} \Psi_{\tau 0}^D \Pi_{\tau 1}}{r + \delta \Psi_{\tau 0}^P}. \tag{72}
\end{aligned}$$

The joint solution of the two equations is fully described in the next theorem.

Proposition 13. *The value function of a firm is given by*

$$V_\tau(m, n, \rho, \theta) = \rho + m \cdot \Pi_{\tau 1} + (n - m) \cdot \Pi_{\tau 0}$$

where

$$\begin{pmatrix} \Pi_{\tau 1} \\ \Pi_{\tau 0} \end{pmatrix} = A^{-1} \begin{pmatrix} \pi - \text{sgn} a_{\tau 1} + \gamma_{\tau 1} \Psi_{\tau 1}^C \phi \pi + \delta T_{\tau 1}^P - \iota_{\tau 1} T_{\tau 1}^D \\ -\text{sgn} a_{\tau 0} + \gamma_{\tau 0} \Psi_{\tau 0}^C \phi \pi + \delta T_{\tau 0}^P - \iota_{\tau 0} T_{\tau 0}^D \end{pmatrix} \tag{73}$$

$$A = \begin{pmatrix} r + \delta\Psi_{\tau 1}^P + \lambda\Psi_{\tau 1}^Q - \iota_{\tau 1}\Psi_{\tau 1}^D & -\lambda\Psi_{\tau 1}^Q \\ -\iota_{\tau 0}\Psi_{\tau 0}^D & r + \delta\Psi_{\tau 0}^P \end{pmatrix}$$

and $sgna_{\tau m} = w[c_i(\iota_{\tau m}) + c_\gamma(\gamma_{\tau m}) + c_\ell(\ell_{\tau m}; \tau)]$.

Startups optimal policies

The startups FOCs are exactly the same as for incumbents on a product line without a monopoly, except that they don't yet know which type τ they will be when they become an incumbent (this status gets drawn upon the discovery, but prior to the litigation).

The FOCs are:

$$c'_i(\iota_S) = \sum_{\tau} \phi_{\tau} \{ \Pi_{\tau 1} \Psi_{\tau 1}^D(\ell_S) - T_{\tau}^D(\ell_S) \} \quad (74)$$

$$c'_\gamma(\gamma_S) = \Psi_S^C(\ell^E) \phi \pi \quad (75)$$

$$c'_\ell(\ell_S; \underline{\tau}) = \iota_S \sum_{\tau} \phi_{\tau} \left\{ \Pi_{\tau 1} \frac{\partial}{\partial \ell} \Psi_{\tau 1}^D(\ell_S) - \frac{\partial}{\partial \ell} T_{\tau}^D(\ell_S) \right\} \quad (76)$$

B.9 Equilibrium firm, markup distribution

Observing that all policies and equilibrium objects do not depend on the firm-size distribution conditional on the mass of firms of each type, K_{τ} and the share of those type with monopolies ν_{τ} , in this subsection we characterize the steady state distribution of firms across firm and monopoly status types.

Let $\Gamma_{\tau\tau'}(m, m') = 1 - \mathbb{1}_{\text{trial}}(\tau, \tau', m) J(\ell_{\tau' m'}, \ell_{\tau 1})$ denote the probability of a disruptive innovation not being blocked conditional on the incumbent is of type (τ, m) and the defendant is of type τ', m' (where in the case of an entrant, $m' = E$).

First, as the only way to have a quality leading product team without a monopoly is for a copycat to have destroyed it, and the only way for these product teams to be destroyed is a new quality innovation on the product, the net flows into $(\tau, 0)$ is given by

$$0 = \lambda \nu_{\tau} K_{\tau} \left(\sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^Q \bar{J}(\ell_{\tau 1}, \ell_{\tau' m'}) \right) - \delta (1 - \nu_{\tau}) K_{\tau} \left(\sum_{\tau'} \sum_{m'} \theta_{\tau' m'}^P \Gamma_{\tau\tau'}(0, m') \right). \quad (77)$$

The net flows into $(\tau, 1)$ are a bit more complicated since entrants discovering new innovations and becoming type τ will also contribute to the flows in, and flows out either stem from destruction of copycat or other type quality innovation.

$$\begin{aligned}
0 = & -\lambda\nu_\tau K_\tau \left(\sum_{\tau'} \sum_{m'} \theta_{\tau'm'}^Q \nu_{\tau'}(m') \bar{J}(\ell_{\tau 1}, \ell_{\tau' m'}) \right) - \delta \nu_\tau K_\tau \left(\sum_{\tau'} \sum_{m'} \theta_{\tau'm'}^P \Gamma_{\tau\tau'}(1, m') \right) \\
& + (1 - \nu_\tau) \iota_{\tau 0} K_\tau \left(\sum_{\tau'} \sum_{m'} \theta_{\tau'm'}^D \Gamma_{\tau'\tau}(m', 0) \right) + \nu_\tau \iota_{\tau 1} K_\tau \left(\sum_{\tau'} \sum_{m'} \theta_{\tau'm'}^D \Gamma_{\tau'\tau}(m', 1) \right) \\
& + \phi_\tau K_{E'E} \left(\sum_{\tau'm'} \theta_{\tau'm'}^D \Gamma_{\tau'\tau}(m', E) \right).
\end{aligned}$$

Define $\delta_{\tau m}^* = \delta \left(\sum_{\tau'} \sum_{m'} \theta_{\tau'm'}^P \Gamma_{\tau\tau'}(m, m') \right)$ as the effective destruction rate of type τ , monopoly status m , and define ι_τ^* and η_τ^* as the effective (non-blocked) entry rates given the innovator is of type τ

Using the definition of δ_τ^* , λ_τ^* , and similarly defining ι_τ^* and η_τ^* as the effective (non-blocked) entry rates given the innovator is of type τ , the above can be re-expressed more compactly as $0 = -\lambda_\tau^* \nu_\tau K_\tau - \nu_\tau \delta_{\tau 1}^* K_\tau + \iota_\tau^* K_\tau + \phi_\tau \eta_\tau^*$ or solving for K_τ :

$$K_\tau = \frac{\phi_\tau \eta_\tau^*}{\nu_\tau \delta_{\tau 1}^* + \lambda_\tau^* \nu_\tau - \iota_\tau^*}. \quad (78)$$

Similarly, from (77) we have that the steady state share of monopolies for type τ is:

$$\nu_\tau = \frac{1}{\frac{\lambda_\tau^*}{\delta_{\tau 0}^*} + 1}. \quad (79)$$

That is, the steady state monopoly share in type τ is determined by the relative rate of destruction of monopolies by copycats relative to the disruptive innovation of non-monopoly products. The monopoly share is 1 if $\lambda^* = 0$ and is decreasing in the relative rate of copycat destruction of τ with a monopoly to creative destruction for type τ without a monopoly.

C Appendix - Supplemental Proofs

C.1 Proof of Theorem 1

Proof. Theorem 2.1(i):

Observe that if $a_1 = 0$ then the incumbent and entrant FOCs are identical and given the strict concavity of the objective function, these conditions are necessary and sufficient and uniquely pin down their investment policies. With $a_1 > 0$, $\Gamma'(\ell) = \Lambda'(\ell) = \Gamma'_S(\ell) = \Lambda'_S(\ell_S) = a_1$.

Noting symmetry amongst incumbent firms, and given that only incumbent firms can be innovated on, $\Gamma(\ell) = \mathbb{E}[a_0 + a_1(\ell - \hat{\ell})] = a_0$ and $\Gamma_S(\ell_S) = \mathbb{E}[a(\ell_S - \hat{\ell})] = a_0 + a_1(\ell_S - \ell)$. Slightly differently, as $\frac{\eta}{\delta}$ is the probability of an

entrant innovating on an incumbent product, the probability of an incumbent losing an infringement suit as a plaintiff depends on whether the innovator is an incumbent or entrant. That is, again using symmetry amongst incumbent firms, $\Lambda(\ell) = a_0 + \frac{\eta}{\delta}a_1(\ell_S - \ell)$.

Thus, plugging these results into the FOCS yields for the incumbents:

$$\begin{aligned} a_0 \cdot \Pi &= c'_i(\iota) \\ a_1[\iota + \delta] \cdot \Pi &= c'_\ell(\ell) \end{aligned}$$

and

$$\begin{aligned} [a_0 - a_1(\ell - \ell_S)] \cdot \Pi &= c'_i(\iota_S) \\ a_1 \cdot \iota_S \cdot \Pi &= c'_\ell(\ell_S). \end{aligned}$$

Suppose by contradiction that $\iota \leq \iota_S$ then $c'_i(\iota) < c'_i(\iota_S)$ but then the LHS of the R&D FOCs imply that $a_0 - a_1(\ell - \ell_S) > a_0$, and thus $\ell < \ell_S$. But since $\delta > \iota_S + \iota$, the LHS of the incumbent IPP investment FOC $a_1[\iota + \delta] \cdot \Pi > a_1\delta \cdot \Pi > a_1\iota_S\Pi =$ LHS of entrant IPP FOC, but the RHS of the IPP investment FOC with $\ell < \ell_S$ requires $c'_\ell(\ell_S) > c'_\ell(\ell)$. Contradiction.

Theorem 2.1(ii):

Taking $\theta = (a_0, a_1)$ as exogenous parameters, the equilibrium conditions are summarized by the FOCs above, equilibrium firm value Π , and the general equilibrium conditions, $\delta = \iota + K_S\iota_S$, $\eta = \iota_S K_S$.

Total differentiating the FOCs and Π yields the following system of equations

$$\begin{aligned} d\iota \cdot c''_i(\iota) &= da_0\Pi + d\Pi a_0 \\ d\ell \cdot c''_\ell(\ell) &= da_1(\iota + \delta)\Pi + d\Pi a_1(\iota + \delta) + d\iota a_1\Pi\left(1 + \frac{\partial\delta}{\partial\iota}\right) + d\iota_S K_S a_1\Pi \\ d\iota_S \cdot c''_i(\iota_S) &= da_0\Pi - da_1(\ell - \ell_S)\Pi + d\Pi[a_0 - a_1(\ell - \ell_S)] \\ d\ell_S \cdot c''_\ell(\ell_S) &= da_1\iota_S\Pi + d\iota_S a_1\Pi + d\Pi a_1\iota_S \end{aligned}$$

Substituting in the $d\Pi$ equation into the FOCs, stacking these equations together yields a 4×1 system $\Delta F(y; \theta)$ with endogenous objects, $y = (\iota, \ell, \iota_S, \ell_S)$.

Appealing to the Implicit Function Theorem, yields the solution for the comparative statics:

$$\left[\frac{dy}{da_0}, \frac{dy}{da_1} \right] = -dF_y^{-1} dF_x$$

where

$$dF_x = \begin{pmatrix} \Pi - \frac{\Pi a_0 (\delta - \iota)}{r^*} & \frac{K_S \Pi a_0 \iota_S (l - \ell_S)}{r^*} \\ -\frac{\Pi a_1 (\delta + \iota) (\delta - \iota)}{r^*} & \Pi (\delta + \iota) + \frac{K_S \Pi a_1 \iota_S (\delta + \iota) (l - \ell_S)}{r^*} \\ \Pi - \frac{\Pi (a_0 - a_1 (l - \ell_S)) (\delta - \iota)}{r^*} & \frac{K_S \Pi \iota_S (a_0 - a_1 (l - \ell_S)) (l - \ell_S)}{r^*} - \Pi (l - \ell_S) \\ -\frac{\Pi a_1 \iota_S (\delta - \iota)}{r^*} & \Pi \iota_S + \frac{K_S \Pi a_1 \iota_S^2 (l - \ell_S)}{r^*} \end{pmatrix}$$

and

$$dF_y^{-1} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 \\ Z_{21} & Z_{22} & Z_{23} & 0 \\ Z_{31} & 0 & Z_{33} & 0 \\ Z_{41} & 0 & Z_{43} & Z_{44} \end{pmatrix}$$

where

$$Z_{11} = -\frac{r^*}{\Pi a_0 + c_l''(\iota) r^*}$$

$$Z_{21} = \frac{K_S \Pi^2 a_0 a_1 - K_S \Pi^2 a_1^2 l + K_S \Pi^2 a_1^2 \ell_S + \Pi a_1 c_l''(\iota_S) \delta - 2 \Pi a_1 c_l''(\iota_S) r^* + \Pi a_1 c_l''(\iota_S) \iota}{c_l''(\iota_S) c_\ell''(\ell) (\Pi a_0 + c_l''(\iota) r^*)}$$

$$Z_{22} = -\frac{1}{c_\ell''(\ell)}, \quad Z_{23} = -\frac{K_S \Pi a_1}{c_l''(\iota_S) c_\ell''(\ell)}, \quad Z_{31} = \frac{\Pi (a_0 - a_1 l + a_1 \ell_S)}{c_l''(\iota_S) (\Pi a_0 + c_l''(\iota) r^*)}, \quad Z_{33} = -\frac{1}{c_l''(\iota_S)}$$

$$Z_{41} = \frac{\Pi^2 a_1^2 \ell_S - \Pi^2 a_1^2 l + \Pi^2 a_0 a_1 + \Pi a_1 c_l''(\iota_S) \iota_S}{c_l''(\iota_S) c_\ell''(\ell_S) (\Pi a_0 + c_l''(\iota) r^*)}, \quad Z_{43} = -\frac{\Pi a_1}{c_l''(\iota_S) c_\ell''(\ell_S)}, \quad Z_{44} = -\frac{1}{c_\ell''(\ell_S)}$$

After some straightforward algebra and simplification, the results follow. \square

C.2 Proof of Theorem 6

Proof of 6 and related results. Similarly, from our firm value function decomposition (without copycats) we have the stock price of a firm is

$$V(n, \rho) = n\Pi + \rho.$$

Using the restriction of no copycat innovation, we can solve for individual firm dynamics

$$\dot{p}_{\tau,n}(t; n_0) = (n-1)p_{\tau,n-1}(t)\iota_\tau^* + (n+1)p_{\tau,n+1}(t)\delta_\tau^* - (\iota_\tau^* + \delta_\tau^*)np_{\tau,n}(t).$$

Firms with no products exit with $\dot{p}_{\tau,1}(t; n_0) = \delta_\tau^* p_{\tau,1}(t; n_0)$.

Considering the lifecycle of firm, who is born with one product, $p_{\tau,n}(t) = p_{\tau n}(t; 1)$. For convenience I will drop the dependence on τ and distinction between ι^* and ι since there is no ambiguity here. Following the arguments of Klette & Kortum (2004), this differential system has a unique solution given by

$$p_0(t) = \frac{\delta}{\iota} \psi(t), \quad \psi(t) = \frac{\iota(1 - e^{-(\delta-\iota)t})}{\delta - \iota e^{-(\delta-\iota)t}}$$

$$p_1(t) = [1 - p_0(t)][1 - \psi(t)]$$

and $p_n(t) = p_{n-1}(t)\psi(t)$ for any $n > 1$. By induction this implies the distribution of firm size conditional on surviving to age t is geometric $\frac{p_n(t)}{1-p_0(t)} = [1 - \psi(t)]\psi(t)^{n-1}$.

With this, we can return to the determination of the dynamics of the markups / value of the firm in our setting. Define $G_t^V = \frac{V_t - V_0}{V_0}$ as the growth

rate of the stock (value) of a firm starting with n_0 product lines and no royalties, we then have the conditional growth rate on a firm starting with n_0

$$E[G_t^V | n_0] = e^{-(\delta-\iota)t} - 1 + \frac{E[\rho_t | n_0]}{n_0 \Pi}.$$

The evolution of net royalties ρ is given by the settlements $b(\cdot)$ transacted upon a successful disruptive innovation. In general, whether (and what size) a settlement b is depends on the counterparties and their respective monopoly status. For illustrative purposes, consider a firm which upon innovating pays b^- and upon being innovated upon, receives b^+ . Let A_t denote the Poisson process (rate $n_0\iota$) counting the number of product lines they innovate on up to t and D_t the Poisson process (rate $n_0\delta$) counting the number of product lines innovated upon them. Then starting at $\rho_0 = 0$,

$$\rho_t = \begin{cases} -A_t b^- + D_t b^+ & D_s - A_s < n_0 \forall s \leq t \\ -A_{\hat{t}} b^- + D_{\hat{t}} b^+ & \hat{t} = \min\{s : D_s - A_s \geq n_0\} \end{cases}. \quad (80)$$

Unfortunately, we lack an exact analytic characterization for $E[\rho_t | n_0]$. However, we are able to characterize a closely related object which abstracts from the downward censored truncation. That is, consider $\hat{\rho}_t$ as given by $-A_t b^- + D_t b^+$ if $N_t > 0$ and $\hat{\rho}_t = 0$ if $N_t = 0$ with A_t, D_t independent of N_t . Then

$$E[\hat{\rho}_t | n_0] = E[-A_t b^- + D_t b^+ | n_0, N_t > 0] Pr(N_t > 0 | n_0) = n_0 t [-\iota b^- + \delta b^+] (1 - p_0(t)^{n_0})$$

where $1 - p_0(t)^{n_0} = 1 - [\frac{\delta}{\iota} \psi(t)]^{n_0}$.

First observe that for n_0 large, the survival truncation consideration is negligible. Second, note for $n_0 = 1$, $t[1 - \frac{\delta}{\iota} \psi(t)] = t \frac{(\delta-\iota)e^{-(\delta-\iota)t}}{(\mu-\lambda)e^{-(\mu-\lambda)t}} \xrightarrow{t \rightarrow \infty} 0$, hence it follows that the expected royalties $\hat{\rho}_t$ is finite. Finally note that for small horizons t and large n_0 , the differences between $\hat{\rho}_t - \rho_t$ vanish. Returning to the firm value growth expression, using this $\hat{\rho}_t$ approximation we have

$$E[G_t^V | n_0] = e^{-(\delta-\iota)t} - 1 + \frac{1}{\Pi} t [-\iota b^- + \delta b^+] (1 - p_0(t)^{n_0})$$

thus expected firm stock value growth is no longer independent from firm size n_0 . Further since $\delta > \iota$ then if $b^+ > b^-$ we have expected firm stock value growth increasing in firm size n_0 .

□